Relative importance of explanatory variables: An annotated bibliography

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For 'Statistical Issues in the Social Sciences', Stockholm, October 1998

In the list below, an item with no annotation is one which I know contains some methodological discussion on relative importance, but which I have not yet seen. DF

References

Achen, C. H. (1982). *Interpreting and Using Regression.* Sage, Beverly Hills.

'Who are the most important people in the world? What is the most important part of an automobile? Questions like these are apt to be answered with another question: Important for what? Without a criterion for importance, the inquiries are meaningless.' Goes on to distinguish three notions of importance, namely 'theoretical importance' (measured by β_x), 'level importance' (measured by $\beta_x\mu_x$) and 'dispersion importance' (measured by $\beta_x\sigma_x/\sigma_y$). Of the last, Achen says: 'although almost no one is substantively interested in it, many social scientists use it as their sole importance measure.' He suggests standardization by the (supposed fixed) range of a variable, rather than by its s.d., to achieve comparability across samples.

Bring, J. (1994a). How to standardize regression coefficients. *The American Statistician*, **48**:209–213.

Suggests use of partial rather than marginal standard deviations when standardizing regression coefficients, an idea mentioned also in Healy (1990).

Bring, J. (1994b). Relative importance of factors affecting blood pressure. *Journal of Human Hypertension*, **8**:297. (Letter to the Editor).

Points out erroneous reasoning in some silly published comparisons drawn from stepwise regression output.

Bring, J. (1994c). Standardized regression coefficients and relative importance of pain and functional disability to patients with rheumatoid arthritis. *Journal of Rheumatology*, **21**:1774–1775. (Letter to the Editor, with reply).

Criticizes comparison of standardized regression coefficients for the familiar reason that they depend on the x variances, not just on the relationship between the x's and y.

Bring, J. (1994d). *Variable importance and regression modelling.* PhD thesis, Dept of Statistics, Uppsala University.

Bring, J. (1996). A geometric approach to compare variables in a regression model. *The American Statistician*, **50**:57–62.

Shows how to interpret geometrically the standardized coefficients, studentized coefficients, and decompositions of R^2 in multiple regression. Gives an example in which the decomposition supported by Pratt (1987) seems unappealing.

Budescu, D. V. (1993). Dominance analysis: A new approach to the problem of relative importance of predictors in multiple regression. *Psychological Bulletin*, **114**:542–551.

Reviews various standard approaches. Then says 'Most of the approaches reviewed...lead to conclusions...not invariant to subset selection.' ... 'Two researchers who are interested in the relative importance of x_1 and x_2 may reach different conclusions depending on the other predictors they include in their models. This situation reflects, in part, a true state of affairs induced by the nature of the relationships between the different predictors involved. However, it would be to everyone's advantage to have a method of determining importance that is invariant under all subset selections.'
Budescu's method says that x_1 dominates x_2 if its partial correlation with y is greater regardless of the variables (from among those available — whatever that means!) that are controlled for: the result is typically a partial ordering of the x variables.

Cox, D. R. and Wermuth, N. (1996). *Multivariate Dependencies.* Chapman and Hall, London.

Especially Section 4.5. Distinguish between 'internal' and 'external' standardization, where internal means standardization determined mainly or entirely from the data under analysis. Careful discussion of the usual kind of (internal) standardization and its limitations for interpretation.

Darlington, R. B. (1968). Multiple regression in psychological research and practice. *Psychological Bulletin*, **69**:161–182.

Amongst other comments on this issue, Darlington is critical of the method later supported strongly in Pratt (1987), calling it 'meaningless'—though without much real justification.

Darlington, R. B. (1990). *Regression and Linear Models.* McGraw-Hill, New York.

Ehrenberg, A. S. C. (1990). The unimportance of relative importance. *The American Statistician*, **44**:260.

A dismissive comment on Kruskal and Majors (1989). 'I think...only unsophisticated people try to make such assessments [of relative importance]'

Genizi, A. (1993). Decomposition of R^2 in multiple regression with correlated regressors. *Statistica Sinica*, **3**:407–420.

Proposes a decomposition of R^2, different from that of Pratt (1987) (to which Genizi mildly objects because its components can be negative valued). Of the Pratt axioms, Genizi focuses on 'orthogonal compatibility', meaning that for a set of regressors partitioned into mutually uncorrelated subsets, the sum of components within any subset should be R^2 for that subset. Genizi's proposed decomposition is based on a specially constructed orthonormal basis for the space of all regressors.

Goldberger, A. S. and Manski, C. F. (1995). Review article: *The Bell Curve* by Herrnstein and Murray. *Journal of Economic Literature*, **33**:762–776.

Criticizes Herrnstein and Murray's assertions on relative importance of IQ and SES, which rest on comparison of standardized regression coefficients in logit models. 'Standardization in the manner of HM—essentially using "beta weights"— has been a common practice in sociology, psychology and education. Yet it is very rarely encountered in economics ... is worse than useless ... yields misleading inferences.' Similar criticism of measures based on variance explained. If relative importance is to have public policy meaning, the units of measurement must be money: 'It would be meaningful to say that IQ is more important than SES if spending the [fixed] sum on IQ improvement rather than SES improvement were to yield a larger expected change in some outcome of interest.'

Green, P. E., Carroll, J. D., and De Sarbo, W. S. (1978). A new measure of predictor variable importance in multiple regression. *Journal of Marketing Research*, **15**:356–60.

Greenland, S., Schlesselman, J. J., and Criqui, M. H. (1986). The fallacy of employing standardized regression coefficients and correlations as measures of effect. *American Journal of Epidemiology*, **123**:203–208.

'The usual methods of standardizing coefficients...distort the assessment of effects because they confound the effect of a risk factor with the standard deviations of the factor and the disease.' Key point is lack of comparability between studies of the same effects but where the standard deviations are different. Critical remarks also about use of 'variance explained' in causal analysis as being irrelevant (e.g., to caseload, where the attributable fraction is more relevant). 'In summary, standardized regression coefficients...have no meaningful biologic or public health interpretation as measures of effect.'

Healy, M. J. R. (1990). Measuring importance. *Statistics in Medicine*, **9**:633–637.

A very thoughtful essay. Says that 'the sizes of effects are best measured experimentally, but their relative importance in practice must take the population frequencies into account.' Defends standardization of coefficients thus: 'Suppose we take two successive items from the population and get their x-values. The differences we observe will on average be proportional to the standard deviations, so that it makes sense to equate such differences when assessing their effects.' To compare effects conditional on other variables in the model, Healy suggests standardization using conditional dispersion, as later elaborated by Bring (1994a). He also makes a comment about interaction, which is potentially important but largely ignored in this literature.

Heckman, J. J. (1995). Review article: Lessons from *The Bell Curve*. *Journal of Political Economy*, **103**:1091–1120.

Similar objections to those of Goldberger and Manski on HM's measurement of the relative importance of IQ and SES for outcomes such as poverty: '...the appropriate economic measure of importance of two variables is the relevant marginal cost for achieving a given change in the outcome.' ... 'Any [relative importance] measure that is divorced from cost considerations is hard to interpret.'

Hooker, R. H. and Yule, G. U. (1906). Note on estimating the relative influence of two variables upon a third. *Journal of the Royal Statistical Society*, **69**:197–200.

King, G. (1986). How not to lie with statistics: avoiding common mistakes in quantitative political science. *American Journal of Political Science*, **30**:666–687.

Criticizes standardization of coefficients. Makes the rather strong statement that 'Standardization does not add information. If there was no basis for comparison prior to standardization, then there is no basis for comparison after standardization.'

Kruskal, W. H. (1984). Concepts of relative importance. *Qüestiió*, **8**:39–45.

Ridicules the criterion which was elegantly justified later by Pratt (1987).

Kruskal, W. H. (1986a). Relative importance of determiners (in chinese). *Journal of the Chinese Statistical Association*, **24**:11042–11060.

Kruskal, W. H. (1986b). Terms of reference: Singular confusion about multiple causation. *Journal of Legal Studies*, **15**:427–436.

Kruskal, W. H. (1987). Relative importance by averaging over orderings. *The American Statistician*, **41**:6–10.

Corrected 1987 vol 41, p341, to acknowledge priority of Lindeman et al., who make essentially the same suggestion. A recipe for removing dependence on the order of introduction of variables when measuring proportionate reduction in R^2: just average over all possible orderings. In essence, re-invents the same suggestion made in Lindeman et al. (1980).

Kruskal, W. H. (1989). Hooker and Yule on relative importance: A statistical detective story. *International Statistical Review*, **57**:83–88.

Tries to unravel what Hooker and Yule meant to measure relative importance by (necessary since their account is confusing). Concludes that they used $\beta_x\mu_x$, the 'level importance' favoured by economists.

Kruskal, W. H. and Majors, R. (1989). Concepts of relative importance in recent scientific literature. *The American Statistician*, **43**:2–6.

A rather diffuse survey of published scientific articles with the words 'relative importance' or similar in the title. Dismay that 20% of the papers they found used statistical significance to measure importance.
Kruskal here moderates his view of the Pratt (1987) approach, which had previously 'seemed arbitrary', but now, following Pratt's axiomatic justification, rates as 'a development that merits careful study'.

LEWIS-BECK, M. S. (1977). The relative importance of socioeconomic and political variables for public policy. *American Political Science Review*, **71**:559–566.

Argues for comparison of effect coefficients (direct plus indirect effects), rather than regression coefficients or correlations, to measure relative importance of variables in path models.

LINDEMAN, R. H., MERENDA, P. F., AND GOLD, R. Z. (1980). *Introduction to Bivariate and Multivariate Analysis.* Scott Foresman, Glenview, IL.

Page 120, with example on pages 125-127: essentially the same idea as Kruskal (1987), averaging reductions of variance over all orderings. Priority acknowledged by Kruskal in his 1987 correction.

PEDHAZUR, E. J. (1982). *Multiple Regression in Behavioral Research: Explanation and Prediction.* Holt, Rinehart and Winston, New York, 2nd edition.

PLATT, W. G. AND PLATT, C. A. (1990). Measuring relative variable importance. In *ASA Proceedings of the Business and Economic Statistics Section*, pages 248–253. American Statistical Association.

PRATT, J. W. (1987). Dividing the indivisible: using simple symmetry to partition variance explained. In Pukkila, T. and Puntanen, S., editors, *Proceedings of the Second International Tampere Conference in Statistics*, pages 245–260.

'[In multiple regression] it is natural and not unusual to try to measure relative importance without explicitly introducing further specifics. In this spirit, we shall put forth some properties or axioms that one would expect of any general procedure and show that they lead to a unique measure.' The result is to partition R^2 *into contributions* $\{\beta_x \sigma_x \rho_{xy}\}$*, with* ρ_{xy} *the ordinary correlation between* x *and* y*. Pratt also shows that the competing suggestion of Kruskal (1987) violates two of his axioms, most crucially that the relative importance it attaches to* x_1 *depends on the coordinate system used for* $x_2, \ldots, x_p$*.*

SCHEMPER, M. (1990). The explained variation in proportional hazards regression. *Biometrika*, **77**:216–218.

SCHEMPER, M. (1992). Further results on the explained variation in proportional hazards regression. *Biometrika*, **79**:202–204.

SCHEMPER, M. (1993). The relative importance of prognostic factors in studies of survival. *Statistics in Medicine*, **12**:2377–2382.

The last of these three papers by Schemper uses an analogue (developed in the earlier two), for survival analysis, of proportion of variance explained. Ranks variables by both their 'marginal' and 'partial' contributions. Describes a bootstrap method for inference on the resultant ranking(s).

Silber, J. H., Rosenbaum, P. R., and Ross, R. N. (1995). Comparing the contributions of groups of predictors: which outcomes vary with hospital rather than patient characteristics? *Journal of the American Statistical Association*, **90**:7–18.

With, say, death as outcome, consider a logit model

$$\log\left(\frac{p_i}{1-p_i}\right) = \alpha + \pi_i + \phi_i$$

where $\pi_i = \sum \beta_k x_{ki}$ *and* $\phi_i = \sum \gamma_m h_{mi}$ *are respectively the total effect of patient characteristics and of hospital characteristics. Silber* et al. *define*

$$\omega = \frac{\text{variance of } \pi_i}{\text{variance of } \phi_i}$$

as measure of relative contribution to variation in the response. Obviously not restricted to logistic regression.

Snedecor, G. W. and Cochran, W. G. (1976). *Statistical Methods* (6th Edn.). Iowa State University Press, Ames.

A clear, concise treatment (in Section 13.7) of the main methods, namely standardized coefficients and decomposition of R^2. *On the dependence of contributions to* R^2 *on the order in which variables are introduced: '[In some applications] there may be a rational way of deciding the order in which the* X*'s should be brought into the regression, so that their contributions to* $\sum y^2$ *add up to the correct combined contribution. In his studies of the variation in the yields of wheat grown continuously on the same plots for many years at Rothamsted, Fisher [in* The Design of Experiments*] postulated the sources of variation in the following order: (1) A steady increase or decrease in level of yield, measured by a linear regression on time; (2) other slow changes in yields through time, represented by a polynomial in time with terms in* T^2, T^3, T^4, T^5*; (3) the effect of total annual rainfall on the deviations of yields from the temporal trend; (4) the effect of the distribution of rainfall throughout the growing season on the deviations from the preceding regression.'*

Soofi, E. S. and Retzer, J. J. (1995). A review of relative importance measures in statistics. In *ASA Proceedings of the Section on Bayesian Statistical Science*, pages 66–70. American Statistical Association.

I have not managed to find this yet. An online abstract by the same authors, for a conference paper titled 'The Issue of Relative Importance of Variables in Statistical Decision Models', reads as follows: 'In a survey of scientific literature, Kruskal & Major (1989) have noted widespread misuse of significance testing for quantification of relative importance. We review some methods that are recently proposed for measuring relative importance of variables. We propose a Bayesian framework for dealing with the problem as a scientific basis for some recently suggested methods.'

STAVIG, G. S. (1977). The semistandardized regression coefficient. *Multivariate Behavioral Research*, **12**:255–258.

Simply a description of the multiplication of β_x by σ_x to achieve the same effect as standardization (to unit variance) of variable x. This is called 'semistandardization' because it does not eliminate the units of y.

THEIL, H. (1987). How many bits of information does an independent variable yield in multiple regression? *Statistics and Probability Letters*, **6**:107–108.

THEIL, H. AND CHUNG, C. F. (1988). Information-theoretic measures of fit for univariate and multivariate linear regressions. *The American Statistician*, **42**:249–252.

These two papers build on Kruskal's 1987 approach (averaging over all orders), but measuring each variable's importance by the number of bits of information contributed by the variable to the whole. In essence, this is Kruskal's method but with the averaging done on a logarithmic (base 2) scale.

WARD, J. H. (1969). Partitioning of variance and the contribution or importance of a variable: A visit to a graduate seminar. *American Educational Research Journal*, **6**:467–474.

Ridicules the same criterion so elegantly justified later by Pratt (1987).

WILLIAMS, E. J. (1979). Postscript to 'Linear hypotheses: regression'. In Kruskal, W. and Tanur, J. M., editors, *International Encyclopedia of Statistics*, pages 537–541. Macmillan, London.

Dismissive of variable importance assessments through partitioning of effects, except when variables are orthogonal. 'In general the only realistic interpretation of a regression relation is that the dependent variable is subject to the combined effect of a number of variables. Any attempt to take the interpretation further [by partitioning] can lead only to misinterpretation and confusion.'