WESS ECONOMETRICS

Exercise Sheet 0

1. A small commuter airline flies planes that can seat up to 8 passengers. The airline has determined the probability that a ticketed passenger will not show up for a flight is 0.15. For each flight, the airline sells tickets to the first 10 people placing orders. The probability distribution for the number of tickets sold per flight is given in the table below:

No. of tickets	6	7	8	9	10
Probability	0.20	0.35	0.20	0.15	0.10

(a) What is the mean number of tickets sold per flight? What is the variance of the number of tickets sold?

(b) If tickets cost £200, what is the expected revenue per flight and what is the variance of the revenue.

(c) Assuming independence between the number of tickets sold and the probability a passenger shows up. For what proportion of flights does the number of ticketed passengers exceed the number of seats (excess demand). Calculate the expected value for the excess demand

(d) Write out the joint probability density function for the excess demand and the number of tickets sold. Calculate the covariance between the excess demand and the number of tickets sold.

2. Define the probability density of the events X_1 and X_2 as:

		x_1			
		1	2	3	4
x_2	1	0.00	0.30	0.20	0.10
	2	0.15	0.15	0.10	0.00

(a) Calculate (i) $\Pr(X_1 > 2)$ and (ii) $\Pr(X_2 = 1)$.

(b) Calculate (i) $\Pr(X_1 > 2 \mid X_2 = 1)$ and (ii) $\Pr(X_2 = 1 \mid X_1 > 2)$.

(c) Calculate (i) $\Pr(X_1 - X_2 = 0)$ and (ii) $\Pr(X_1 + X_2 \geq 4)$.

(d) A random sample of 2 is taken from the variable X_1 write out the joint probability distribution of these two random variables.

(e) Write out the probability distribution of the sample mean from the random sample of 2 obtained in (d).

3. The random variables X and Y have the following bivariate probability distribution:

		x		
		-1	0	1
	0	0.15	0.1	0.15
y	1	0.2	0.0	0.15
	2	0.15	0.1	0.0

(a) Write out the marginal distributions of *X* and *Y*.

(b) Calculate the E(*X*) and E(*Y*).

(c) Calculate the V(*X*) and V(*Y*) and cov(*X*,*Y*).

(d) Calculate the E(*Y*|*X*=-1) and V(*Y*|*X*=-1).

(e) Calculate the $\text{cov}(X, Y \mid Y \geq 1)$.

4. Let the random variable, Z, follow a standard normal distribution:

(a) Find $P(Z > -1.00)$

(b) Find $P(-1.70 < Z < 1.20)$

(c) Find a such $P(Z < a) = 0.70$

(d) Find a such $P(Z < a) = 0.25$

5. A lecturer found that time spent by students on an exercise sheet follow a normal distribution with mean 60 minutes and standard deviation 18 minutes.

(a) The probability is 0.9 that a randomly chosen student spends more than how many minutes on this exercise sheet.

(b) The probability is 0.8 that a randomly chosen student spends less than how many minutes on this exercise sheet.

(c) Two students are chosen at random. What is the probability that on average they spend at least 75 minutes on the exercise sheet?