PLAN, SIPHONING, AND CORRUPTION IN THE SOVIET COMMAND ECONOMY

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Abstract

This paper reconsiders Andrei Shleifer and Robert Vishny's suggestion that a socialist industry will always prefer to cut both price and output relative to a market–clearing equilibrium in order to maximise bribe income. The evidence from recent archival studies of the Soviet economy does not support this conjecture. To understand the evidence we present an analytical framework within which a plan–setter and an effort–setter interact, subject to a hard resource constraint, to determine real output and hidden inflation simultaneously. We find that managers who use resources gained corruptly were enabled to produce more real output with less hidden inflation and fulfil the plan more honestly as a result. We find clear rationales for plan–setters to have tolerated corruption and siphoning while maintaining plan tension, and we associate reduced plan tension in the 1970s with the spread of disloyal behaviours.

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Plan, Siphoning, and Corruption in the Soviet Command Economy

Introduction

In a much cited article Andrei Shleifer and Robert W. Vishny (1992) presented a simple model to explain the shortages and excess demands characteristic of state–socialist command economies. In their framework the main feature of a "socialist industry" is monopoly: the industry may comprise one firm or many but, if many, then the latter behave collectively like a monopolist, collusion being enforced by the fundholding ministry. The industry equates marginal revenue with marginal cost. But the state fixes prices, meets all production costs, and collects all revenue accruing at official prices. Therefore it is the official price, not production costs, that defines the industry's internal cost schedule. The industry can gain a net revenue only by extracting a bribe from consumers on top of the official price. The industry sets its output where the official price equals marginal revenue from bribes. Compared with any given market–clearing combination of price and output, the industry always wants to cut both price and output. Below we call this the Shleifer–Vishny conjecture.

One motivation professed by Shleifer and Vishny was a desire to explain pervasive consumer shortages without reference to the "soft budget constraint" hypothesis advanced by Kornai (1980) or to aggregate demand and repressed inflation: "In Kornai's model it is not so much that goods are underpriced, but that the income of the buyers is effectively infinite. This model may be appropriate for some intermediate goods. But households face hard budget constraints, and therefore the systematic shortages of many consumer goods remain a puzzle".[1] In fact a theoretical solution to this puzzle had long been available in the Sovietological literature (Kaser, 1975; Birman, 1980; Kornai, 1980): "siphoning". A soft budget constraint for firms could result in shortages for consumers if firms' demand for inputs spilled over into retail outlets, siphoning resources intended for final consumption back into intemediate use. However, the existence of siphoning remained conjectural, especially since it required firms to be able to mobilise cash illicitly. Even if siphoning occurred its scale and significance remained doubtful. If it was significant, it was unclear why the authorities tolerated it.

More recently Mathias Dewatripont and Eric Maskin, Yingyi Qian, and Byung–Yeon Kim have provided all the pieces necessary for this puzzle to be solved. Dewatripont and Maskin (1995) showed how centralization of credit leads to a soft budget constraint in the context of sunk costs and contract renegotiation. Within a similar framework Qian (1994) showed that when state–owned enterprises compete with households for goods that may be used in both consumption and production, non–market clearing prices may improve efficiency by allowing household consumption to crowd out some bad projects that would otherwise proceed. Finally, Kim (2000) showed from postwar archival records that Soviet firms and budgetary organisations did engage in siphoning, that there were unofficial demand spillovers in addition to those that were officially sanctioned, and that siphoning made a substantial contribution to repressed inflation and a growing monetary overhang in the Soviet retail market. In particular, he found that in two postwar subperiods of reduced containment of inflation in factory prices enterprises used their additional liquidity to raise their purchases in the retail market more rapidly.

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[1] Wintrobe (1998: 198) echoed this doubt: firms' soft budget constraint explains why "firms always want more inputs than are available. […] However, this still does not explain shortages in consumer markets, which were equally legendary in the former Soviet Union".

In short, one original motivation behind the Shleifer–Vishny conjecture has been eliminated. Moreover, the restatement of the soft budget constraint hypothesis must weaken the analytical basis of the Shleifer–Vishny conjecture since the two are not consistent: in the Shleifer–Vishny story firms limit outputs to increase shortages and extract bribes, whereas in the story of the soft budget constraint they demand inputs and limit outputs to conserve effort.

At the same time, the Shleifer–Vishny conjecture has had two important merits. First, it has brought the issue of price–setting by enterprises in command economies into sharper focus. With rare exceptions the formal models of the state–socialist firm that are to be found in the traditional Sovietological literature ignored this issue, assuming that enterprises faced prices that were exogenously fixed.[2] If enterprises were free to set their own prices as they wished, how would they do so? The Shleifer–Vishny conjecture suggested that they would set them as low as possible so as to collect the maximum bribe. Second, the Shleifer–Vishny conjecture clearly contains more than a grain of truth: many who lived in the shortage economy found ways of deriving some corrupt benefit from its rigidities, and bribery was one of the most common methods of reallocating commodities when supplies were short. Did it follow that the rigidities themselves were created intentionally for the purpose of distributing the bribes?

In this paper we proceed as follows. Part 1 uses the results of recent investigations in the Russian state archives to establish some relevant stylised facts. In particular, the orientation of Soviet producers toward their own–product plan prices was exclusively inflationary, being motivated by the desire for an easier plan. Further, producers strove continually to obtain above–plan liquidity for use in siphoning; this extra liquidity could take the form of either higher prices or side payments. In Part 2 we present a simple model of a resource–constrained firm under command arrangements in which producers' allocation of effort interacts with the output target set by the planner. Part 3 extends the model to allow the firm to gain hidden revenues that interact with its resource constraint through siphoning. At first we treat these hidden revenues as exogenously given from some previous period. Part 4 extends our analysis to siphoning in continuous time. At each point we consider the rationale for managers to engage in corruption and for planners to tolerate their behaviour. Part 5 concludes.

1. What the archives show

1.1. Plan prices

Shleifer and Vishny's story implies that firms always preferred lower prices to higher ones. But there is absolutely no evidence of this preference ever being expressed through firms' investments in lobbying or negotiation. On the contrary, the evidence that firms tried continually to negotiate prices upwards is abundant and overwhelming.

To navigate this subject it is necessary to distinguish plan or estimate prices, official factory wholesale prices, and retail prices (for more detail see Nove, 1977). Accounting plan prices (*neizmennye* or *sopostavimye tseny*) were used as a fixed standard of value in formulating plans for heterogeneous products denominated in rubles and evaluating their fulfilment. Estimate prices (*smetnye tseny*) were used for the same purpose in the case of unique construction projects. Official factory prices were authorised by the centre, usually on the basis of actual direct costs plus an allowance for overheads and, once authorised, were supposed to remain unchanged.

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[2] See for example Ames (1965). A notable feature of Shleifer and Vishny's (1992) model is that their planner is able to control prices perfectly if she wishes, yet does not control quantities at all.

Factory prices were those actually received by producers for output and credited to their financial profit–and–loss accounts; the factory price with a further allowance for transport and distribution costs (the "factory wholesale" price) was charged to users including, in the case of consumer goods, the retail trade distribution system. Accouting plan prices were usually based on factory prices of some base year which was updated infrequently and could be long distant, unless the product concerned had been introduced or upgraded at a more recent date, in which case the factory price of that date was used. Finally, where products were designated for retail trade they also carried a retail price; the difference between the retail price and the factory wholesale price was the trade markup plus a variable–rate turnover tax (or minus a subsidy).

Consider first the importance of plan prices to the enterprise. Contrary to the conventional stereotype of an economy subject to direct quantity regulation, we now know that nearly all plan targets in the Soviet economy were denominated in rubles. The most important control figures decided by the Politburo, the annual investment plan and the defence budget, including the plan for military procurements from industry, were measured in rubles. So were almost all important plan targets that had binding significance on production ministries and enterprises. Those few targets fixed by the centre in physical units, for example tons of steel, usually had an indicative status which was more than propaganda but less than operational command. Thus, the Politburo might announce a five–year plan target for steel tonnage; still, the directive plan for the steel industry for the year emerged in rubles. At lower levels plans for the gross value of output of sectors or enterprises might be accompanied by side conditions for physical assortment, but the latter were inevitably controlled with a lower degree of stringency.

The percentage degree of plan fulfilment helped to determine the private rewards available to producers. The value of output reported from below was compared with the value of output planned from above. Traditional payment schemes involved a substantial jump in the premium as 100 per cent was achieved, giving producers a strong incentive to fulfil the plan or overfulfil it by a small margin. As a result, producers also faced strong incentives to report output at higher prices than those assumed in the plan. At a higher price level they could secure the rewards associated with plan fulfilment with less real output and less productive effort, thus improving their ratio of reward to effort at the authorities' expense.

The authorities, on the other hand, had a strong incentive to resist this pressure. Inflation in plan fulfilment figures meant less real output and a reduction in the real rents at the disposal of the centre. A mechanism designed with the obvious intention of ruling out fulfilment of the plan through inflation was the fixing of plan prices in relation to a base year. Thus production plans were denominated in "unchanged" prices of a previous benchmark year; for example, 1926/27 was used for the period from 1928 to 1950; from 1952 the prices of 1950 were used, and so on.

The authorities' problem was to prevent inflation of plan prices. If producers could hide inflation from planners, or secure their collusion in hiding it, then producers could fulfil the plan with less real output and secure the associated rewards with less productive effort. The basis of producers' discretion was their grasp of the initiative in price–setting for new, improved, or unique products and, for construction units, in contracting for unique projects. For such products and projects a benchmark plan price did not exist, had to be invented, and was generally based on the producer's initial experience of unit costs.

Producers' main opportunity to influence plan prices lay therefore in product innovation. If a product had been introduced or upgraded at a date more recent than the base year for plan prices, its factory price from the period of its introduction was used for the plan price. Planners controlled both the assortment profile of the plan and the setting of plan prices with difficulty. Producers were therefore enabled to exploit product innovation to achieve a given gross value of output in plan rubles with less effort in three respects (Harrison, 1998). First, when nominal unit costs were rising,

they skewed the assortment profile of output towards newer products. Second, they inflated the costs at which new and upgraded products were priced. Third, they simulated innovation with no other intention than to free themselves from established plan prices (Berliner, 1952 and 1976). The result was a upward drift in plan prices that was concealed from the planners at the time but detectable after the event (Harrison, 2000).

1.2. Factory prices and liquidity

In the stereotype of the Soviet system propagated by its founders, what mattered was real resources: the plan steered the real economy, and money followed the plan. From the start, however, reality was different (Gregory and Tikhonov, 2000). The danger was not just one of private embezzlement as Shleifer and Vishny supposed; even loyal agents of the Soviet economic system were strongly motivated to acquire financial surpluses on behalf of their enterprises (Belova, 2001). The uses of enterprise liquidity were many but in particular to provide for incentive payments to workers and supply agents (*tolkachi*) and to engage in siphoning the resources away from the retail market that were required by the enterprise to fulfill its plan.

To create reserves of liquidity safe from confiscation by the state budgetary authorities, enterprises had to secure hidden revenues over and above officially listed factory prices. The desire for liquidity was strongly felt in enterprises' price–setting behaviour. The opportunity for factory price inflation arose during the negotiation of contracts between suppliers and users which translated aggregate plans at the level of production ministries into specific assignments and delivery obligations. It suited both planners and producers that plans were issued only in a highly aggregated form; this freed planners from responsibility for issuing specific assignments and gave producers maximum freedom of action in deciding how to meet them (Belova and Gregory, 2001). In a seller's market the inevitable result was an annual ritual played out between ardent suitors (purchasers) and reluctant brides (producers), which became known as the "contracts campaign". During this process suppliers used all sorts of strategems to extract advantage from potential purchasers. In the defence industry, for example, in addition to unauthorised increases in prices of established products we find demands for illegal advance payments, demands for contracts in which prices were specified "provisionally" and subject to review in the light of actual costs, exaggeration of costs, refusal to permit their verification, withholding of evidence concerning actual costs, delaying coming to terms until well into the contract period, and refusal to come to an agreement at all unless concessions were made (Harrison and Simonov, 2000).

The inflationary proclivities of producers were taken for granted at the highest level. When Stalin's Politburo discussed the year's investment plan it was understood that ambitious real plans converted into rubles would be partly eroded by higher construction costs: the building industry would respond to a larger budget by raising costs above estimate prices (Davies, 2001). Consequently the annual investment plan was set, and at times limited, in the light of this perception.

In summary the archives provide us with compelling and robust evidence of producers' preferences with regard to own–product prices. These were uniformly inflationary. Even a single example of a producer seeking a lower official price has yet to be found.[3] In the next section we present an analytical framework within which this behaviour may be understood.

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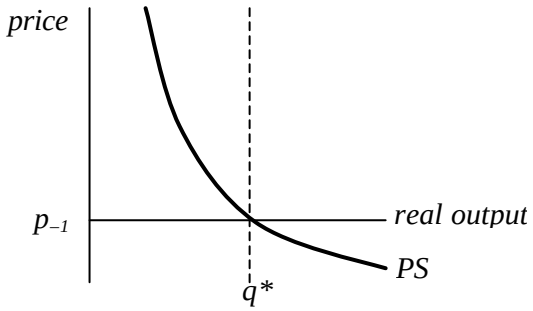
[3] In fact wholesale and retail prices fell rarely, and then only as a result of administrative fiat. Policies of general price deflation were pursued on two occasions. In the early years of World War II the authorities subjected weapon prices to substantial downward pressure to force productivity gains from mass production to be reflected in lower budgetary costs of military procurement (Harrison, 1996). Again,

2. Plan–Setting and Effort–Setting

2.1. Plan–Setting

In the Soviet command economy for each period and each firm planners set a target for the real gross value of output. This target was expressed in plan rubles, being calculated as a quantity vector q^* multiplied by a vector of official prices p_{-1} that held in the previous or base period. In practice, however, planners could not effectively compel firms to fulfil the plan at base–period prices. They tried to authorise only those changes in product–unit prices that left the level of prices per unit of characteristics unchanged, but the stream of continuous alterations in the product assortment and product characteristics left planners unable to detect the hidden inflation associated with simulated product innovation. Inflation was concealed and the planners were fooled when the firm pushed up product–unit prices faster than the value to the user of any improvement in product characteristics, or when product–unit prices remained constant despite some unreported deterioration of product quality.

Figure 1. The plan–setting curve



As a result the planners had to be satisfied *ex post* with any real output vector q that, combined with a new current price vector p set by firms and subject to planners' limited scrutiny of price alterations, matched the ruble total set by them *ex ante*:

1. $$q = \frac{1}{p} \cdot \left(p_{-1} \cdot q^* \right).$$

This equation is represented in figure 1 by the *PS* or plan–setting curve, which was unit–elastic and passed through p_{-1}, q^* when $p = p_{-1}$ and there was no hidden inflation. In general we will only be concerned with that part of the *PS* curve that lies above the horizontal line marking $p = p_{-1}$. The vertical axis in figure 1 represents the true price level, not the official one, and includes the inflation that that the planners failed to detect. The section of the *PS* curve above the p_{-1} line therefore shows the rate at which producers could trade real output for hidden inflation while continuing to satisfy the plan. Similarly, the horizontal axis measures not official real output but true real output, corrected for the exaggerated claims that producers made for the quality of their products in order to fool the plan–setter.

2.2. Effort–Setting

From the firm's point of view output was costly, but so was the concealment of inflation, because both required the exertion of effort. Suppose the firm's utility to have been based on its members' wage income w and leisure l:

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in the early 1950s the authorities pursued a policy of stabilising money wages and allowing living standards to rise by forcing down retail prices (Davies, 1958). These were discretionary acts of high–level policy, and tell us nothing about producers' preferences or motivations.

2. $u = u(w, l), \quad u'(w, l) > 0, \quad u''(w, l) < 0$.

If the firm satisfied the planners' target it received a fixed wage *w*; otherwise, it received nothing: this is the classical Soviet incentive scheme in a simplified form (Nove, 1958). We assume that leisure and income interact, so the firm required at least some of both. In order to have any income at all the firm had first to satisfy the output target, and then it could also maximise leisure; the firm behaved lexicographically although its utility was not lexicographic. The significance of a "quiet life" for Soviet enterprises can be found in studies as far back as that of Berliner (1952). The firm maximised leisure subject to a resource constraint that we shall treat initially as hard:

3. $1 - e - i - l = 0$

where *e* is productive effort, *i* is the effort required to simulate innovation and so conceal inflation, and the firm's initial time endowment is normalised to 1.

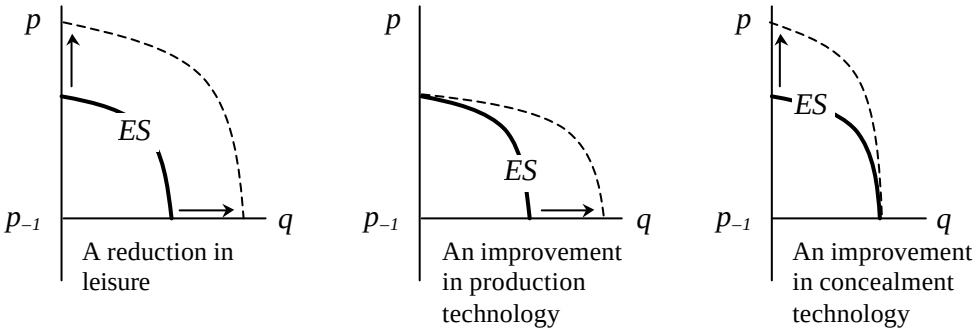
Finally, there were two technologies, one for the production of output *a* and one for the concealment of inflation *x*. Production required effort to be combined with capital k, but for now we will treat capital as an exogenous resource. Inflation concealment required effort alone. In both activities, returns to effort diminished:

4. $q = a(\bar{k}, e), \quad a'(e) > 0, \; a''(e) < 0$

5. $\dfrac{p - p_{-1}}{p_{-1}} = x(i), \quad x' > 0, \; x'' < 0.$

Then, for any given level of overall effort that it chose to set, the firm faced a feasible set of combinations of real output and hidden inflation that was concave to the origin (**see the appendix, proposition 1**). In figure 2 this is shown as the *ES* or effort–setting curve.

Figure 2. The effort–setting curve



For each firm there was a family of *ES* curves, one for each feasible level of effort. As the firm set its effort level higher, the *ES* curve moved outwards in all directions (**proposition 2**). An improvement in production technology shifted the whole *ES* family to the right, raising real output for given effort and given hidden inflation. An improvement in the concealment technology shifted the whole *ES* family upward, raising hidden inflation for given effort and given real output (**proposition 3**).

2.3. The Firm's Equilibrium

The firm's problem, faced with a given *PS*, was to allocate effort between production and inflation concealment such that the plan was fulfilled and effort was set at a minimum. This point is found where the plan–setter's line is tangential to the lowest

available effort–setter's line as in figure 3, and it determines the equilibrium values of real output and hidden inflation simultaneously.

Figure 3. The firm's equilibrium

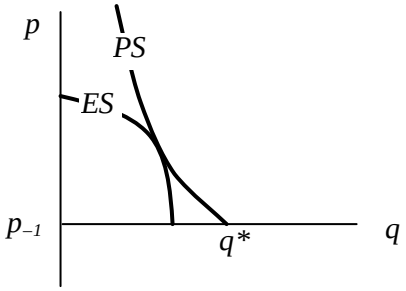
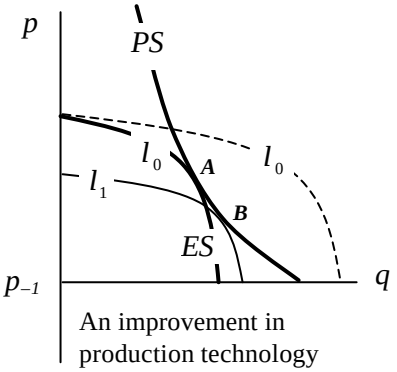


Figure 4. Improvements in technologies



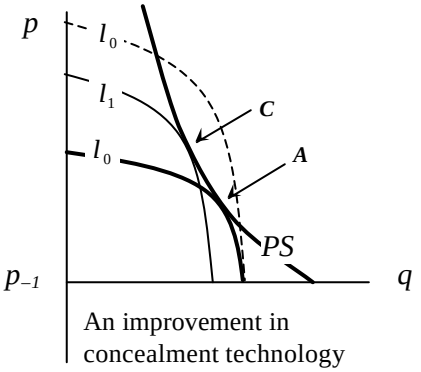
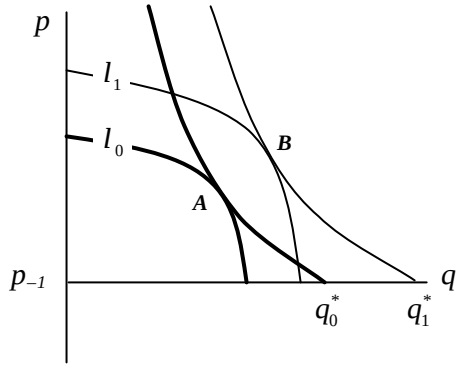


Figure 5. A rise in plan tension



When the properties of this model are explored, it turns out to have the following features. Figure 4 illustrates changes in technologies. In each panel the firm's initial equilibrium is shown at point A. From figure 2, an improvement in production technology right–shifts *ES*, raising real output for given effort $(1-l_0)$ and given hidden inflation. In figure 4, since the *ES* curve for l_0 lies partly outside the *PS* curve, the firm may fall back to a lower *ES* curve with reduced overall effort $(1-l_1)$, where $l_1 > l_0$. The *ES* curve with better production technology and less effort is skewed to the right compared with the old one. Because returns to effort invested in production have risen relative to the returns to effort invested in fooling the planners

the firm also trades along the *PS* curve from A to B, substituting more real output for less hidden inflation.

From figure 2, conversely, an increase in planners' susceptibility to being fooled up–shifts *ES*, raising hidden inflation for given effort and given real output. In figure 4 the firm is again able to fall back to a lower *ES* curve with reduced overall effort $(l_1 > l_0)$, and is also able to substitute concealment effort for productive effort, so at the new equilibrium C there is more hidden inflation and less real output**.**

Figure 5 illustrates a rise in plan tension. Other things being equal, an increase in q^* right–shifts *PS*. The firm is forced to move to a higher *ES* curve. At the new equilibrium B there is less leisure $(l_1 < l_0)$ and more real output. However, diminishing returns to productive effort mean that it would not be efficient for the firm to meet the higher plan solely by increasing real output. Some of the extra effort will go into fooling the planners. Therefore real output rises by less than q^* and there is also more hidden inflation**.**

Simple extensions of the model suggest conditions under which the command system may break down. These conditions include both excessive liberalism and excessive harshness. First, suppose the plan was set at a level that could not be fulfilled with any combination of real output and hidden inflation given the firm's hard resource constraint. Producers would then prefer zero effort, zero reward, and zero utility to the negative utility created by trying and failing to fulfil the plan; there would be no hidden inflation, but output would collapse to the origin. Second, if prices were liberalised within the command system producers would use their increased discretion to climb the *PS* curve, raising prices without limit and cutting real output to a minimum. Sounds familiar? It should: it happened in Russia between 1989 and 1992 (Harrison, 2001).

3. Siphoning and Corruption

3.1. A Single Transaction

The foregoing analysis was carried out under the assumption that the enterprise was subject to a hard resource constraint. In fact, while resources were constrained for the economy as a whole, for the individual firm the resource constraint could be softened by siphoning and the substitution of outside resources for insider effort. Our attempt to capture the outcome will resemble the Shleifer–Vishny story of the bribe in some respects, but in our initial model the bribe is collected in order to fulfil the plan, not to line pockets. The result is that, with plan–setting unchanged, real output rises while both hidden inflation and insider effort fall; however, there is also an increase in the inflation experienced by consumers.

Subsequently we will go on to examine the implications of bribe–taking for personal enrichment, or embezzlement. However, unlike Vishny and Shleifer we do not regard embezzlement as the general case. We will draw a distinction between corruption and disloyalty. All managers were potentially *corrupt*, but only *disloyal* managers used the proceeds for personal enrichment. Think of an agent's loyalty as an investment in the perceived alignment of her objectives with those of superiors and inferiors in the vertical administrative hierarchy. For many purposes it is reasonable to presume that managers, however corrupt, remained rationally loyal. First, managers invested in loyalty to their superiors for the sake of their careers: to get promotion, they needed a record of plan fulfilment. This record also relied on meeting the needs of inferiors and maintaining their cooperation, so managers also had to invest in loyalty to the workforce. Disloyalty in either direction, upward or downward, prejudiced career aspirations. Second, the cash return to disloyalty may have been hard to spend on personal consumption, both because of the seller's market and also because of the need to avoid attracting attention.

Belova (2001) reports what she calls the "honest" manager's dilemma, but in this system no one was completely honest, so we will call it the dilemma of the *loyal* manager. To fulfil the plan, the manager had to compete for scarce supplies that the planners forgot, or promised but did not deliver. To compete, she had to pay. Payments went to private supply agents (*tolkachi*), sideline suppliers of deficit commodities, and workers for the extra effort required to make bricks without straw. For our purposes we will emphasise siphoning: in order to augment their own productive stocks firms entered the retail market as purchasers of commodities intended for purchase by households for personal consumption. For this purpose the loyal manager had to acquire discretionary liquidity by creating revenues that were hidden from the planners. She got the liquidity from her purchasers by securing an above–list price, or advance payment, or bribe, but, in distinction from the the Shleifer–Vishny story, the bribe was paid into the firm's account.

First, rewrite the firm's production function from equation (4), adding Δk to represent the availability of an outside resource that substitutes perfectly for the firm's inside stocks of fixed and working capital:

6. $q = a\left(\bar{k} + \Delta k, e\right), \quad \Delta k > 0, \quad a'(\bar{k} + \Delta k, e) > 0, \quad a''(\bar{k} + \Delta k, e) < 0\,.$

In order to get hold of Δk the firm requires hidden revenue for discretionary use in the secondary market where commodities are traded that are of potential intermediate use. This hidden revenue must be over and above the liquidity officially allocated in the firm's financial plan. The firm extracts this hidden revenue in advance from consumers in its own product market: the hidden revenue gathered in the previous period, b_{-1}, is used to add to the firm's real resources in the current period. We model this as follows: there is a market–clearing price $\hat{p}$ of output that exceeds the official price p at all relevant levels of output. The firm can capture some proportion of the gap in the form of an advance payment, side payment, or markup over its officially listed product price without being detected; for simplicity we can set this proportion at 100 per cent without affecting basic results. The firm uses its hidden revenue as discretionary purchasing power in the secondary market. The effect on the firm's resources is the outcome of a siphoning technology s in which the input is hidden revenue but, since the secondary market too is a seller's market, there are diminishing returns:

7. $\Delta k = s\left(b_{-1}\right), \; s' > 0, \; s'' < 0\,,$

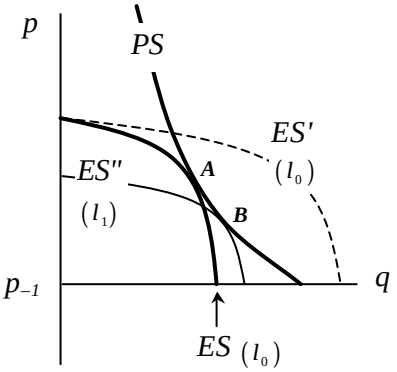
8. $b_{-1} = q_{-1} \cdot \left(\hat{p}_{-1} - p_{-1}\right), \quad q > 0, \quad \hat{p} > p\,.$

How will siphoning affect the *ES* curve? The result will be a new ES' curve that is still concave (**proposition 4**) but right–shifted by comparison with the no–siphoning case (**proposition 5**). For a baseline illustration, consider how the structure of the firm's incentives is changed in what we will call the "single–transaction" case. The single transaction begins in period –1 when the firm sells a commodity in return for a side–payment and recycles it through siphoning to acquire Δk. The same transaction ends in period 0 when the firm uses Δk to satisfy the plan–setter and augment its utility, but the firm makes no provision for repeated siphoning of resources to be used in period 1.Thus the single transaction takes two periods to complete, but the only decision we analyse is the firm's optimisation within the current period.

This case is illustrated in figure 6; it is practically identical with that of an improvement in production technology shown in figure 4. The firm is initially subject to a hard resource constraint. Given plan–setting, the firm is in equilibrium with the solid effort–setting curve *ES* and leisure l_0 at point A. Softening its resource constraint through siphoning enables the firm to produce more for given hidden

inflation and given effort. The dashed *ES'* curve in figure 4 illustrates the production–augmenting effect of siphoning with effort held at l_0. Since *ES'* lies partly outside the firm's *PS* curve, it is not efficient for the firm to hold effort at this level. As long as plan–setting remains unchanged, the firm will prefer to cut effort to l_1, yielding an *ES''* line that, with augmented productive capital, is skewed to the right by comparison with the no–siphoning *ES* curve and touches *PS* at B, a point showing more real output and less hidden inflation.

Figure 6. Siphoning: a single transaction



In short, softening her resource constraint and tolerating corruption helped the loyal manager to fulfil the plan with less effort, less hidden inflation, and more true real output than would have been possible otherwise. This result suggests that not only the firm gained; there was also less fooling of the planners, for whom there was a clear rationale to turn a blind eye to the rule–breaking involved since in the outcome the plan was fulfilled more honestly. The welfare implications for consumers, however, are ambiguous. There was more real output in a gross sense, but more of it was diverted away from consumption and recycled back into production. Moreover, consumers paid more for the output that was supplied to them through some combination of open inflation and covert side payments.

3.2. Loyal and Disloyal Managers

In this economy all managers were self–interested, and all managers were corruptible, but the manager we have described so far remained loyal. Think of an agent's loyalty as measured by the degree of alignment of her objectives with those of superiors and inferiors in the vertical administrative hierarchy. Managers invested in loyalty to their superiors for the sake of their careers: to get promotion, they needed a record of plan fulfilment. This record also relied on meeting the needs of inferiors and maintaining their cooperation, so managers also had to invest in loyalty to the workforce. By accepting illegal side–payments and disbursing them via the siphoning mechanism, the manager fulfilled the plan and at the same time reduced the efforts of the workforce. This manager was therefore loyal to both superiors and inferiors.

Were all managers loyal? For reasons given above we find it reasonable to presume that managers, however corrupt, might rationally choose to remain loyal. Nonetheless disloyal managers existed. If disloyal, they pocketed bribes for personal enrichment, and failed to distribute gains among either planners or workers. They still aimed to fulfill the plan but they were no longer minimising the workers' or their own efforts. It is widely held that disloyalty in this sense increased through time and contributed to the decay of the Soviet command system under Brezhnev (Grossman, 1998). Therefore the implications of disloyalty deserve brief examination.

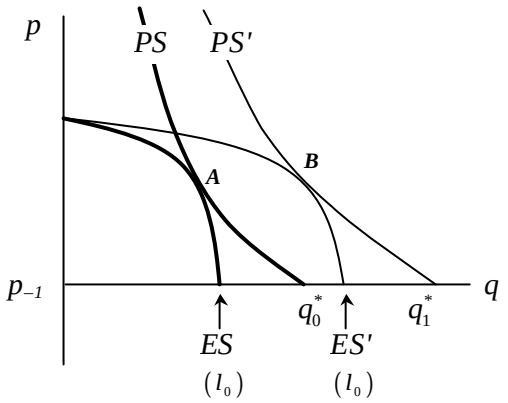
A manager might choose to be either completely or partly disloyal. A *completely disloyal* manager embezzled all side payments for personal enrichment, so there

would have been no siphoning and the firm's resource constraint would remain hard. In practice, we suggest that the scope for managerial disloyalty was always limited by workers' and planners' responses. To explore these responses we specify the distribution of information about disloyal behaviour by managers as follows. Suppose that information about specific disloyal acts was only available to other agents within the firm; above the firm, there was information only about the incidence of corruption possibilities generally. If the manager was disloyal the workers knew, but the planners could only suspect.

With respect to the workforce the manager with a disloyal propensity faced the following problem. She could sufficiently maintain her reputation upwards by fulfilling the plan. But to fulfil the plan the manager needed the cooperation of the workers, who would have knowledge of her disloyalty. The workers might threaten to withhold effort, or alternatively to betray her wrong–doing, as a means of inducing her to share the gains from corruption. In principle this sharing might be done in two ways. First, the corrupt manager might distribute part of the bribe revenue directly to the workforce in cash, but on our understanding of the context and evidence this was detectable, therefore dangerous, and seldom done. Instead, managers bought cooperation by transforming cash rents into additional resources through siphoning and sharing the gain with the workers in the form of leisure. In this form the redistribution of rents was less visible to higher–level audit or even took on a "good" appearance. As a result, for a given plan insiders' effort fell. In short, complete disloyalty was an infeasible choice. *Self–interested managers were always at least partly loyal.*

Consider the problem of the same disloyal manager in relation to plan–setters. Plan–setters did not know firms' intrinsic capacity, nor the extent of disloyalty, nor the extent to which bribes were recycled into siphoning. They knew there was corruption, that siphoning might occur as a result, that siphoning enlarged firms' capacity, and that even disloyal managers would rationally choose to recycle some bribes into siphoning. They knew therefore that when there was siphoning the effort level and the utilisation of capacity would tend to fall with an unchanged plan; this case was illustrated in figure 6. By setting plans "from the achieved level" planners aimed to avoid a slackening of efforts. Thus they would rationally respond to their knowledge of corruption possibilities by tautening the plan.

Figure 7. A rise in plan tension with siphoning



A possible outcome for a single transaction is shown in figure 7. The gain in the firm's capacity for a given level of effort is the shift from *ES* to *ES'*. The increase in plan tension is shown in the shift from q_0^* to q_1^*, and the associated shift from *PS* to *PS'*. In the case of a corrupt but loyal manager this increase exactly captures the firm's additional resources, moving its equilibrium from A to B. At B, effort is the same as at A. Managers' loyalty has resulted in the gains being shared with the

planners through the increase in real output, but real output has risen less than in proportion to the rise in plan tension and there is also higher hidden inflation.

The disloyal manager wished to pocket the gain herself. But she still had to maintain her reputation with plan–setters by fulfilling the plan. Since the planners' knowledge of corruption possibilities led them set a higher plan, the disloyal manager now had to get the workers to work harder. For this she again required the workers' cooperation, and this forced her to share the proceeds of corruption with the workers by engaging in siphoning to enlarge capacity and spread effort. In short, *when managers were corrupt raising plan tension was a mechanism to limit disloyalty*. But the converse was also the case: *reducing plan tension could promote embezzlement by corrupt managers*. This result establishes a clear mechanism that links the spread of corruption in the Soviet economy in the 1970s with the simultaneous reduction of growth targets (on corruption see Grossman, 1977, and on growth targets and growth Schroeder, 1985).

4. Repeated Siphoning

A single–transaction framework does not allow us to address the question prompted by the Shleifer–Vishny model: in the presence of corruption, may the producer gain by restricting output? In the single–transaction model the value of side payments is fixed beforehand. Effort is not optimised from the point of view of supplying the market in such a way as to secure further side payments as a basis for repeated siphoning. However, side–payments depend on the gap between the market–clearing price and the official price. When siphoning is repeated over several periods this gap can be determined simultaneously with the effort–setting decision, since market–clearing and official prices both decline, but at differing rates, as output rises. The official price varies inversely with output along the *PS* curve, the latter being unit price–elastic. The market–clearing price varies inversely with output along the demand curve as the balance in the market shifts from seller to buyer. Therefore, rewrite equations (8) and (9) in continuous time:

9. $\Delta k = s(b), \quad s' > 0, s'' < 0,$

10. $b = q \cdot (\hat{p} - p), \quad q > 0, \quad \hat{p} > p;$

and add a market demand curve, linear for the sake of illustration:

11. $\hat{p} = z - p \cdot q, \quad z, p > 0.$

Consider the behaviour of the firm's hidden revenues as real output increases. We do not model this formally but we observe that it depends on the relative elasticities of the *PS* and market demand curves. The *PS* curve, which could also be termed the planner's demand curve, determines open revenues but is unit–elastic: the firm must achieve a fixed total planned revenue, which in this context becomes a lump–sum tax. Assume for simplicity that the firm succeeds in extracting all the side payments that the market will bear; then, the market demand curve determines the firm's total revenue. It is downward–sloping and linear so its elasticity falls as output rises. Where the elasticity of the market demand curve equals unity, the firm's total revenue is maximised and, since its planned revenues are a lump sum, its hidden revenues are also maximised.

Consider figure 8. The siphoning firm operates in the "shortage" region of the diagram, in the region between $\underline{q}$ and $\bar{q}$ where the market–clearing price exceeds the official price; it is also a necessary condition that both prices exceed the baseline plan price p_{-1}. In this region, assuming that the firm is able to extract the market price on its total output, the gap between the *PS* and *D* (market–demand) curves represents the firm's hidden revenue per unit of output. It maximises its total hidden revenues

midway between $\underline{q}$ and $\bar{q}$, which is also the point where marginal revenue becomes zero and the marginal revenue curve, drawn with slope $-\frac{1}{2p}$, meets the quantity axis (drawn in this figure at $p = 0$, not $p = p_{-1}$). On either side of this point hidden revenues diminish, falling to zero at $\underline{q}$ and $\bar{q}$.

Figure 8. Hidden revenues

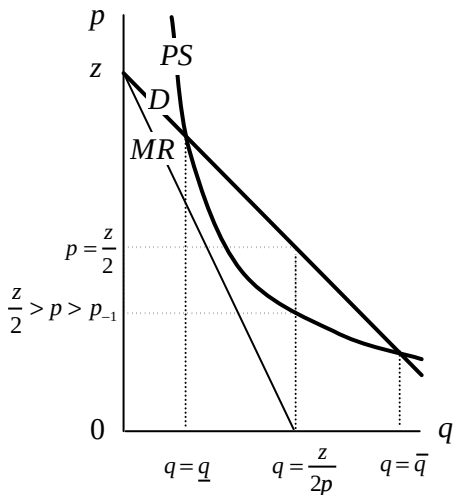
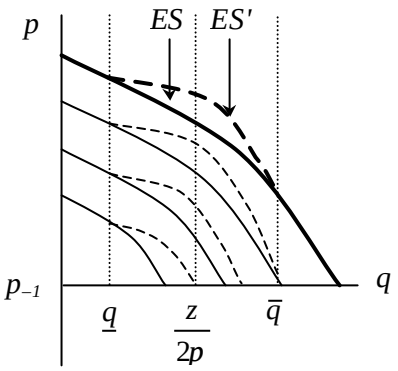


Figure 9. The ES' curve with repeated siphoning



When siphoning is repeated, what is now the effect of endogenising side payments on the ES curve? The effect is as if a blister were to appear on the *ES* curve in the region between $\underline{q}$ and $\bar{q}$. In the single–transaction case (figure 6), the whole *ES* curve was right–shifted in favour of higher real output. When siphoning is repeated and hidden revenues are endogenous, the rightward drift of the ES surface is limited to the region already identified, the drift reaching a maximum extent at the level of output where hidden revenues are maximised. This drift is determined by the *PS* and *D* curves alone, and there is therefore an identical blister on each *ES* curve in the family of curves as shown in figure 9.

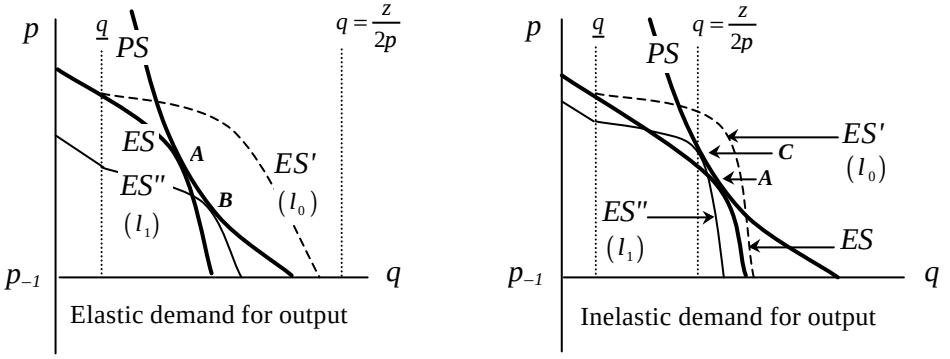
How does repeated siphoning influence the *PS–ES* equilibrium? In principle the effect of siphoning on the firm's initial equilibrium depends on three things: the location of its initial equilibrium, the point–elasticity of market demand for output at that equilibrium, and the size of the siphoning blister. Several cases are possible, including some that do not result in a siphoning equilibrium: for example, a firm

encountering a siphoning blister that is small and far to the left or right of the initial equilibrium might find it unprofitable to siphon. There appear to be two possible cases when the siphoning blister is near enough and large enough for the firm to exploit. The difference between them is the location of the point of maximum hidden revenues relative to the no–siphoning equilibrium or, in other words, the elasticity of the market demand curve relative to that of the *PS* curve at the no–siphoning equilibrium.

To illustrate this point for simplicity rewrite equation (10) such that the relationship between hidden revenues and resources siphoned becomes linear with constant returns. Now siphoning is maximised where hidden revenues are maximised. Thus, when the market demand curve is more elastic than the *PS* curve and hidden revenues are increasing, the siphoning blister increases the slope of the *ES′* curve; conversely, when the market demand curve is inelastic and hidden revenues are falling, the siphoning blister reduces the slope of the *ES′* curve. If *ES′* is made flatter then the siphoning equilibrium will tend to shift in favour of higher output; conversely, if steeper, then the siphoning equilibrium will tend to shift in favour of higher hidden inflation.

These are illustrated in figure 10, with the no–siphoning equilibrium marked at point A in each case. Suppose that, with a low value of p , the market demand curve is still elastic at the no–siphoning equilibrium; then for given output the *ES′* curve will become flatter, and the siphoning equilibrium will tend to shift in favour of higher output. Intuitively, the firm that faces elastic market demand loses hidden revenue by restricting output. Instead, up to a point dictated by diminishing returns to effort and liquidity the firm produces extra output, takes the extra hidden revenue, and uses it to buy extra inputs. For insider effort set at $(1-l_0)$ the *ES* curve expands to *ES′*. This effort is above the efficient level, so the firm cuts effort to the lower *ES″* where effort is $(1-l_1)$ and $l_1 > l_0$. The outcome is an equilibrium at B with less hidden inflation and more real output.

Figure 10. Repeated siphoning



Conversely suppose that, with a higher value of p , the market demand curve is already inelastic at the no–siphoning equilibrium; then for given output the *ES′* curve will be made steeper, and the siphoning equilibrium will tend to shift in favour of higher concealed inflation. The firm facing inelastic demand can raise hidden revenue from side payments by restricting output. This case is more consonant with the stylised facts of a shortage economy proposed by Shleifer and Vishny. But in our case the firm, while restricting output to increase hidden revenues, must still satisfy the planners. It does so by diverting effort released by external resources siphoned from the secondary market to hiding inflation rather than producing output. In figure 10 the outcome at C is more hidden inflation and less real output.

These cases suggest that when siphoning was repeated successive increases in plan tension would no longer necessarily shift the firm's equilibrium towards higher levels of real output. This is because the firm's point of maximum hidden revenues was fixed by the market demand curve. Other things being equal, a rise in plan tension shifted the firm's no–siphoning equilibrium to the right, and at the same time the probability increased that the no–siphoning equilibrium would drift to the right of the point where the firm's hidden revenues were maximised. At this point the firm might rationally choose to curtail any further increase in real output which would drive down hidden revenues and reduce siphoned resources. This placed an ultimate limit on the ability of the planner to raise plan tension so as to maintain productive effort and raise real output when siphoning was repeated.

Other things being equal, however, an increase in plan tension would reduce the scope for both corruption and siphoning. This is evident from figure 8: the firm may extract a net surplus of hidden revenues only when market demand exceeds planned revenues. An increase in the firm's planned output requires the firm to engage in some combination of producing more output and driving up its official price more vigorously, and the result is to limit the firm's corruption opportunities possibilities. In short, when siphoning was repeated and firms responded to the resulting possibilities with adverse effects on real output, regardless of whether managers were loyal or disloyal, *tautening the plan was a mechanism to limit corruption and corruption oriented behaviour*. Conversely, reducing plan tension increased the scope for both corruption and disloyalty.

5. Conclusions

First, we find no evidence that Soviet producers set prices below market–clearing levels so as to collect bribes for personal gain. On the contrary, they engaged in concealed inflation so as to fulfill the plan more easily at prices higher than those foreseen in the plan. Thus the Shleifer–Vishny conjecture does not correspond with significant aspects of producer behaviour under the Soviet command system.

Second, we have presented an analytical framework within which a plan–setter and an effort–setter interact, subject to a hard resource constraint, to determine real output and hidden inflation simultaneously.

Third, there was a clear rationale for plan–setters to permit softening of the firm's resource constraint through siphoning. Managers allocated resources that were gained corruptly to produce more real output with less hidden inflation and fulfill the plan more honestly as a result. Self–interested managers, however corrupt, always remained at least partly loyal to the goals of planners and workers. Moreover, by increasing plan tension plan–setters could influence disloyal managers to recycle more hidden revenues into production. Thus, tautening the plan limited managerial disloyalty.

Fourth, we have qualified these results for the case of siphoning repeated in continuous time. In this case higher real output was not the invariable consequence. But raising plan tension remained an effective mechanism for limiting the scope for corruption and adverse corruption–oriented behaviour.

Finally, our findings reflect on the relationship between corruption and shortage. We are far from convinced by the argument that the shortage economy was created with the purpose of distributing bribes into private pockets. It is clear that corruption was enabled as one consequence of the shortage economy, and that it was tolerated by planners for their own purposes. Corruption was commonly part of a process that aligned the objectives of planners and producers, rather than a process of private enrichment involving disloyalty as well as corruption. However, planners could influence the scope for both corruption and disloyalty through the degree of plan tension, and we argue that reductions in plan tension in the 1970s contributed to the dissipation of rents in the Soviet economy.

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Appendix. The Effort–Setting Curve

Symbols

a	production technology
e	effort in production
i	effort in concealment of inflation
k	capital
l	leisure
p	official price
q	real output
t	plan target
x	inflation concealment technology

Plan–setting and effort–setting

Consider a firm that operates subject to two constraints. The first constraint is the *PS* or plan–setting line: its real output at official prices must match its target in rubles at plan prices, $p_{-1} \cdot q^*$, deflated by the current official price level:

A1. $$q = \frac{1}{p} \cdot \left(p_{-1} \cdot q^* \right).$$

Second is a resource constraint, where the firm's time endowment is normalised to 1:

A2. $$1 - e - i - l = 0.$$

The firm has two activities. Production requires capital and effort. To begin with only effort is variable; effort is also subject to diminishing returns:

A3. $$q = a \cdot \bar{k}^{1-a} \cdot e^a, \quad 0 < a < 1.$$

The concealment of inflation in the price at which the firm's plan target is to be fulfilled requires only effort, again subject to diminishing returns:

A4. $$\frac{p - p_{-1}}{p_{-1}} = x \cdot i^b, \quad 0 < b < 1,$$

where p is concealed from the plan–setter in the current period.

For reasons given in the text the firm's problem, given the plan–setter's line, is reduced to allocating effort between production and inflation concealment so as to fulfil the plan with minimum overall effort. This point is found where *PS*, the plan–setting line (equation A1), is tangential to the lowest effort–setting line or *ES* (from A2, A3, and A4):

A5. $$q = a \cdot \bar{k}^{1-a} \cdot \left[1 - l - \left(\frac{p - p_{-1}}{x \cdot p_{-1}} \right)^{\frac{1}{b}} \right]^a.$$

The *ES* curve

Proposition 1. *The tangential line of the ES curve is negative. In addition, the ES curve is strictly concave: the marginal rate of substitution decreases as p increases.*

Proof:

The partial derivative of q with respect to p is less than zero:

$$\frac{\partial q}{\partial p} = a \cdot \bar{k}^{1-a} \cdot a \left[1 - l - \left(\frac{p - p_{-1}}{x \cdot p_{-1}} \right)^{\frac{1}{b}} \right]^{a-1} \cdot \left(-\frac{1}{b} \right) \left(\frac{p - p_{-1}}{x \cdot p_{-1}} \right)^{\frac{1}{b}-1} \left(\frac{1}{x \cdot p_{-1}} \right).$$

Set $A = \left[1 - l - \left(\frac{p - p_{-1}}{x \cdot p_{-1}} \right)^{\frac{1}{b}} \right] > 0$, and $B = \left(\frac{p - p_{-1}}{x \cdot p_{-1}} \right) > 0$.

Then $\frac{\partial q}{\partial p} = -\frac{a \cdot \bar{k}^{1-a} \cdot a}{x \cdot b \cdot p_{-1}} A^{a-1} \cdot B^{\frac{1-b}{b}}$

and $\frac{\partial q}{\partial p} < 0$.

Moreover, the second derivative of q with respect to p is also negative:

$$\frac{\partial^2 q}{\partial p^2} = -\frac{a \cdot \bar{k}^{1-a} \cdot a \cdot (a-1)}{x \cdot b \cdot p_{-1}} \cdot \left(\frac{1}{x \cdot p_{-1}} \right)^2 \cdot A^{a-2} \cdot \left(-\frac{1}{b} \right) \cdot B^{\frac{1-b}{b}} \cdot \left(\frac{1-b}{b} \right) \cdot B^{\frac{1-2b}{b}}$$, where $a < 1$

and $\frac{\partial^2 q}{\partial p^2} < 0$, suggesting that the *ES* curve is strictly concave to the origin.

Proposition 2. *A reduction in leisure expands the ES curve; the new curve lies outside the old curve at all points.*

*Proof**:***

This proposition is investigated through the effects on the *p*– and *q*–intercepts of the *ES* curve arising from a decline in the value of l. The *p*–intercept is found by setting $q = 0$ in equation A5:

A6. $$p = p_{-1} \cdot \left[x \cdot (1 - l)^b + 1 \right].$$

Similarly the *q*–intercept is found from equation A5 by setting $p = p_{-1}$:

A7. $$q = a \cdot \bar{k}^{1-a} \cdot (1 - l)^a.$$

Other things being equal, when l declines, both the *p*– and *q*–intercepts increase: the *ES* curve expands in all directions.

Proposition 3. *An improvement in production (concealment) technology shifts the ES curve toward output (concealed inflation).*

*Proof**:***

Consider equations A6 and A7. When a increases and other variables are held equal, the *q*–intercept increases while the *p*–intercept stays the same: the *ES* curve is right–shifted toward higher real output. A similar proof applies to the converse case of an increase in x when other variables are controlled.

Effort–Setting and Siphoning: a Single Transaction

Let the firm siphon capital–augmenting resources Δk from the retail market so its production function becomes:

A8. $$q = a \cdot \left(\bar{k} + \Delta k\right)^{1-a} \cdot e^{a} .$$

The *ES* curve with siphoning (the *ES'* curve) becomes:

A9. $$q = a \cdot \left(\bar{k} + \Delta k\right)^{1-a} \cdot \left[1 - l - \left(\frac{p - p_{-1}}{x \cdot p_{-1}}\right)^{\frac{1}{b}}\right]^{a} .$$

Proposition 4. *The ES' curve is concave to the origin: the marginal rate of substitution decreases as p increases.*

Proof:

Because Δk is predetermined, its influence on equation A9 compared with equation A5 is purely scalar. Therefore, the proof suggested for proposition 1 is also applicable to this proposition. In other words, the signs of the first and second derivatives of q with respect to p for the *ES'* curve are the same as those for the *ES* curve.

Proposition 5. *Siphoning shifts the ES curve toward real output.*

Proof:

Because Δk is predetermined, its influence on equation A9 compared with equation A5 is identical to that of capital accumulation or a technological improvement, i.e. an increase in $a \cdot k^{1-a}$. Therefore, the proof suggested for proposition 3 is also applicable to this proposition, writing $a \cdot \left(\bar{k} + \Delta k\right)^{1-a}$ for $a \cdot \bar{k}^{1-a}$ in equation A7.