sAMPle COMPression

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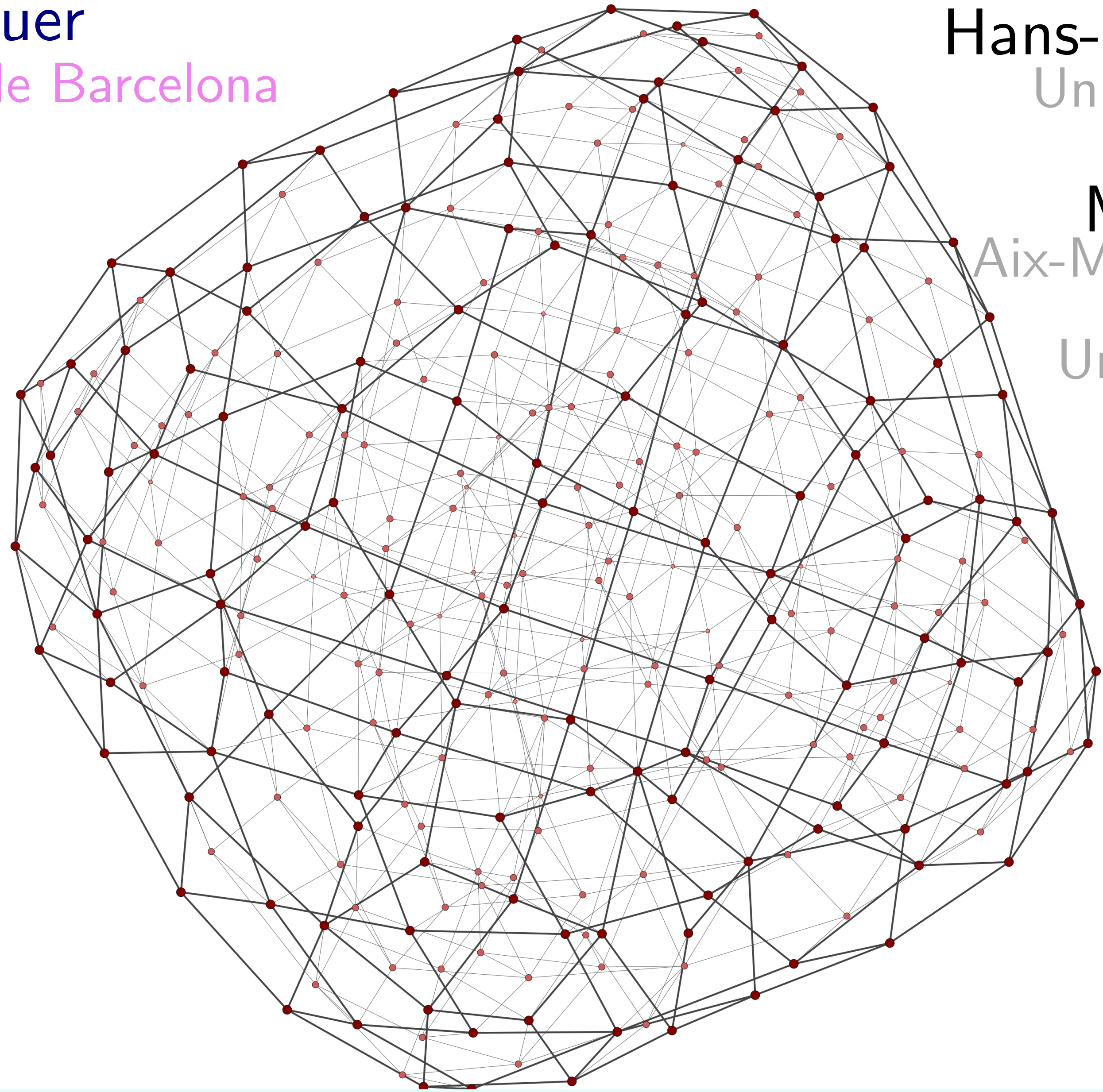
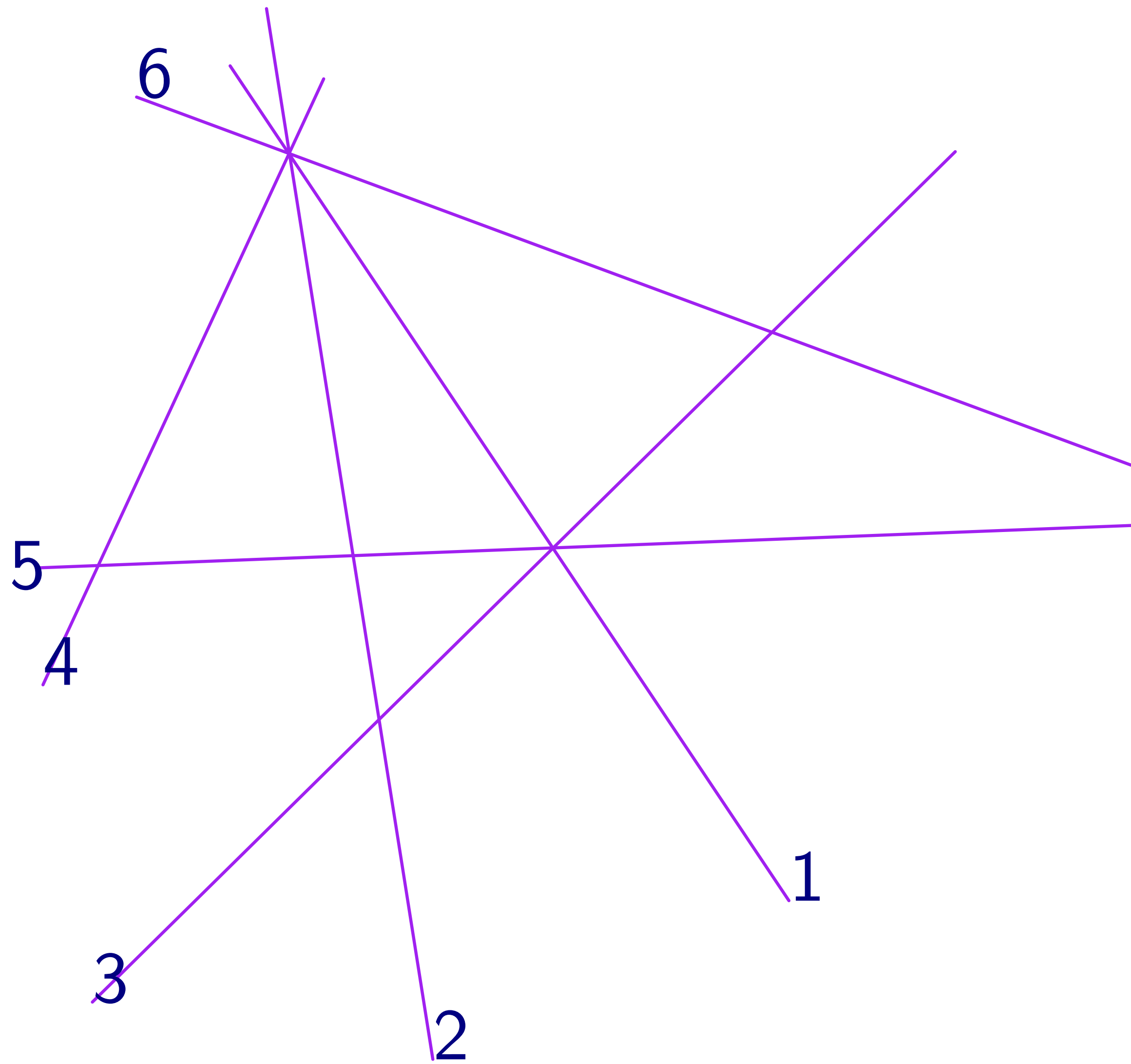
Univerza v Ljubljani



Warwick 07/02/2022

the *realizable* setting

(affine) hyperplane arrangement $\mathcal{H} = \{H_e \mid e \in E\}$ in $\mathbb{R}^d$



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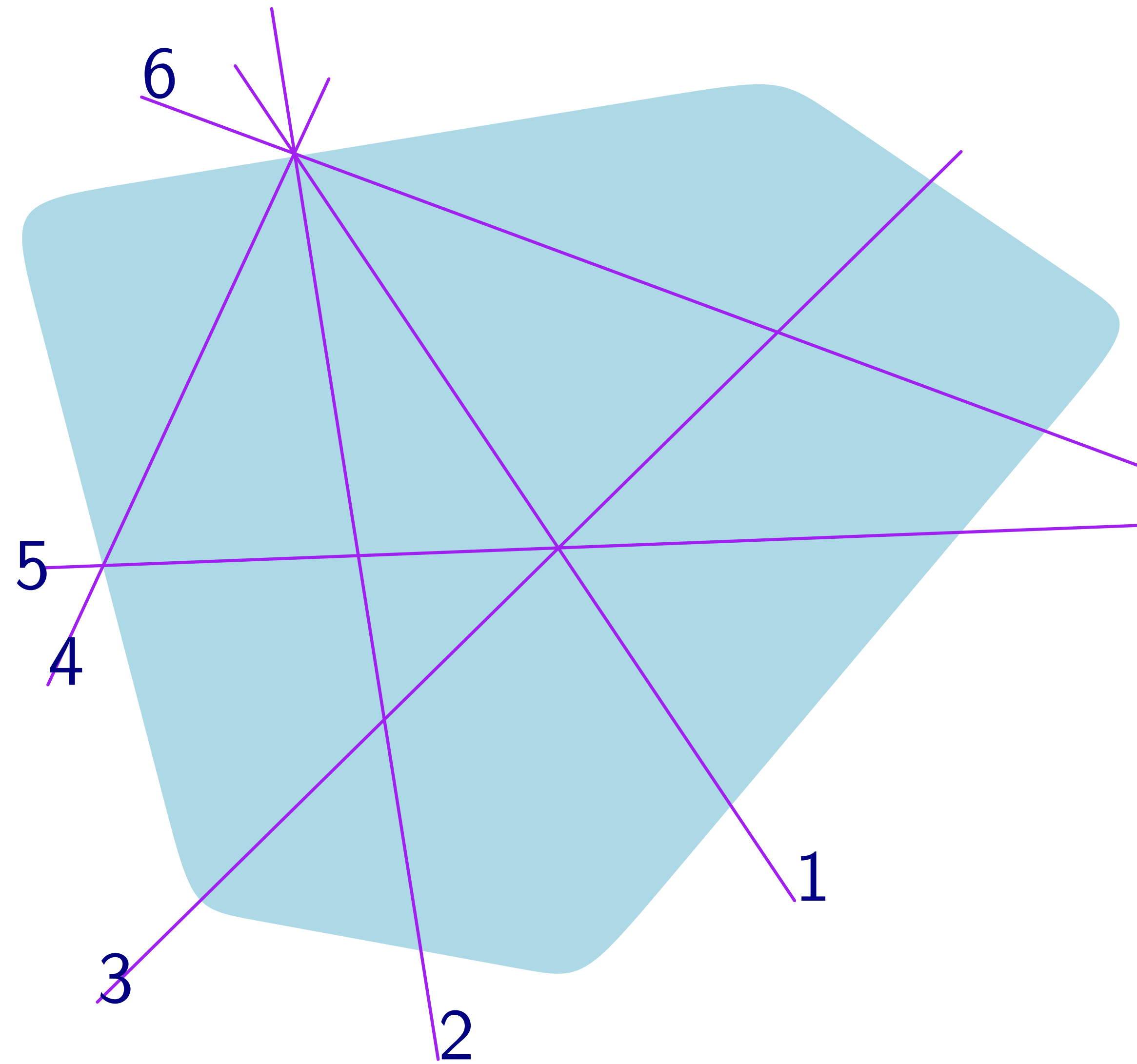
(affine) hyperplane arrangement $\mathcal{H} = \{H_e \mid e \in E\}$ in $\mathbb{R}^d$
intersect with open convex K



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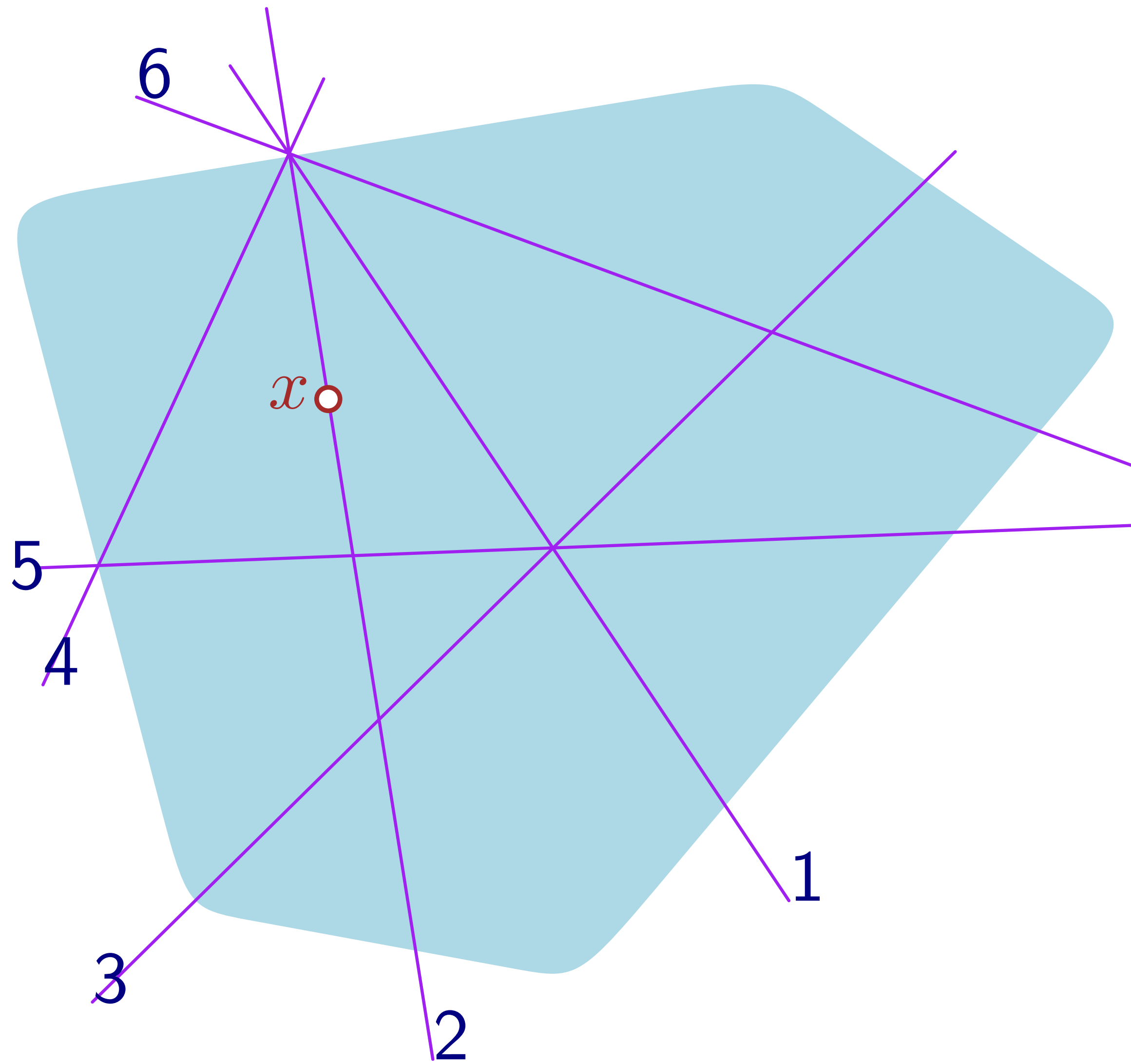
code position $x \in K$ relative to $\mathcal{H}$



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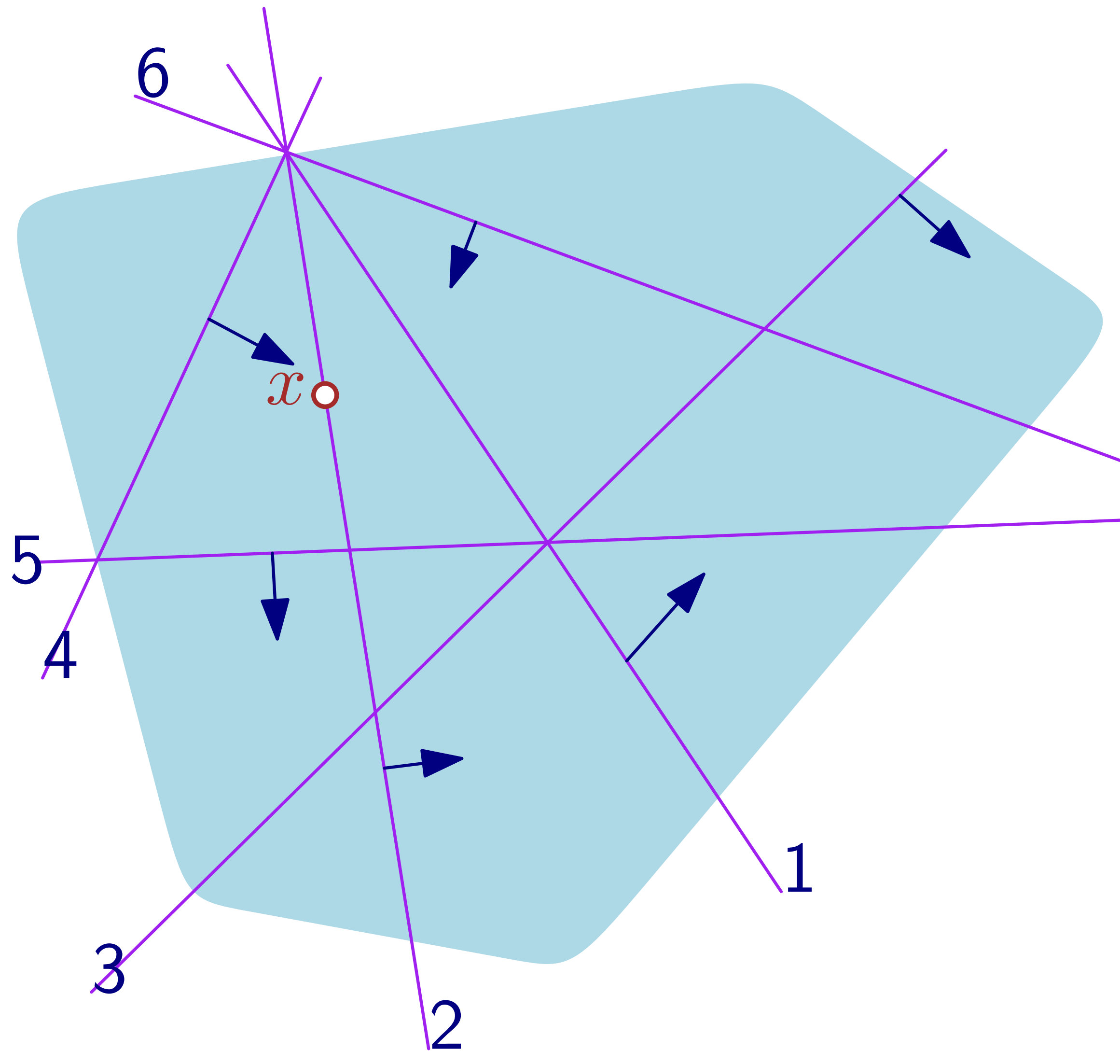
code position $x \in K$ relative to $\mathcal{H}$
$H_e \in \mathcal{H}$ splits K in two halfs (positive and negative)



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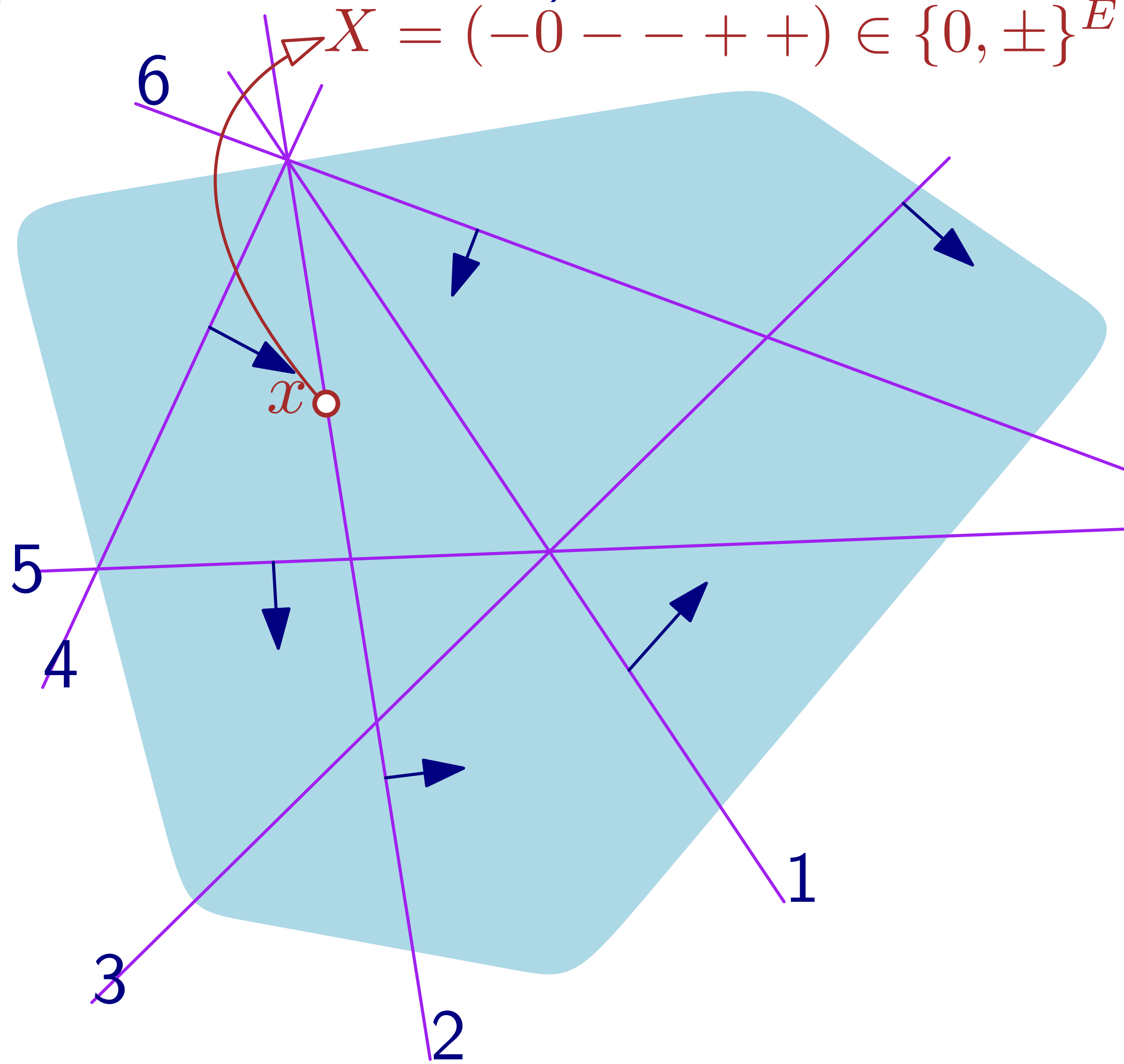

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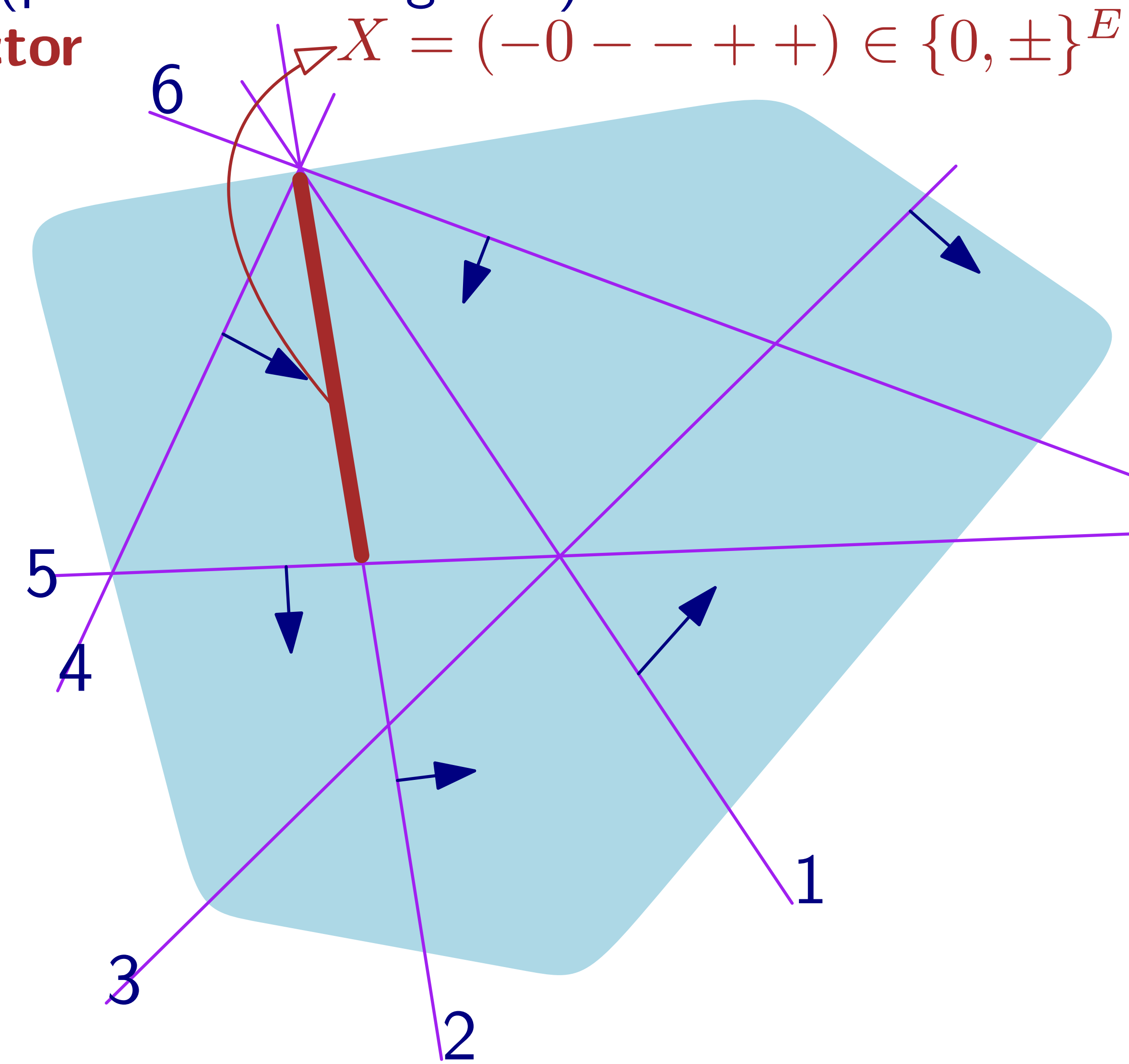
assign to each *cell* a **sign-vector**



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- $H_e \in \mathcal{H}$ splits K in two halfs (positive and negative)
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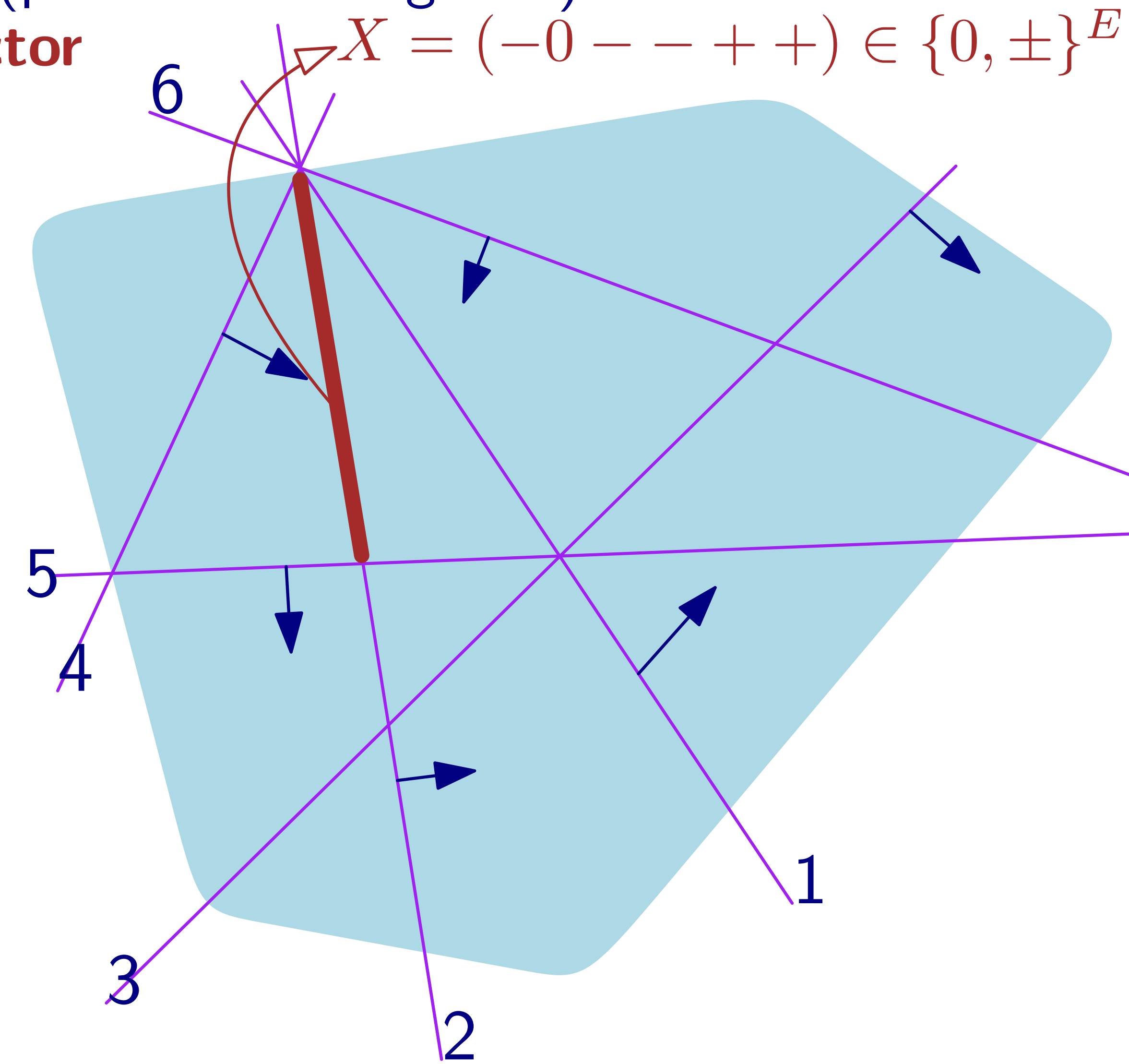
- $\rightsquigarrow$ **covectors** $\mathcal{L} \subseteq \{0, \pm\}^E$



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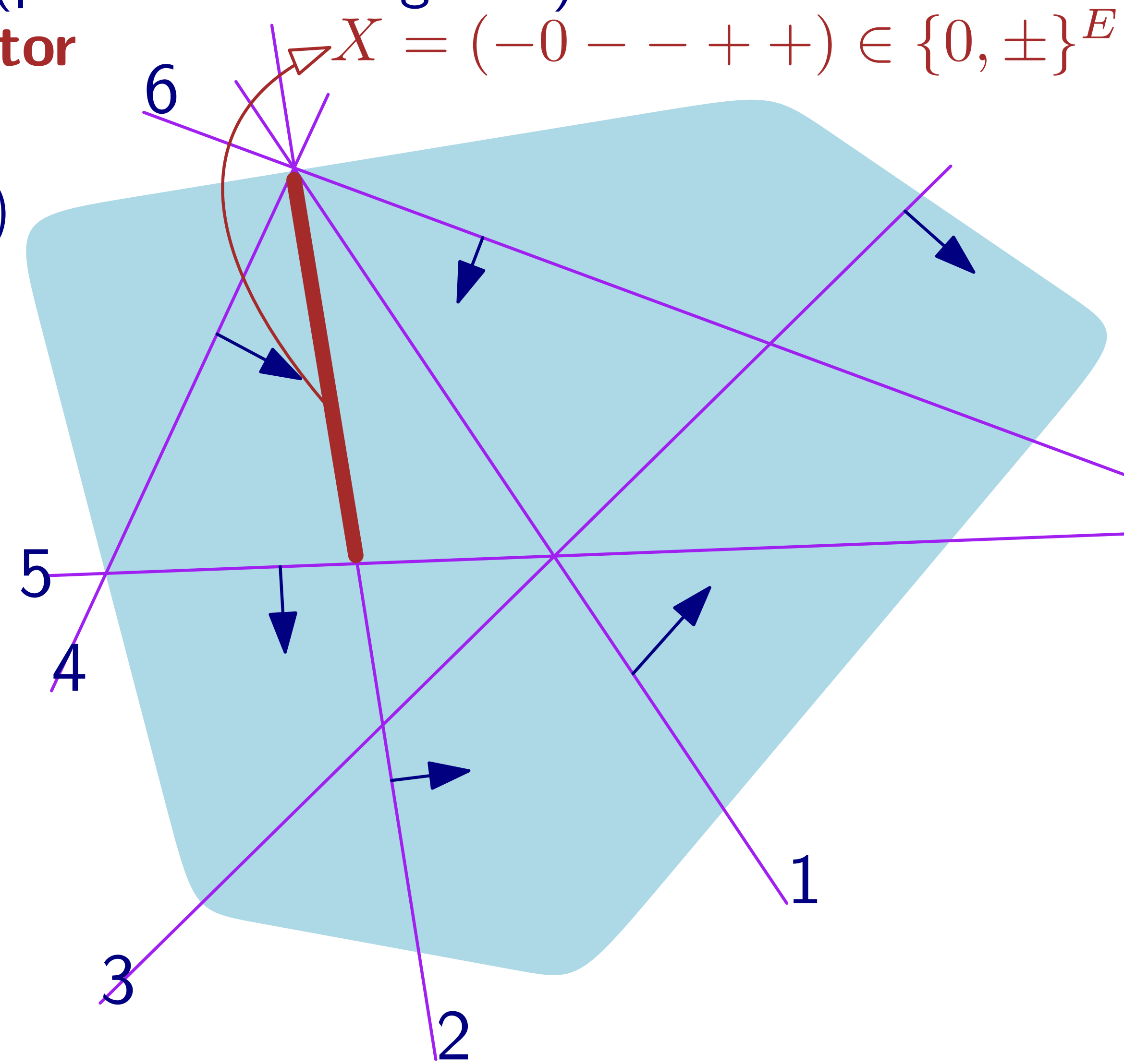

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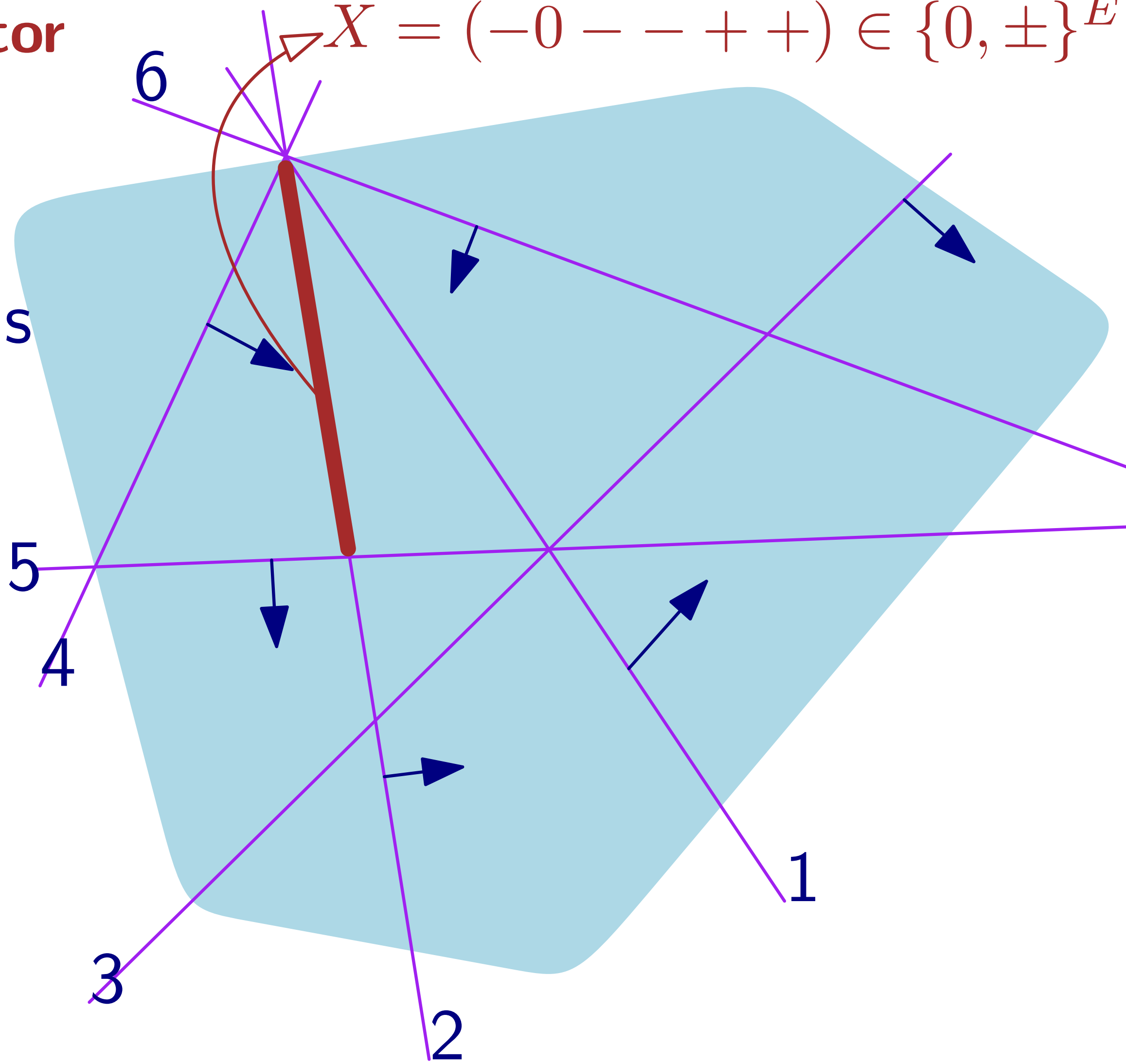
Complex of **O**riented **M**atroids



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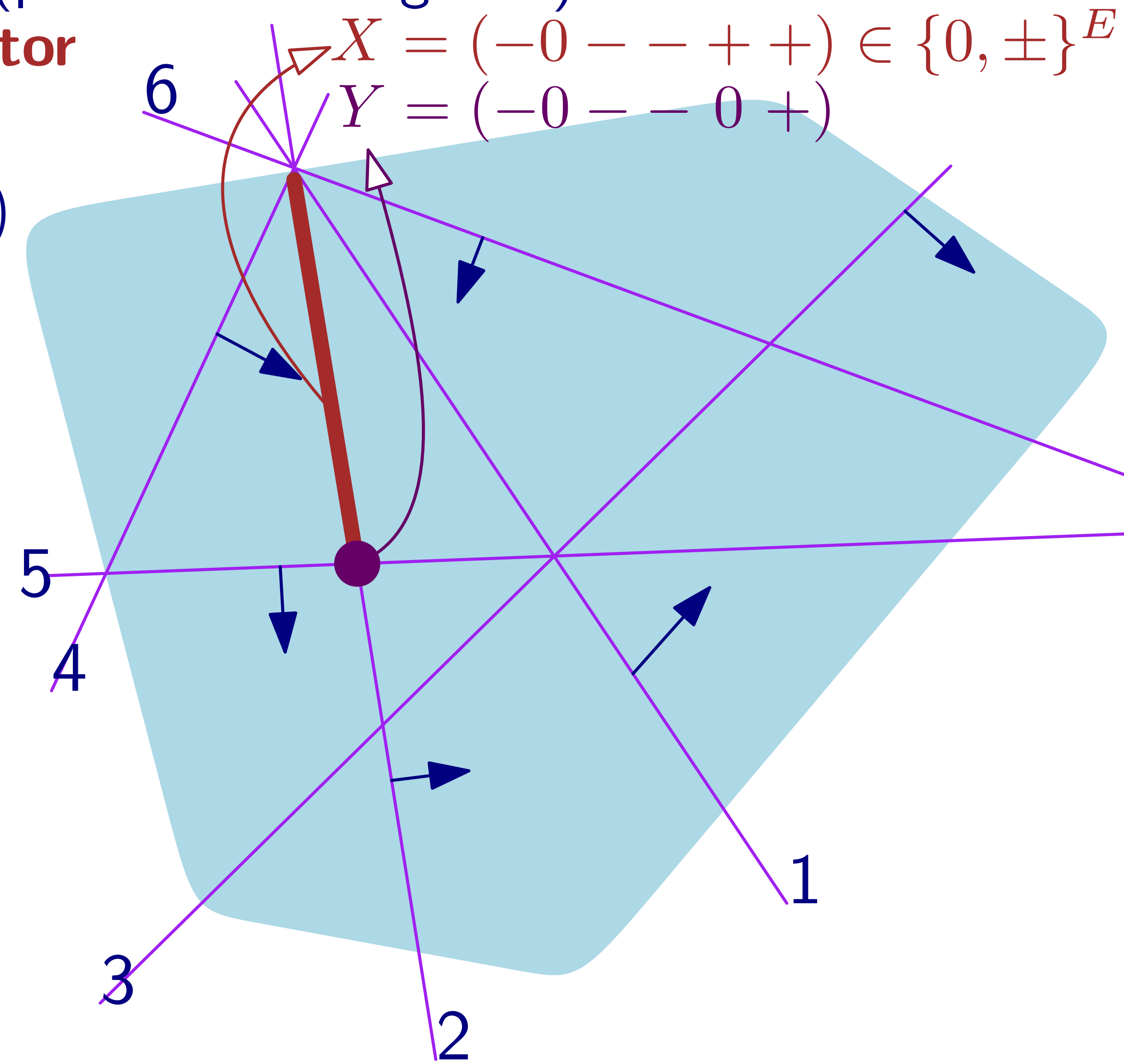

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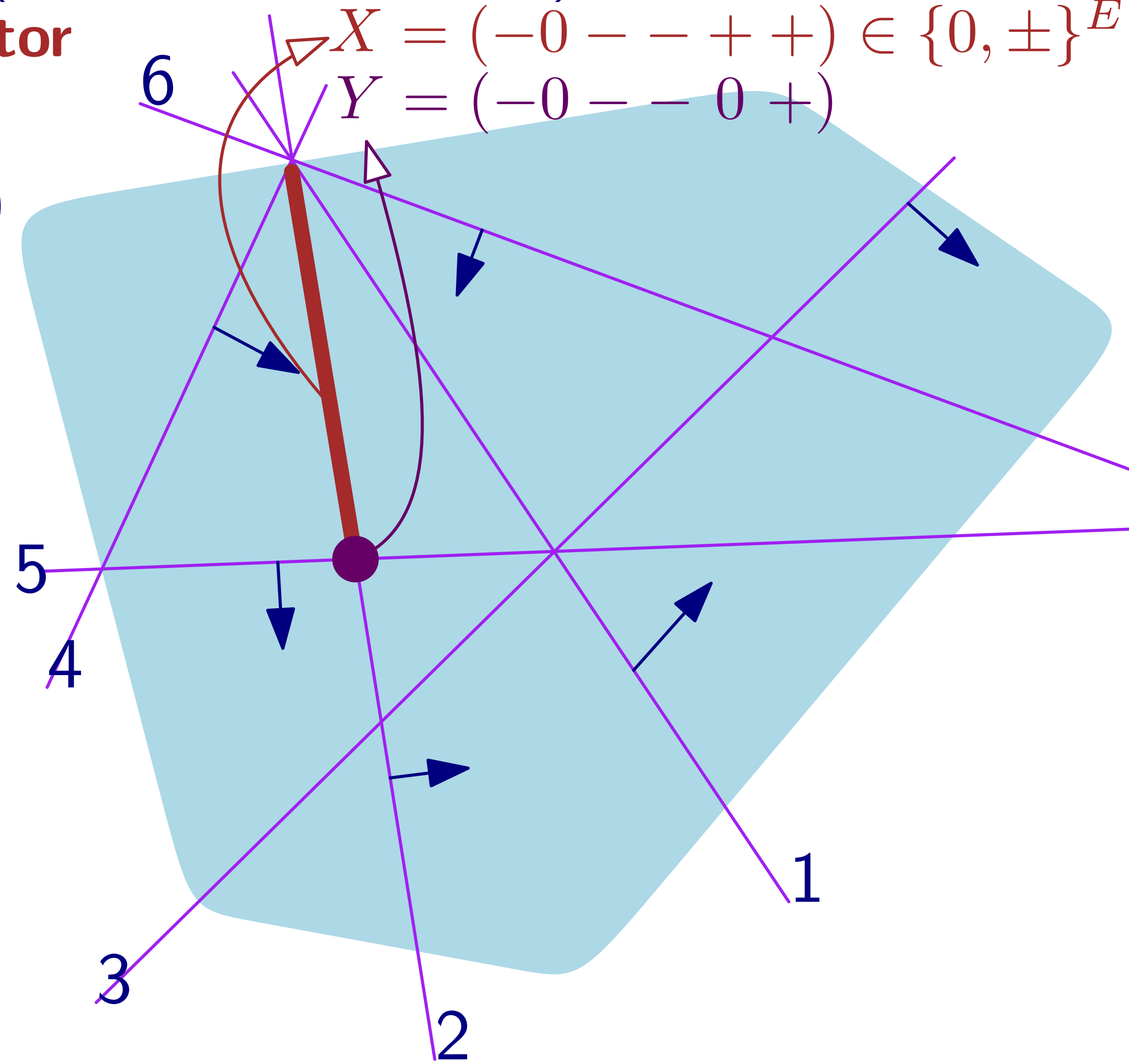
$\rightsquigarrow Y \subseteq X \Leftrightarrow Y \leq X$

as cells as sign-vectors $(0 < +, -)$



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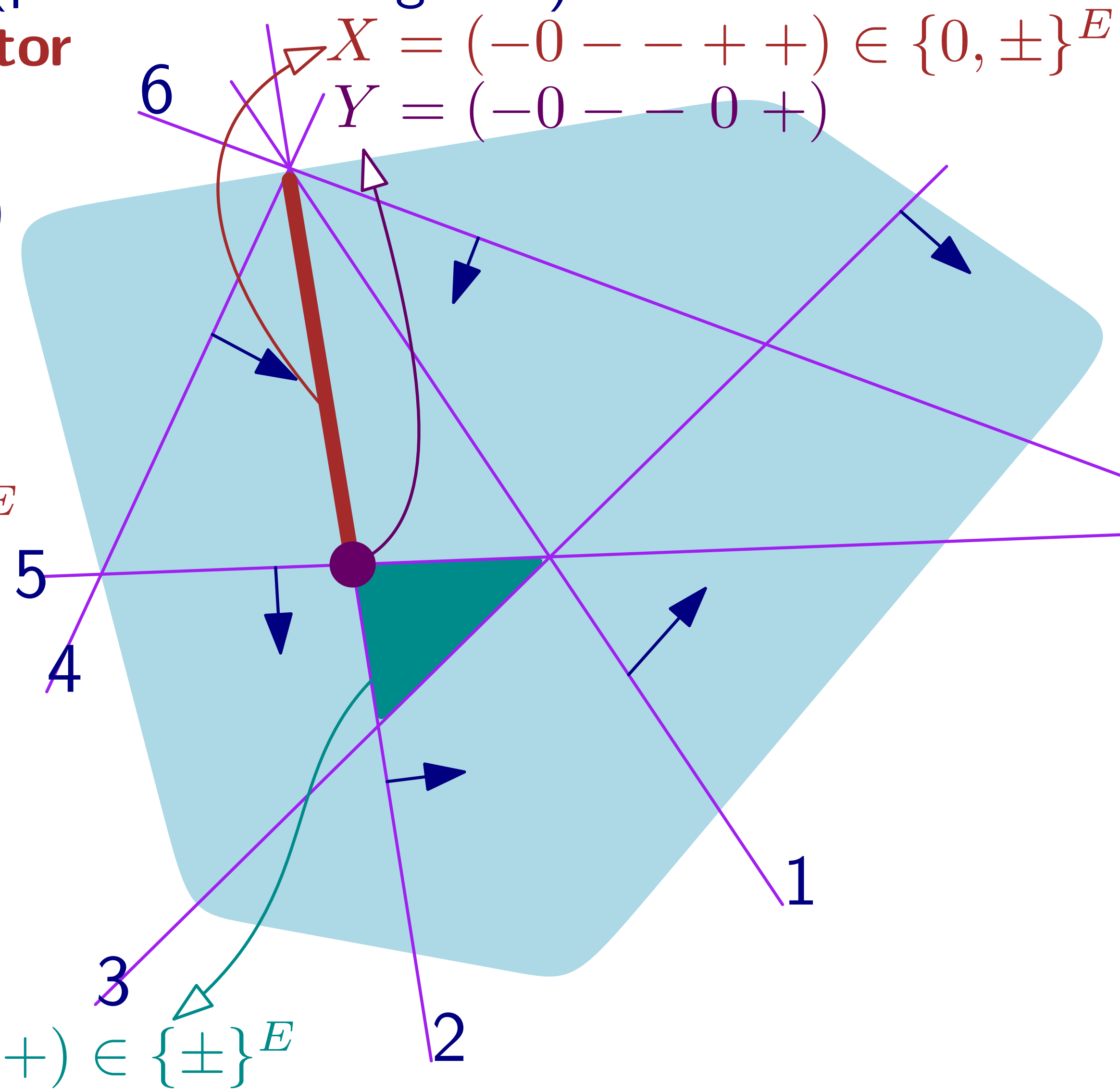

$T = (- + - + + +) \in \{\pm\}^E$

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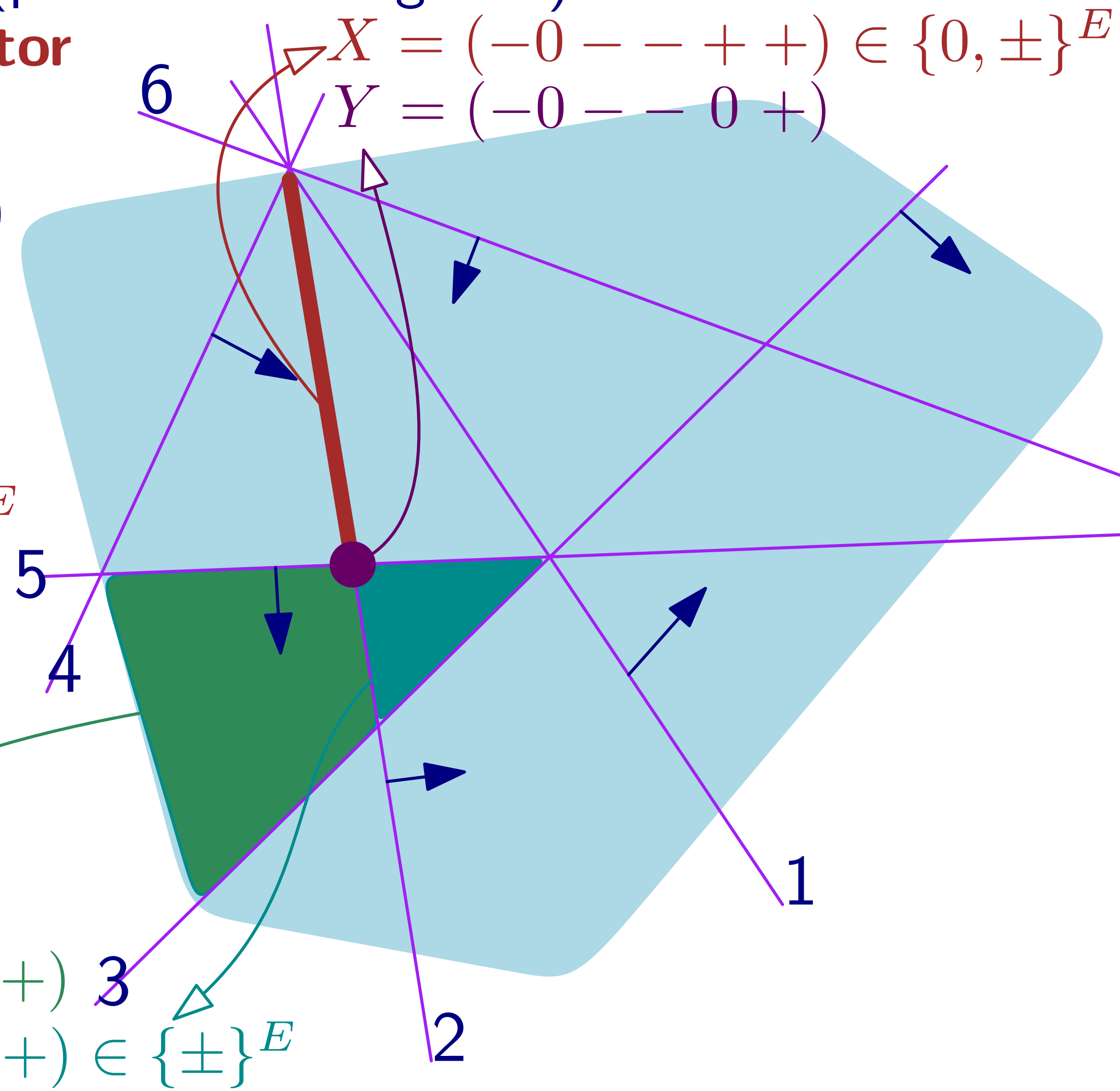

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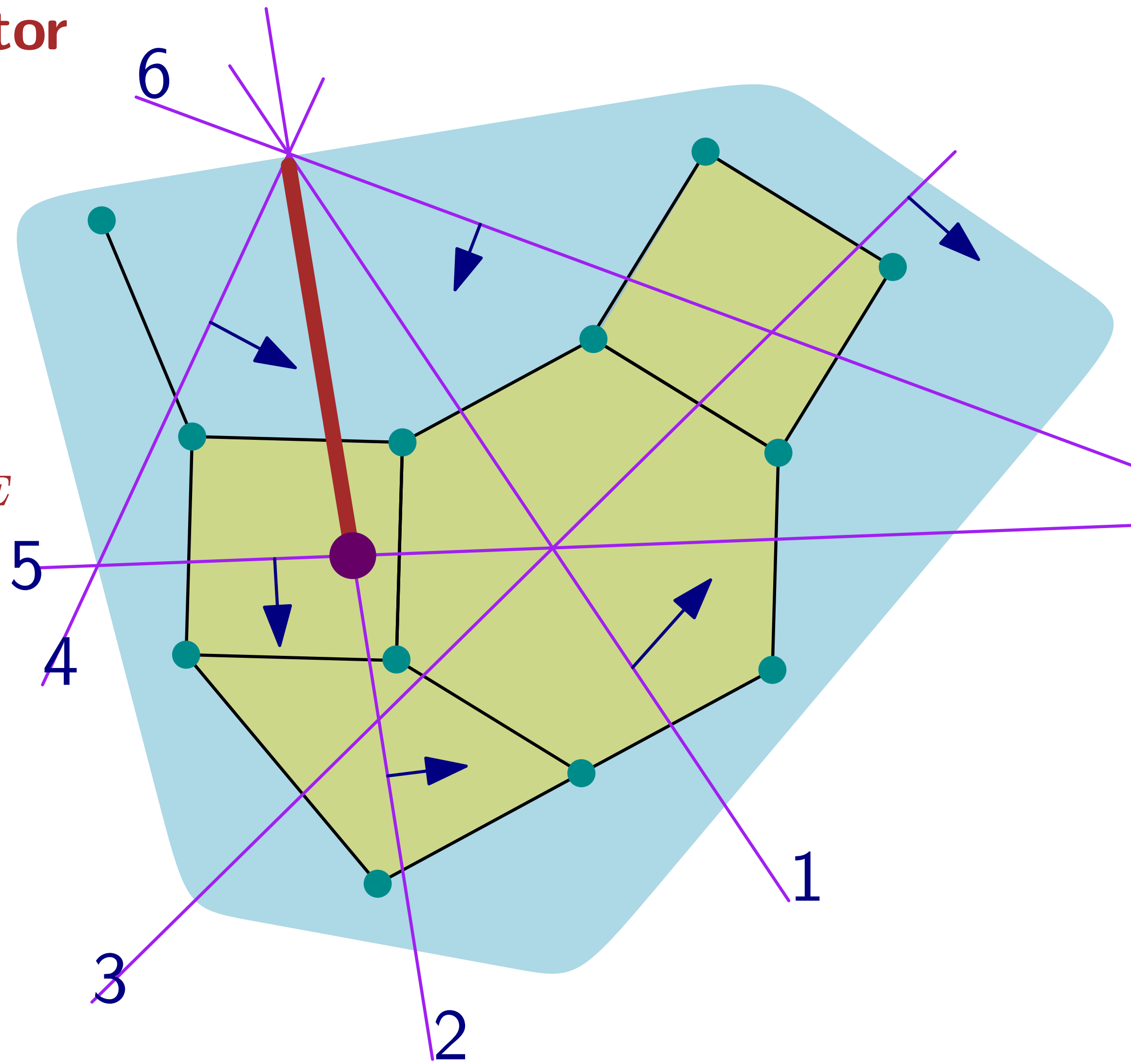

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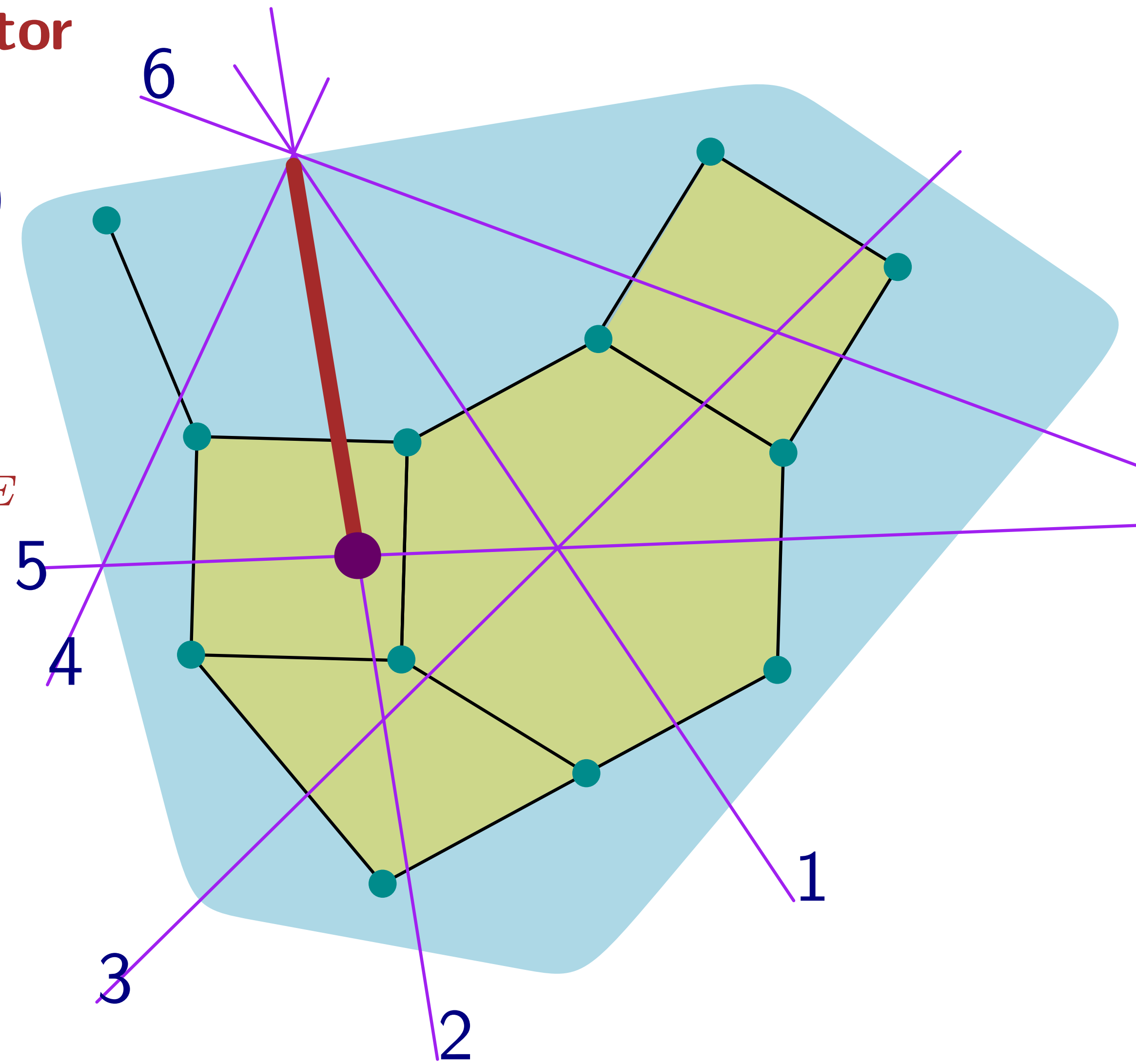

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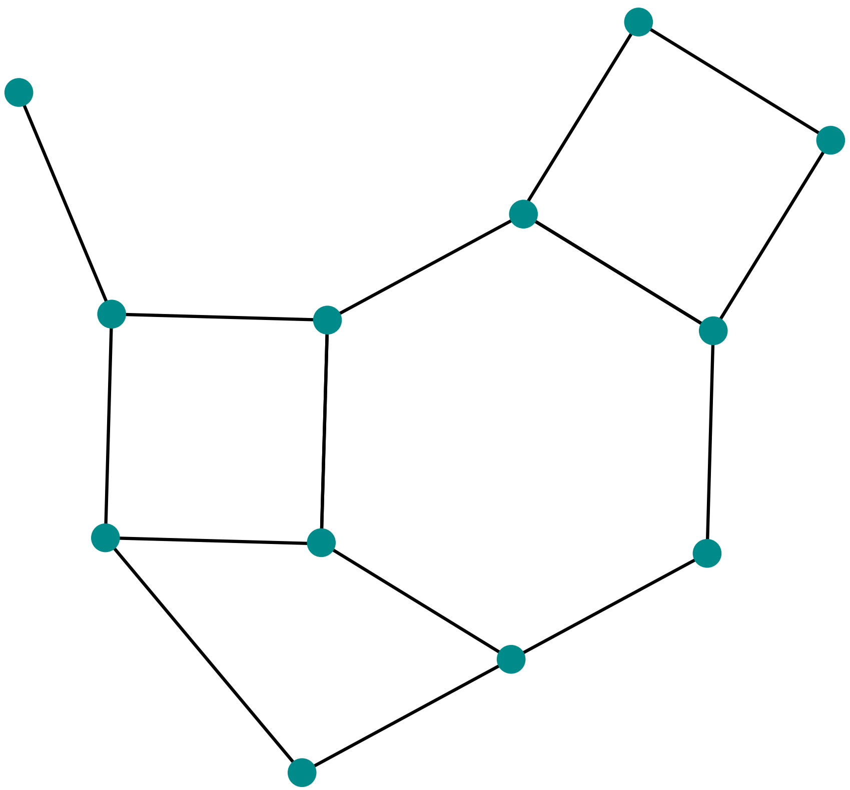

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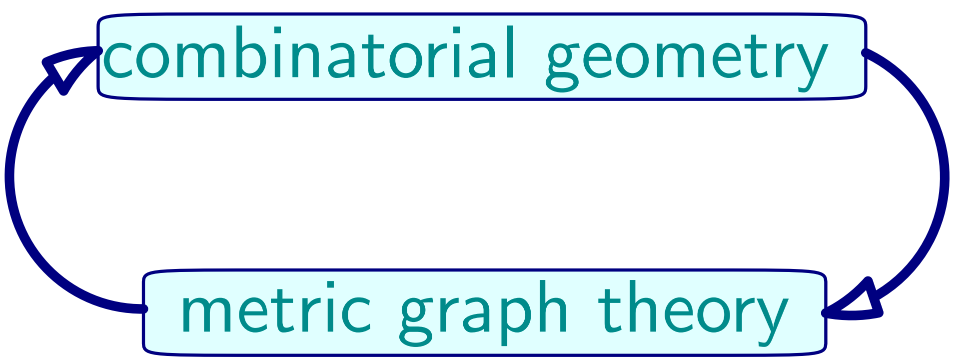

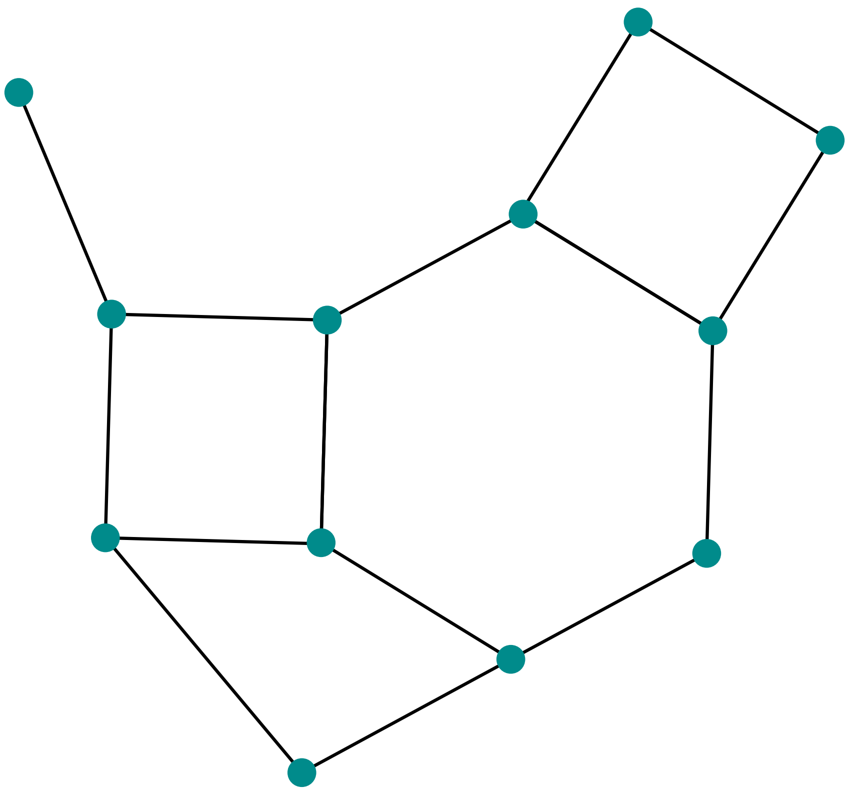

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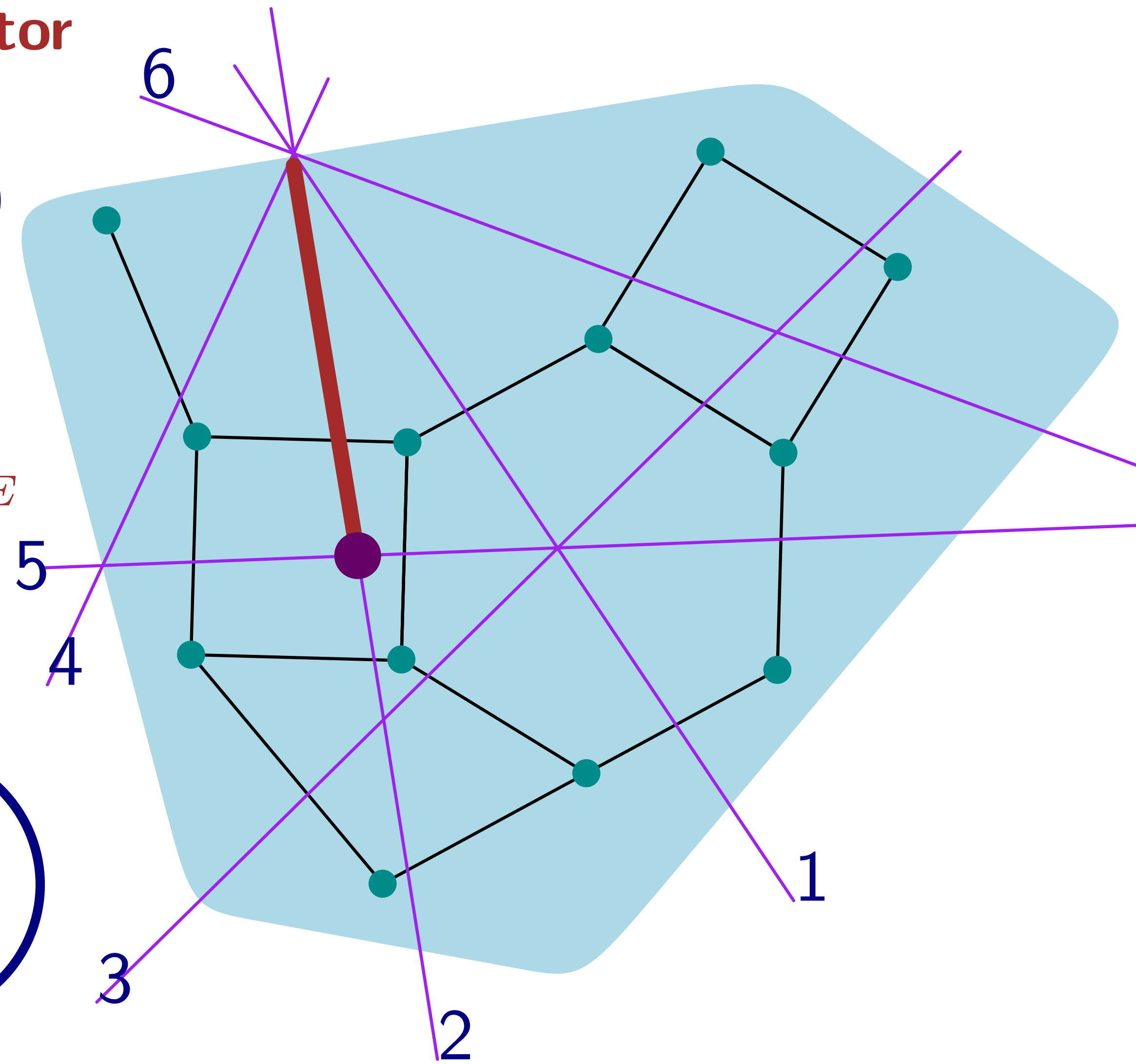
combinatorial geometry

metric graph theory



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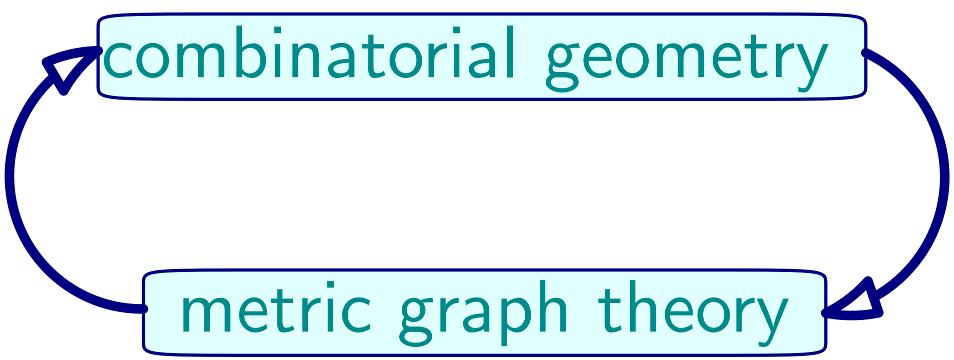

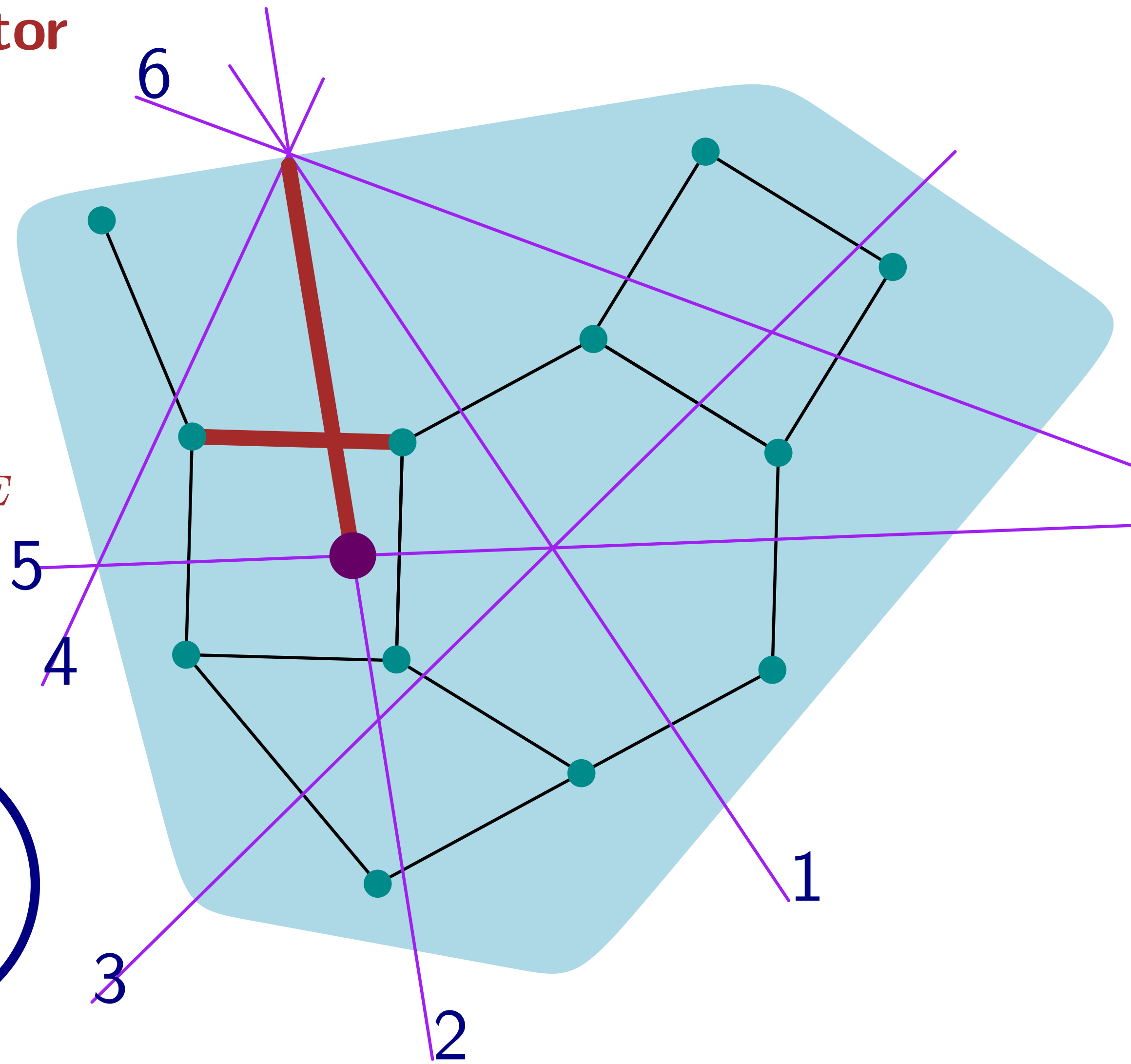

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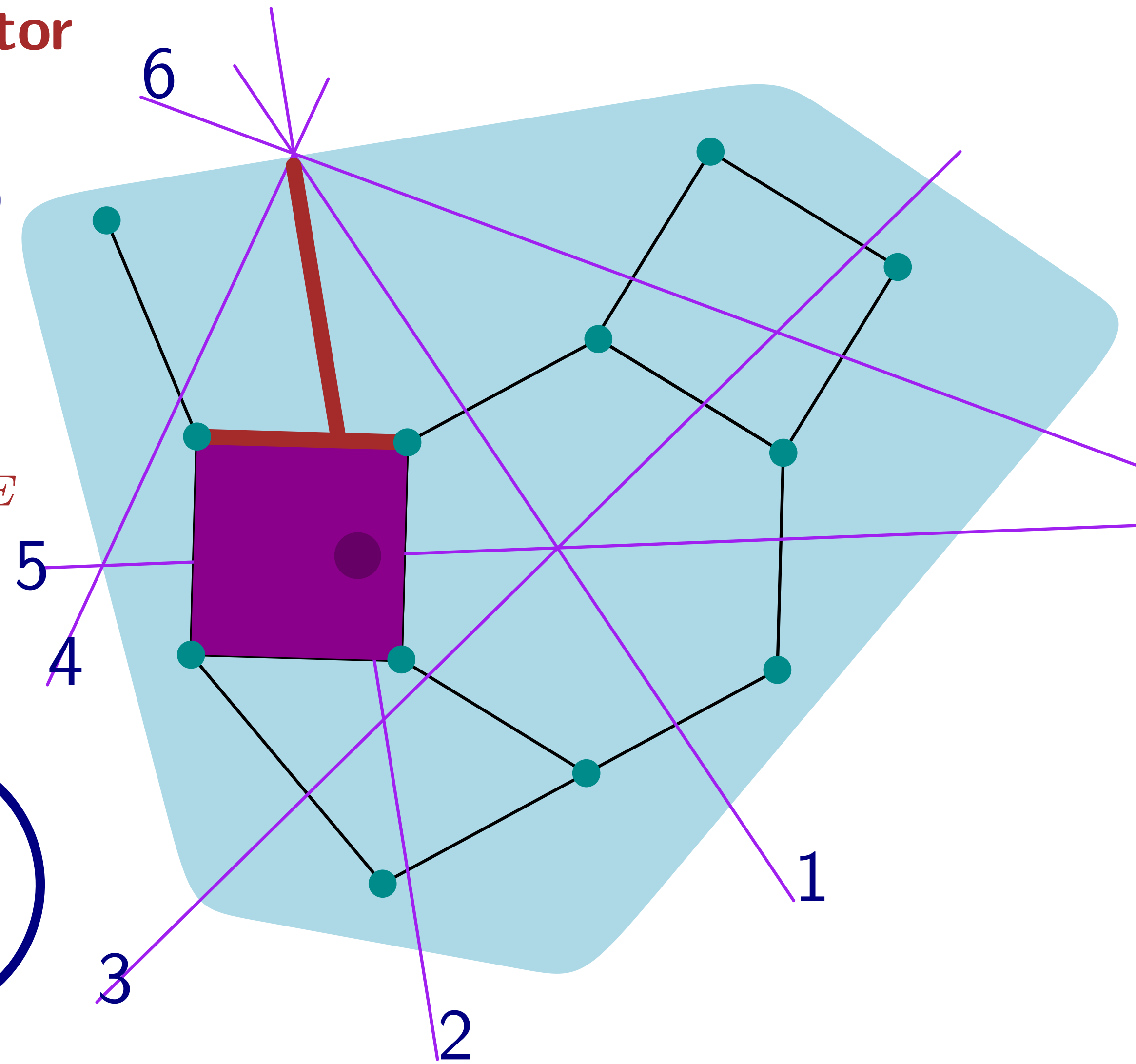

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$\rightsquigarrow Y \subseteq X \Leftrightarrow Y \leq X \Leftrightarrow Y \supseteq X$

as cells | as sign-vectors $(0 < +, -)$ | as subgraphs

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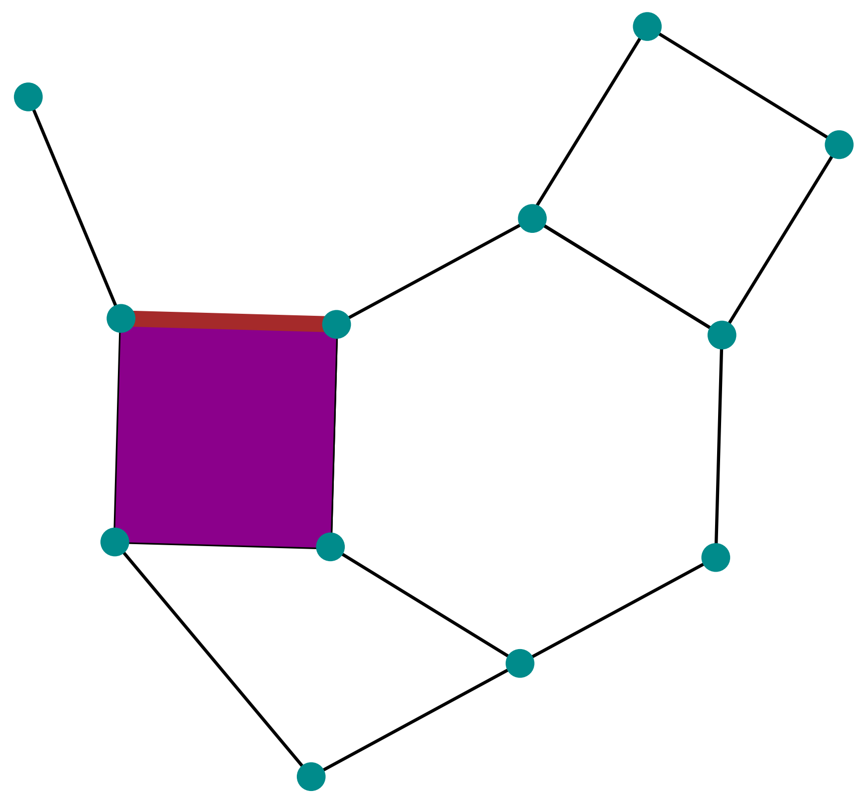
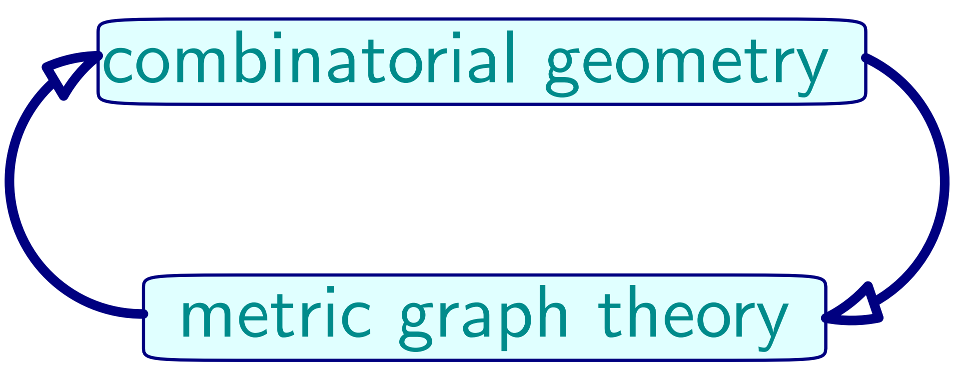
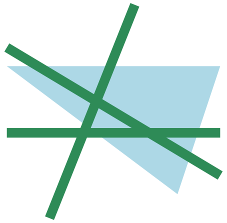
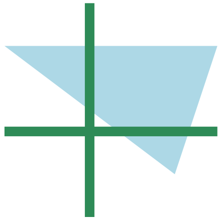




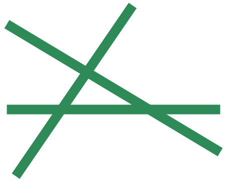
special cases of realizability



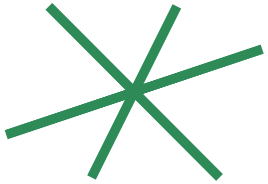
affine arrangement in $\mathbb{R}^d$ intersected with open convex
$\rightsquigarrow$ *complex of oriented matroids (COM)* (Bandelt, Chepoi, K '18)



coordinate hyperplanes in $\mathbb{R}^d$ intersected with open convex
$\rightsquigarrow$ *ample set systems (AMP)* (Lawrence '83)



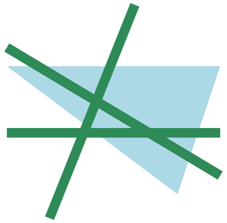
affine arrangement $\mathbb{R}^d$
$\rightsquigarrow$ *affine oriented matroid (AOM)* (Edmonds, Fukuda, Mandel '82)



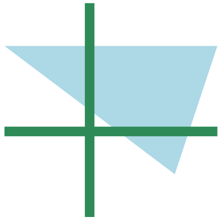
central arrangement in $\mathbb{R}^d$
$\rightsquigarrow$ *oriented matroid (OM)* (Bland, Las Vergnas '78)

special cases of realizability

alcoved triangulations
graphic COMs
acyclic orientations of mixed graphs
ranking COMs and linear extension graphs
Fibonacci cubes



affine arrangement in $\mathbb{R}^d$ intersected with open convex
$\rightsquigarrow$ *complex of oriented matroids (COM)* (Bandelt, Chepoi, K '18)



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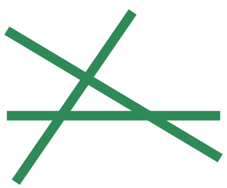

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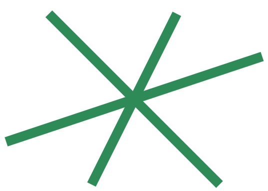
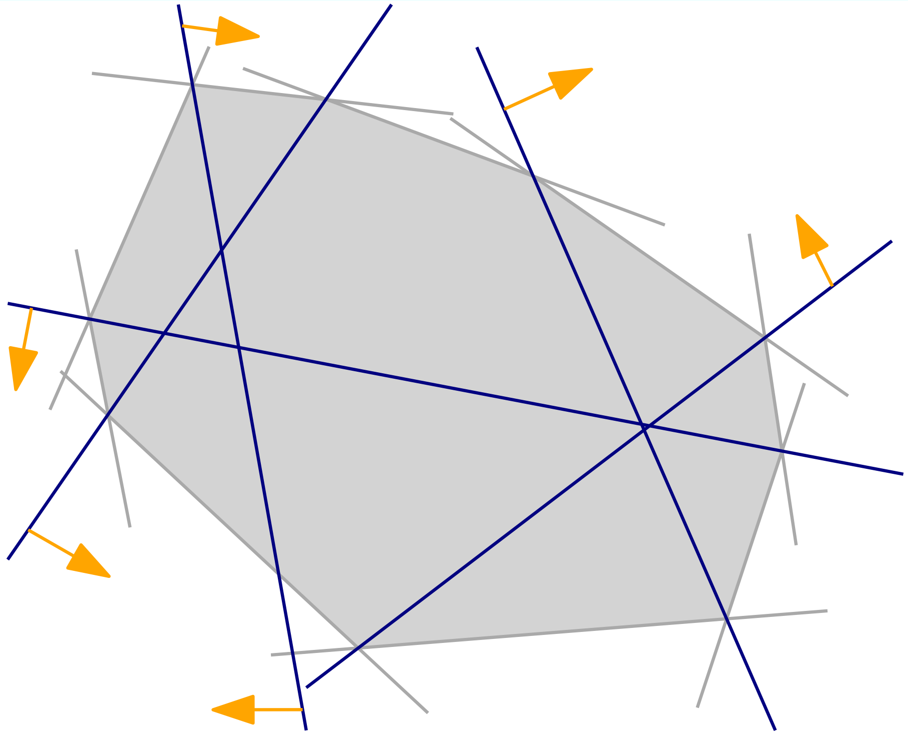


central arrangement in $\mathbb{R}^d$
$\rightsquigarrow$ *oriented matroid (OM)* (Bland, Las Vergnas '78)

axioms for sign vectors



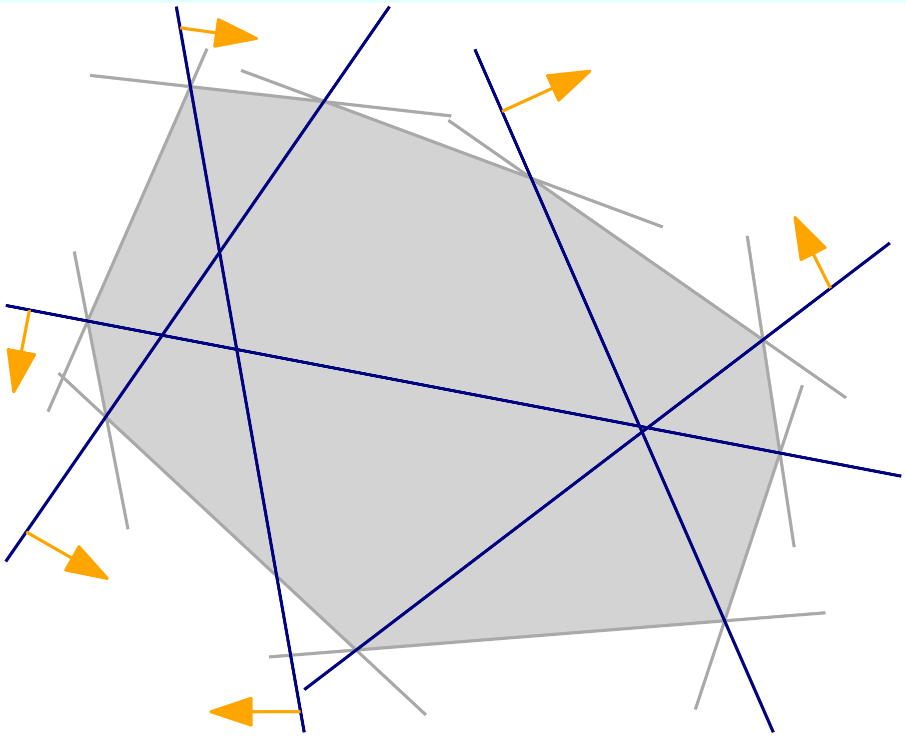
◦ Covector axioms: $\mathcal{M} = (E, \mathcal{L})$ **COM**:

(FS) $\mathcal{L} \circ -\mathcal{L} \subseteq \mathcal{L}$

(SE) $\forall X, Y \in \mathcal{L}$ and $e \in S(X,Y) \exists Z \in \mathcal{L}$:

$Z_e = 0$ and $Z_f = X_f \circ Y_f$ for $f \notin S(X,Y)$.

axioms for sign vectors



(FS) $\begin{pmatrix} 0 \\ + \\ - \\ + \end{pmatrix} \circ (- \begin{pmatrix} + \\ + \\ + \\ + \end{pmatrix}) = \begin{pmatrix} - \\ + \\ - \\ + \end{pmatrix}$

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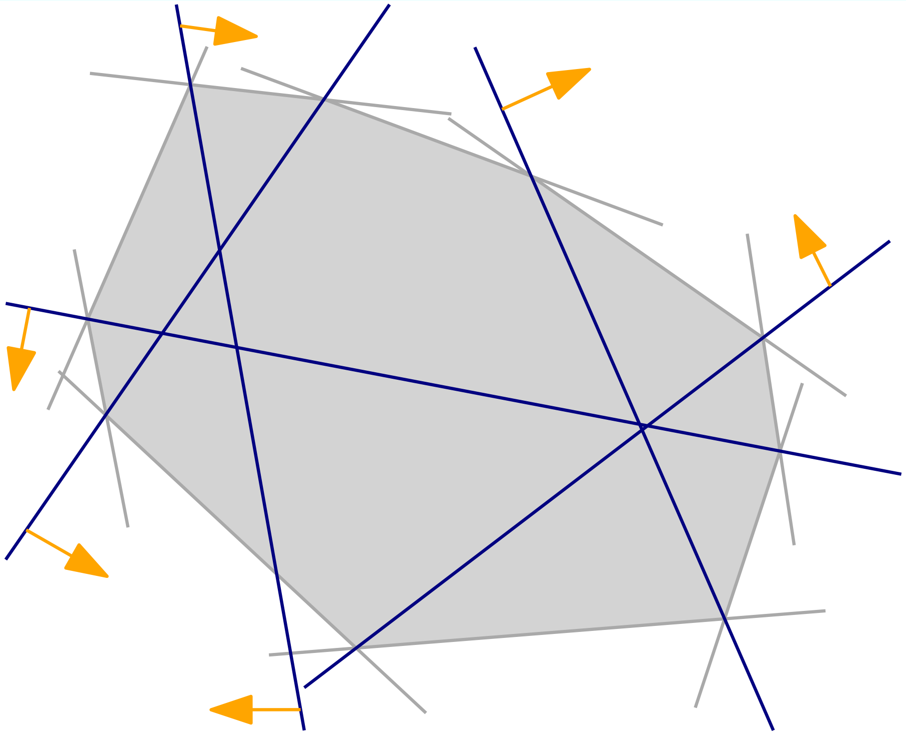
$Z_e = 0$ and $Z_f = X_f \circ Y_f$ for $f \notin S(X,Y)$.

axioms for sign vectors



(FS)

$$\begin{pmatrix} 0 \\ + \\ - \\ + \end{pmatrix} \circ \begin{pmatrix} - \\ - \\ - \\ - \end{pmatrix} = \begin{pmatrix} - \\ + \\ - \\ + \end{pmatrix}$$

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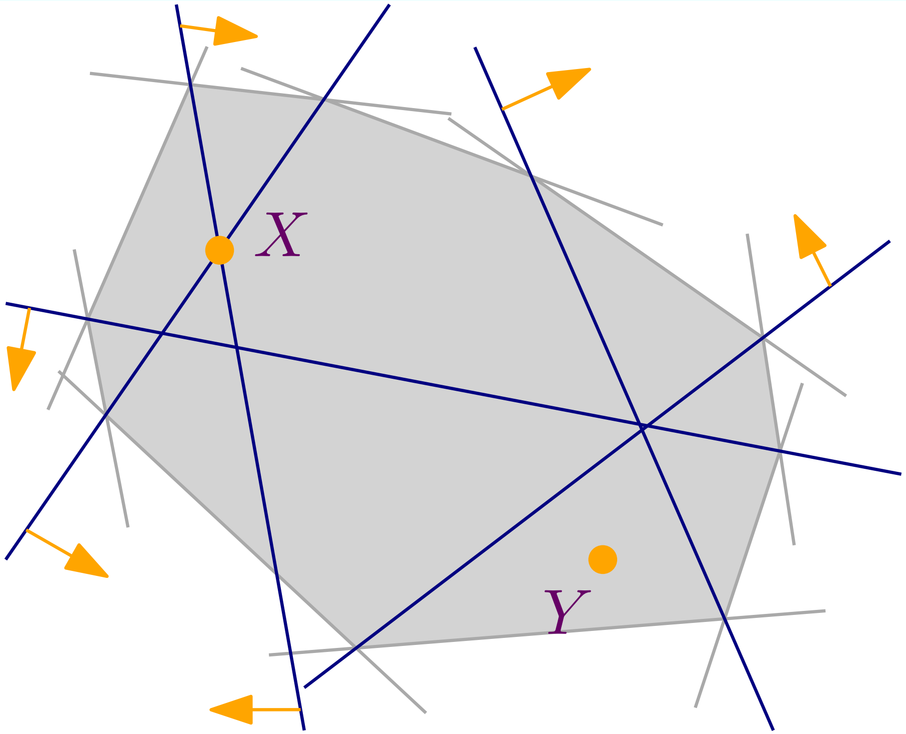

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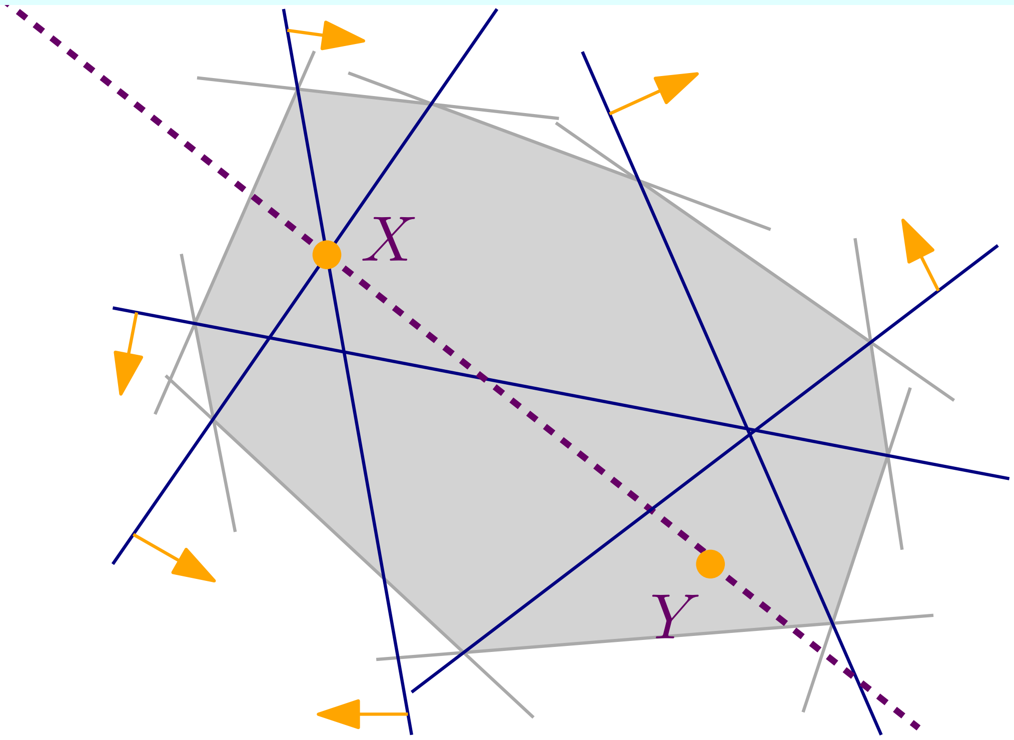

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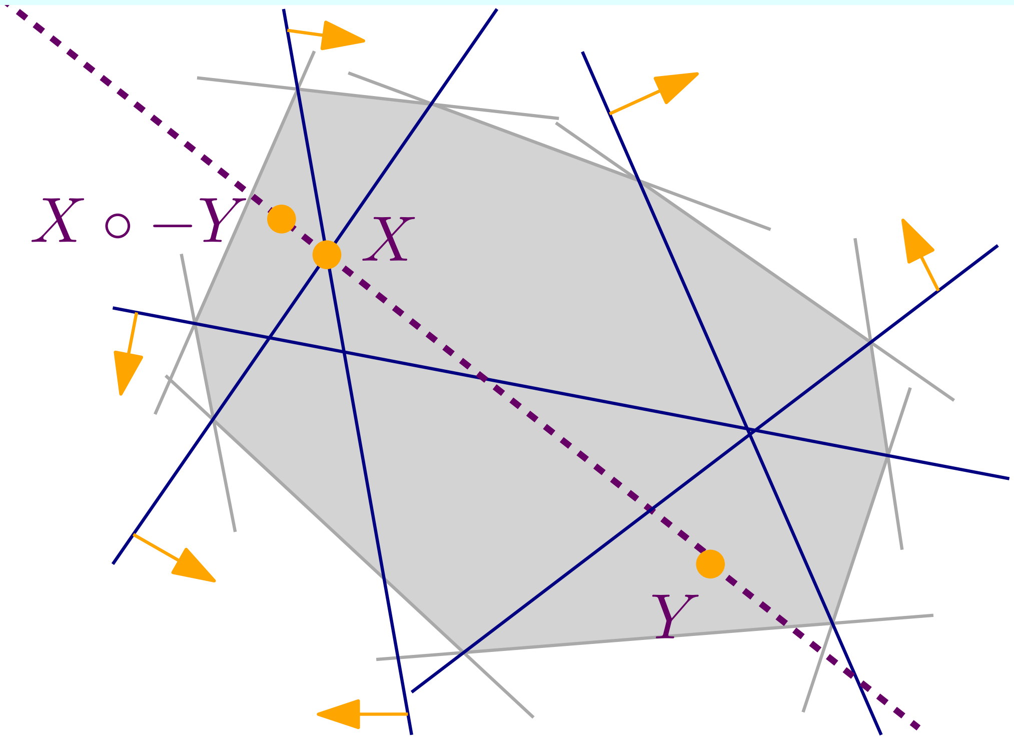

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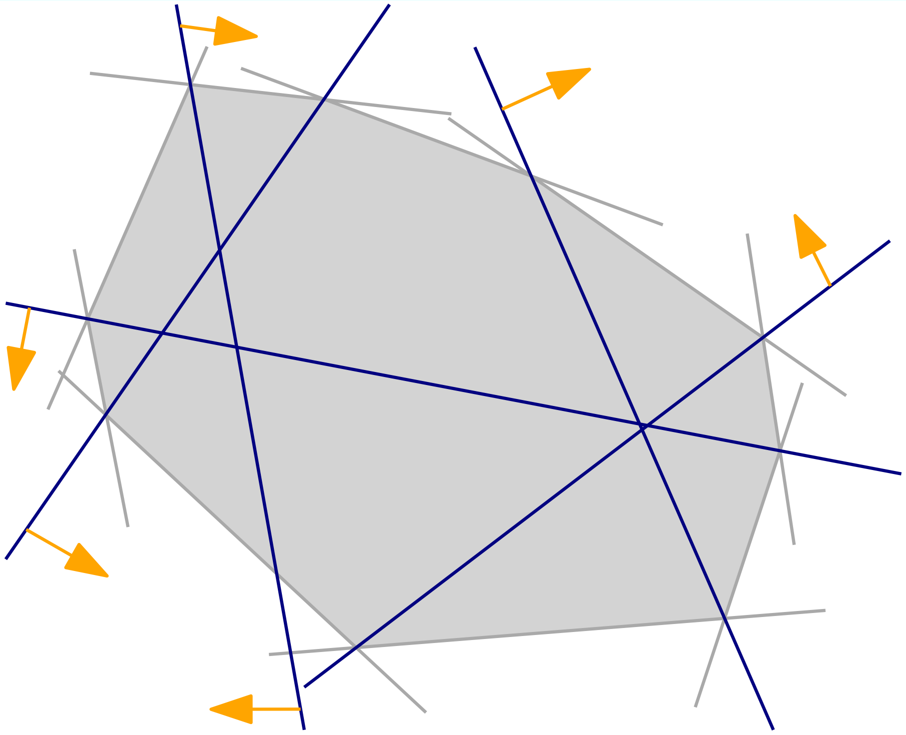

(SE) $\begin{pmatrix} 0 \\ + \\ - \\ + \end{pmatrix}, \begin{pmatrix} - \\ - \\ - \\ - \end{pmatrix}$

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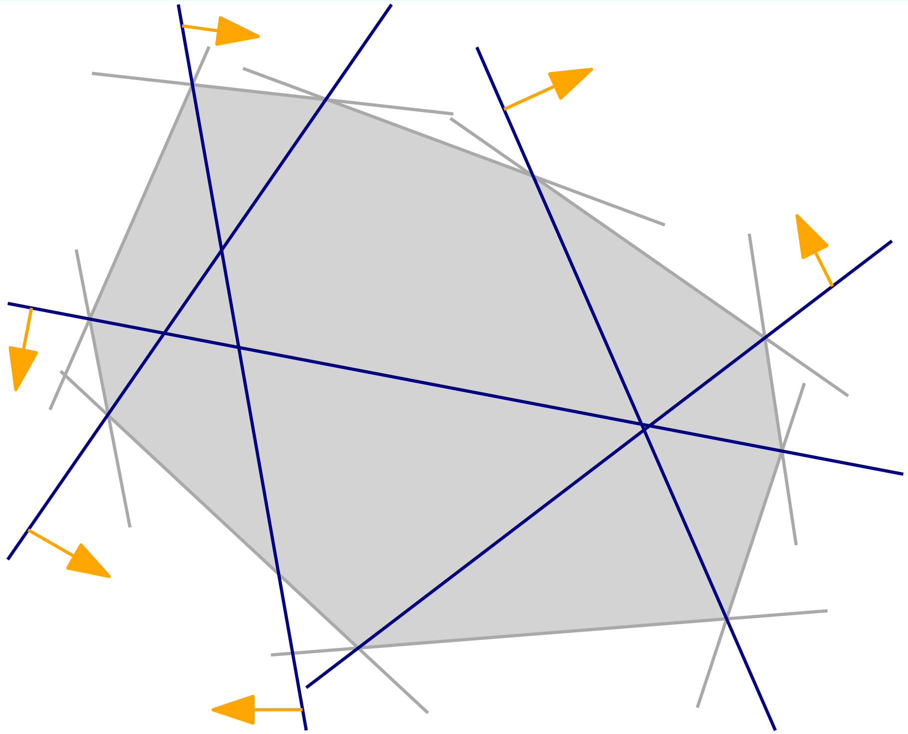
axioms for sign vectors



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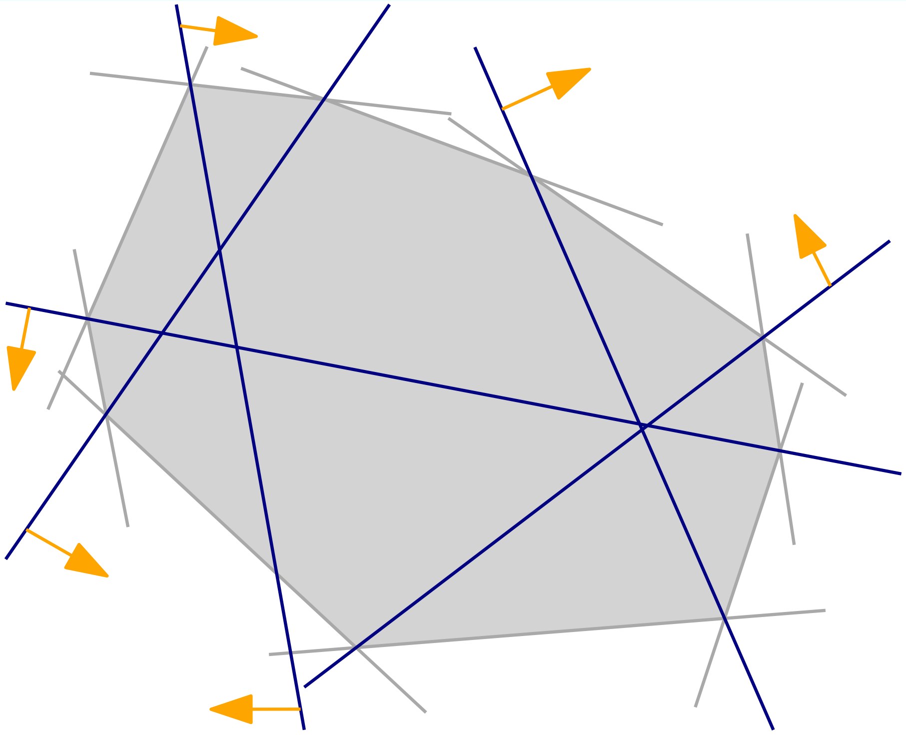

$$\text{(SE)}\quad {\color{red}e}\begin{pmatrix}0\\+\\-\\+\end{pmatrix},\begin{pmatrix}-\\-\\-\\-\end{pmatrix}\quad\rightsquigarrow\quad\begin{pmatrix}-\\0\\-\\?\end{pmatrix}$$

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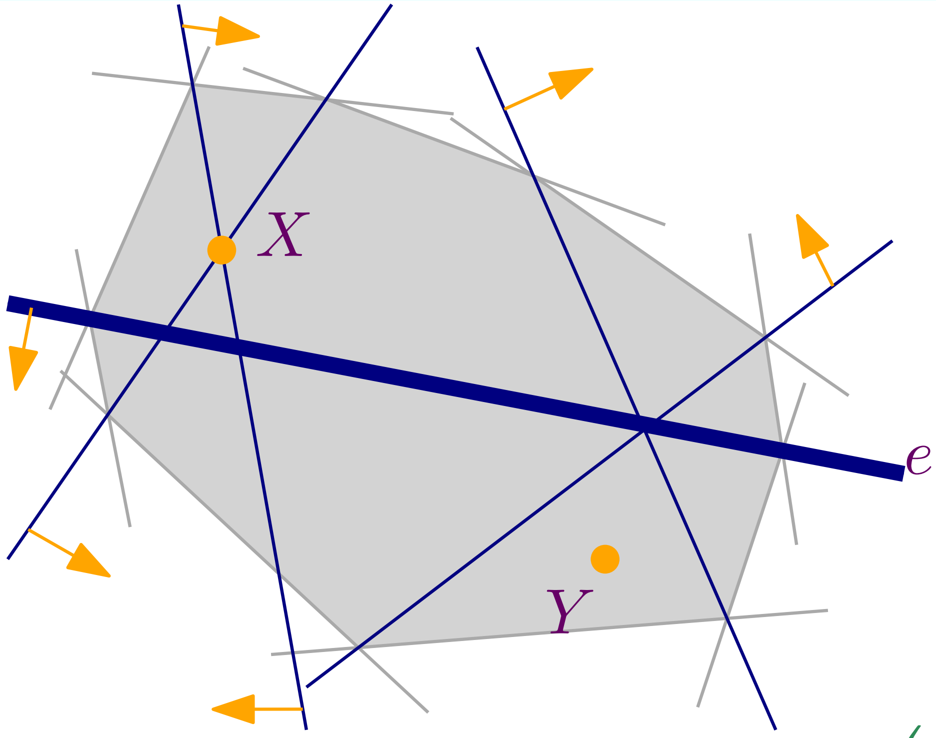

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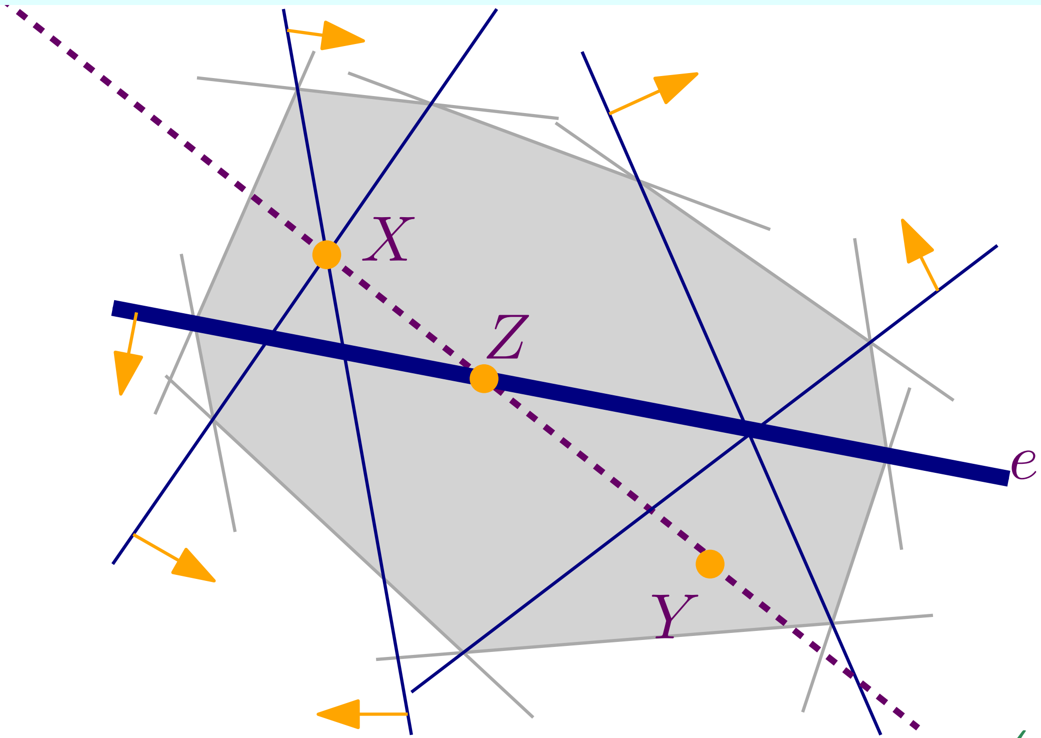

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a common generalization

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◦ Covector axioms: $\mathcal{M} = (E, \mathcal{L})$ OM:

(FS)+(SE) and:

(Z) $\mathbf{0} \in \mathcal{L}$

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(FS)+(SE) and:

(A) *something lengthy*

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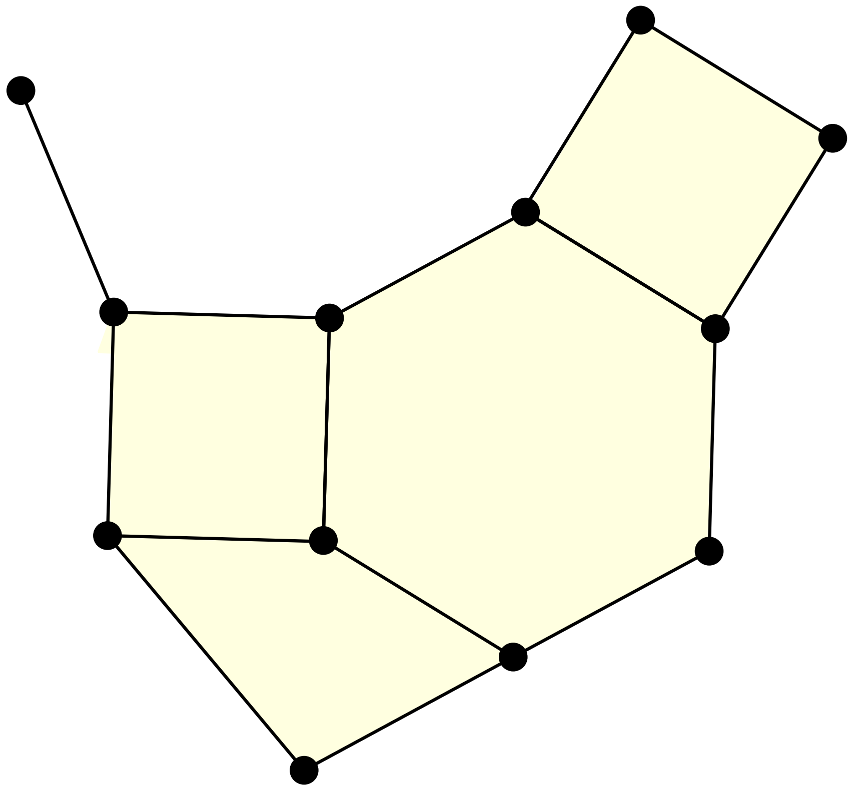
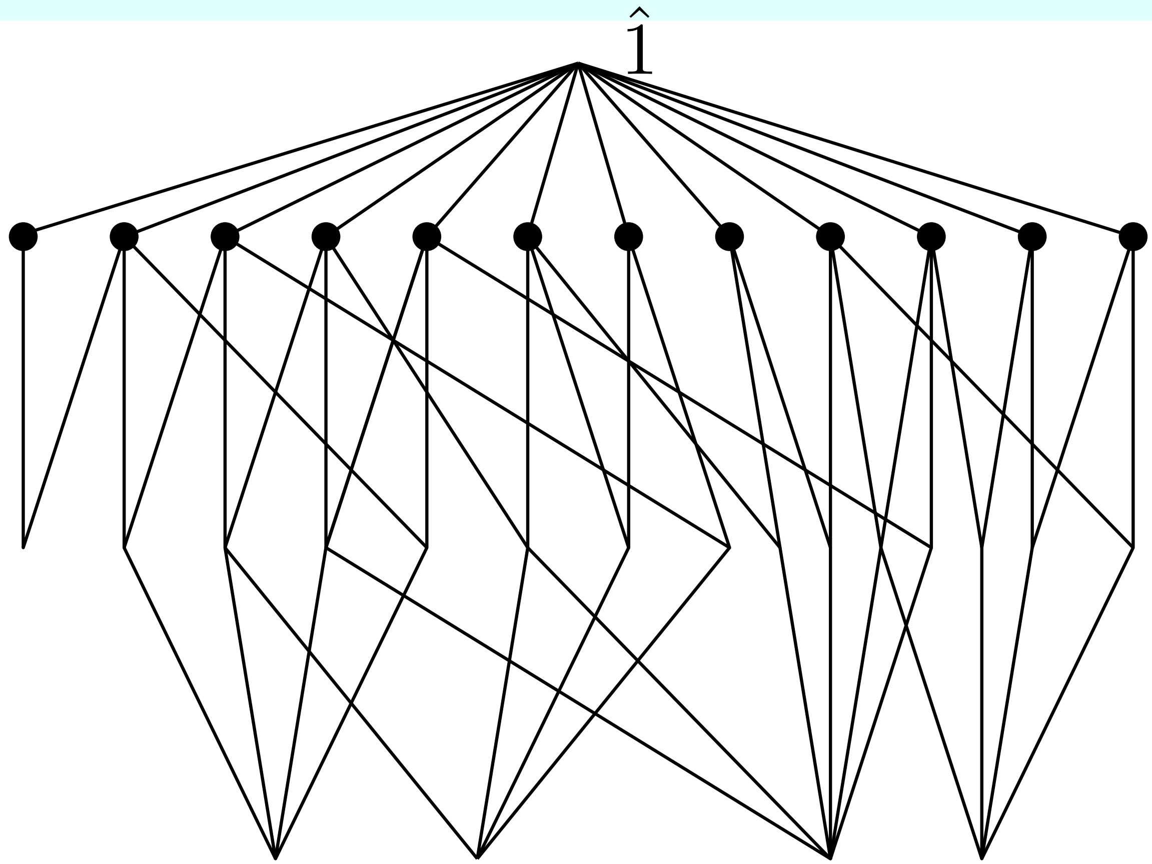
COMs as **C**omplexes of **O**riented **M**atroids





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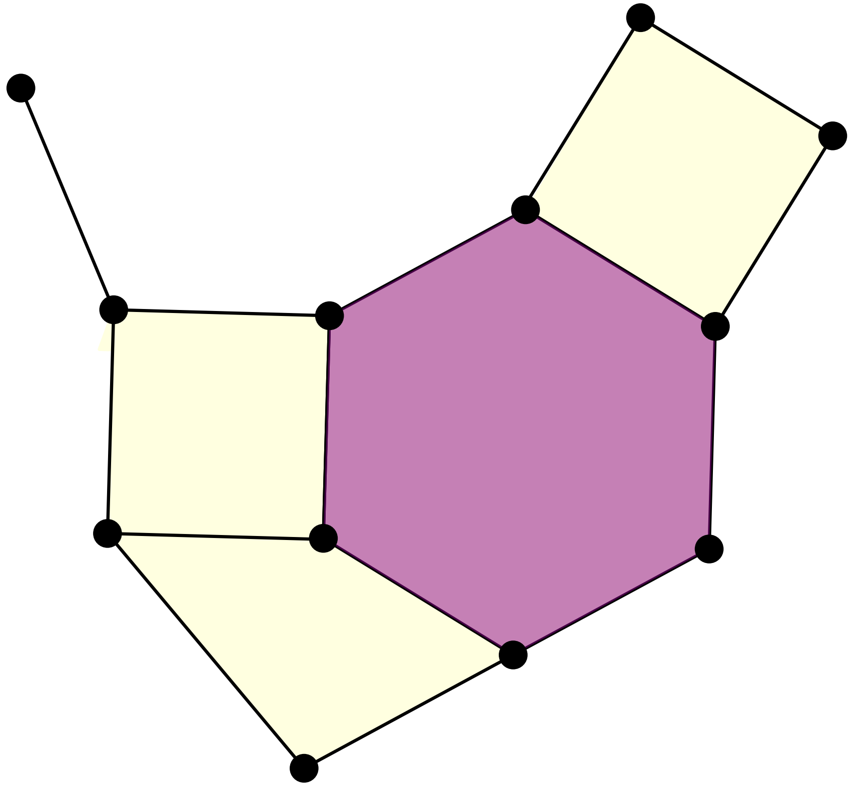

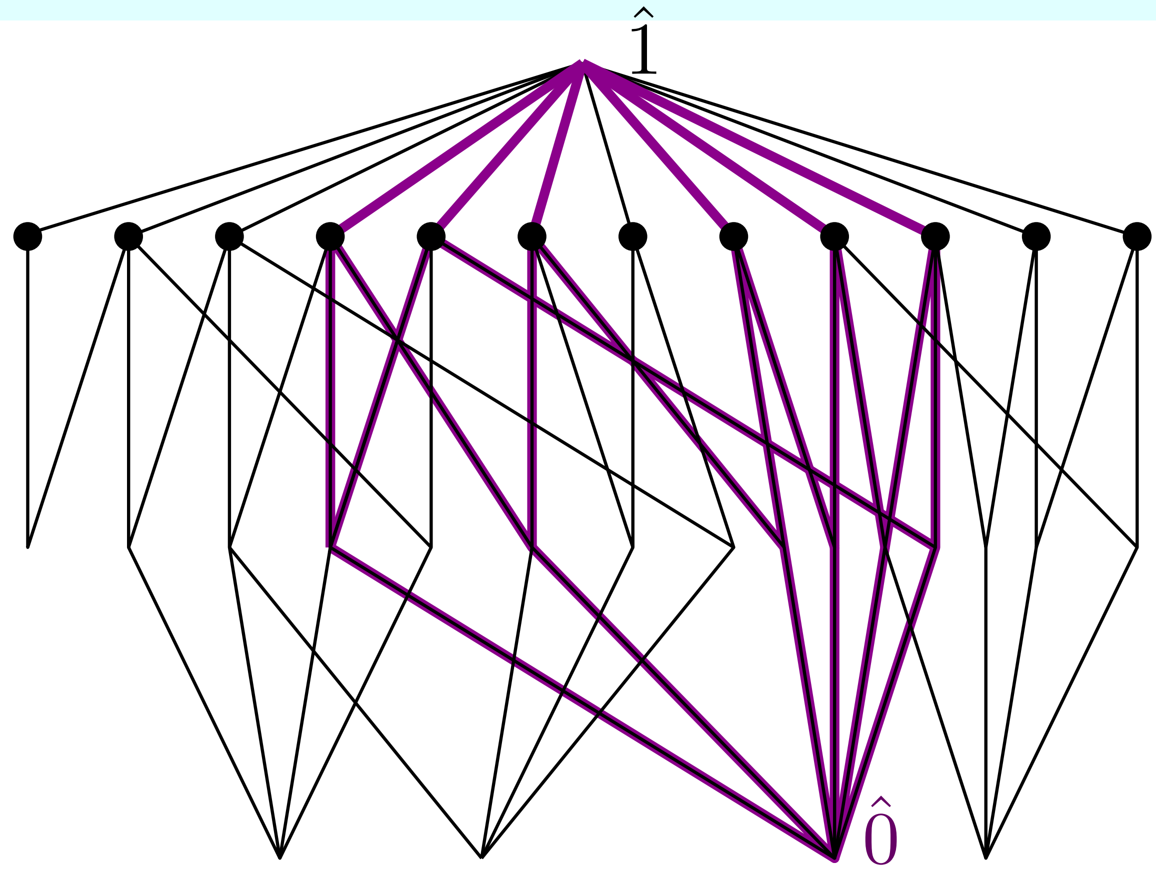

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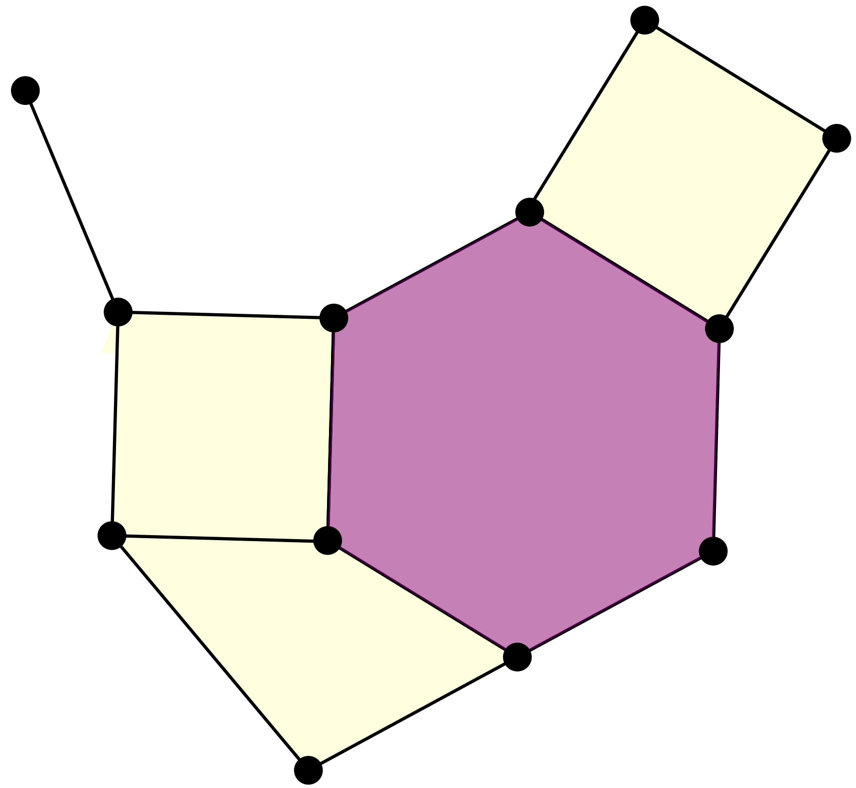
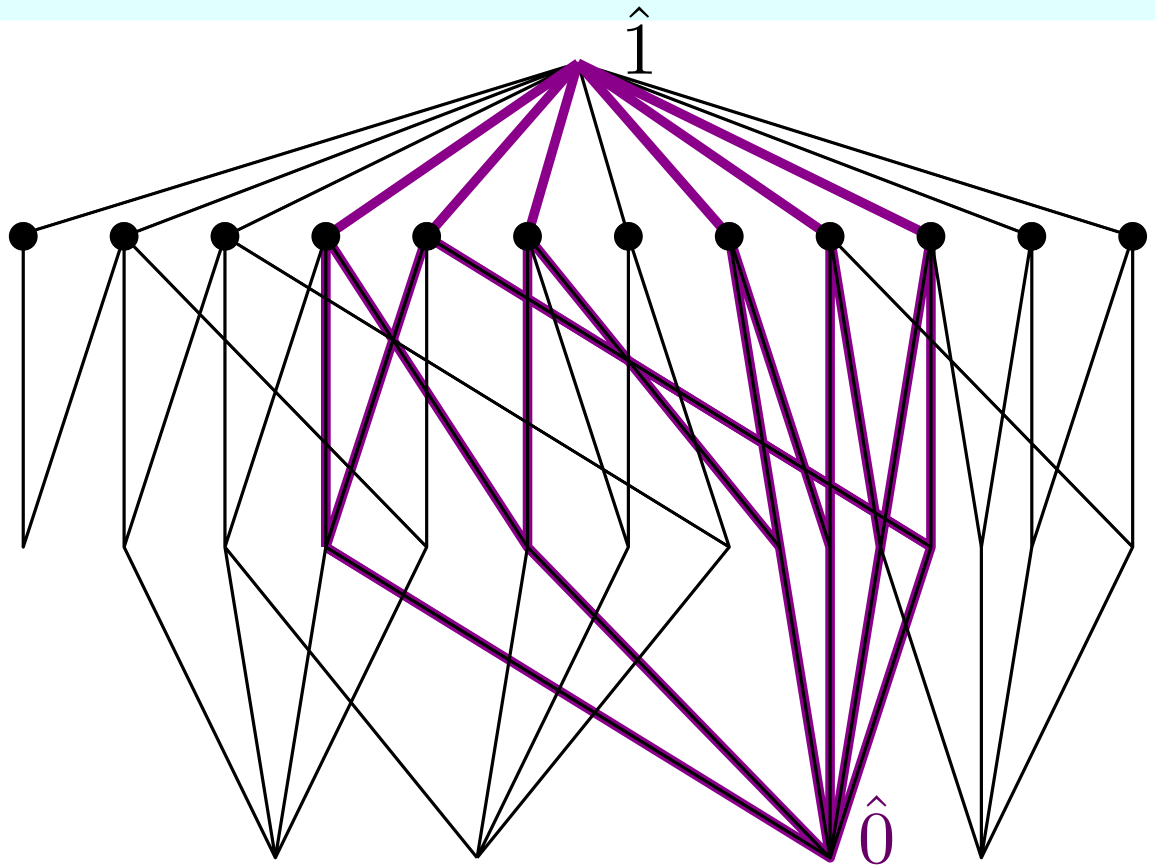


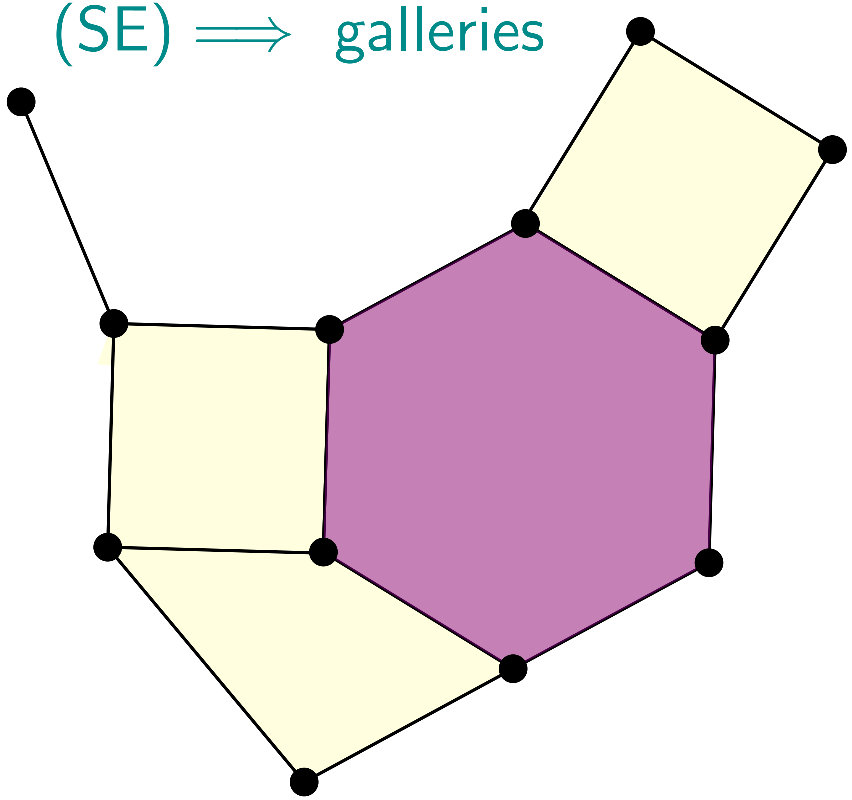


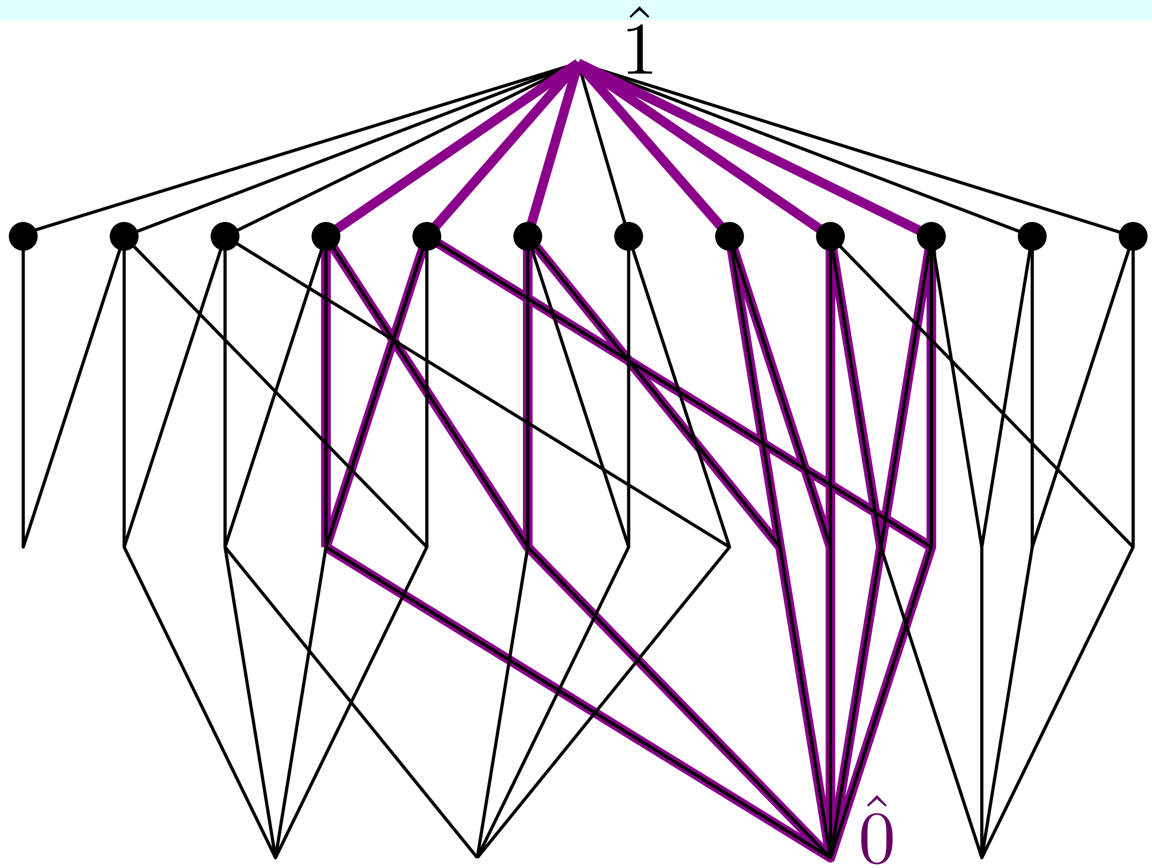
CW left regular bands (Margolis, Saliola, Steinberg '18):
left regular band: idempotent semigroup with $X \circ Y \circ X = X \circ Y$
$\rightsquigarrow$ poset structure: $X \leq Y$ if $X \circ Y = Y$
CW left regular band: principal filters are CW-posets

other examples: complex oriented matroids, interval greedoids

COMs as **C**omplexes of **O**riented **M**atroids

(SE) $\Longrightarrow$ galleries





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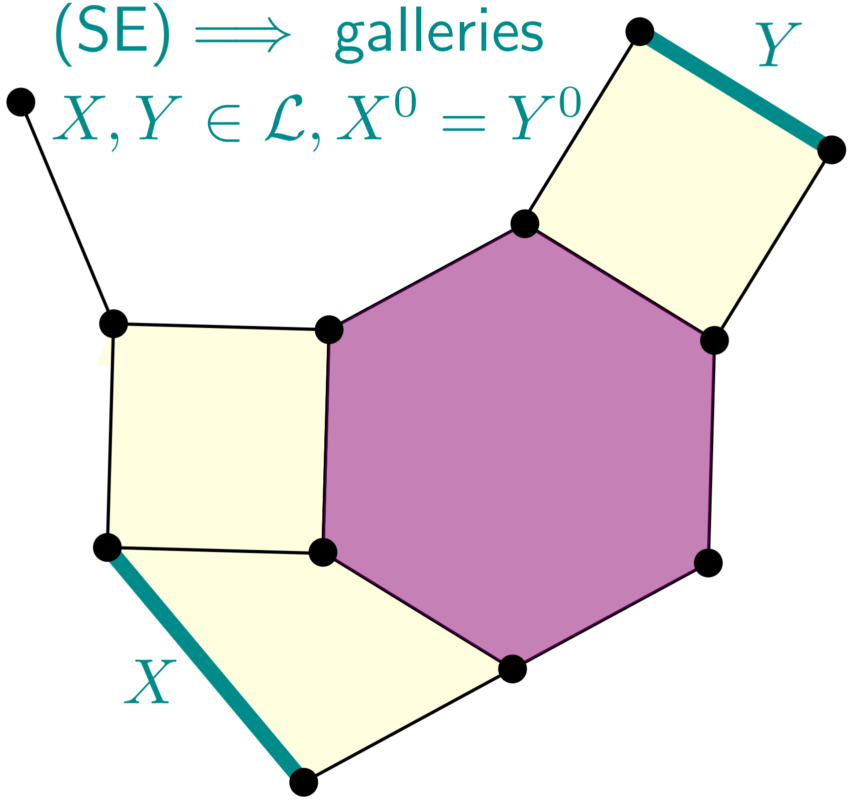

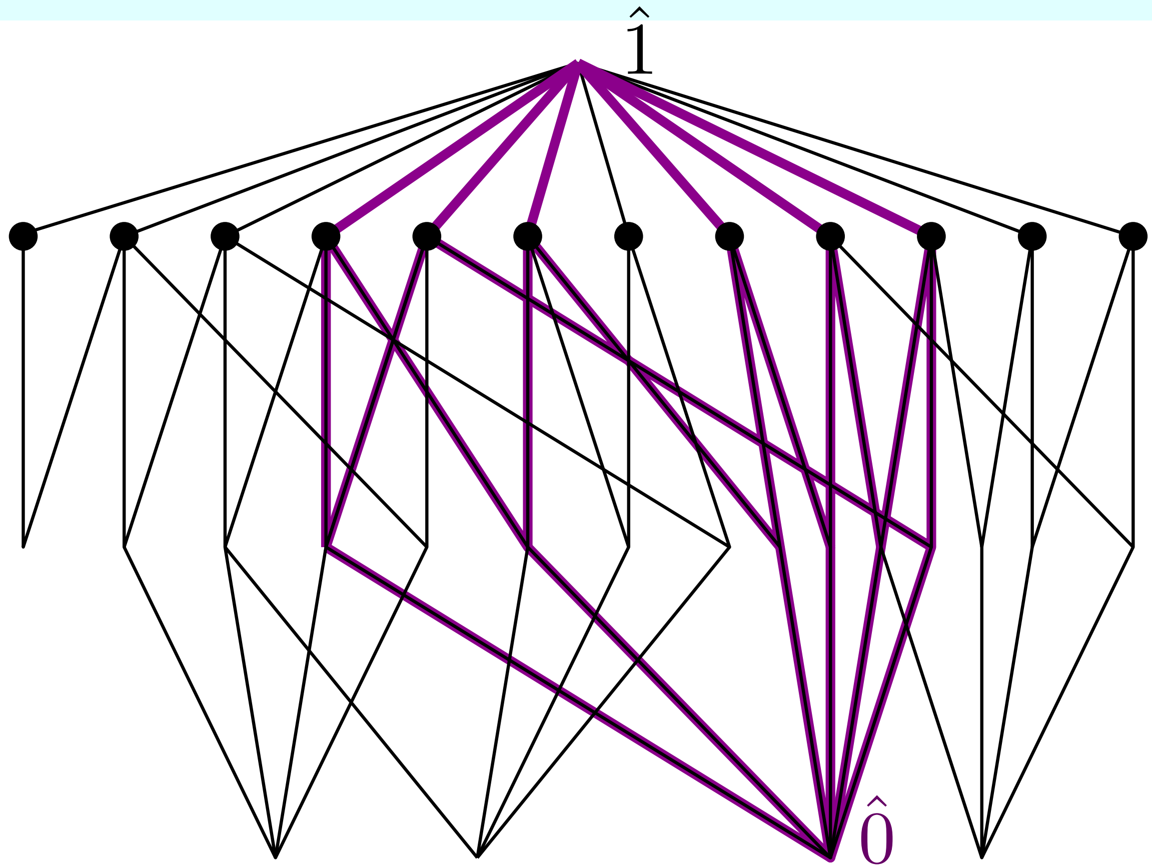

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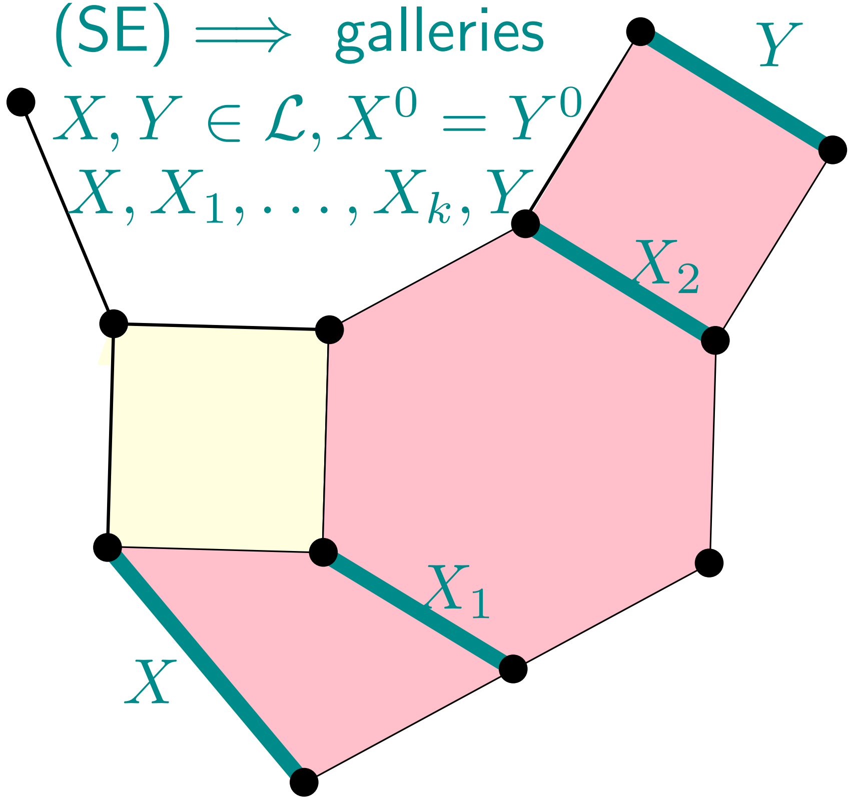
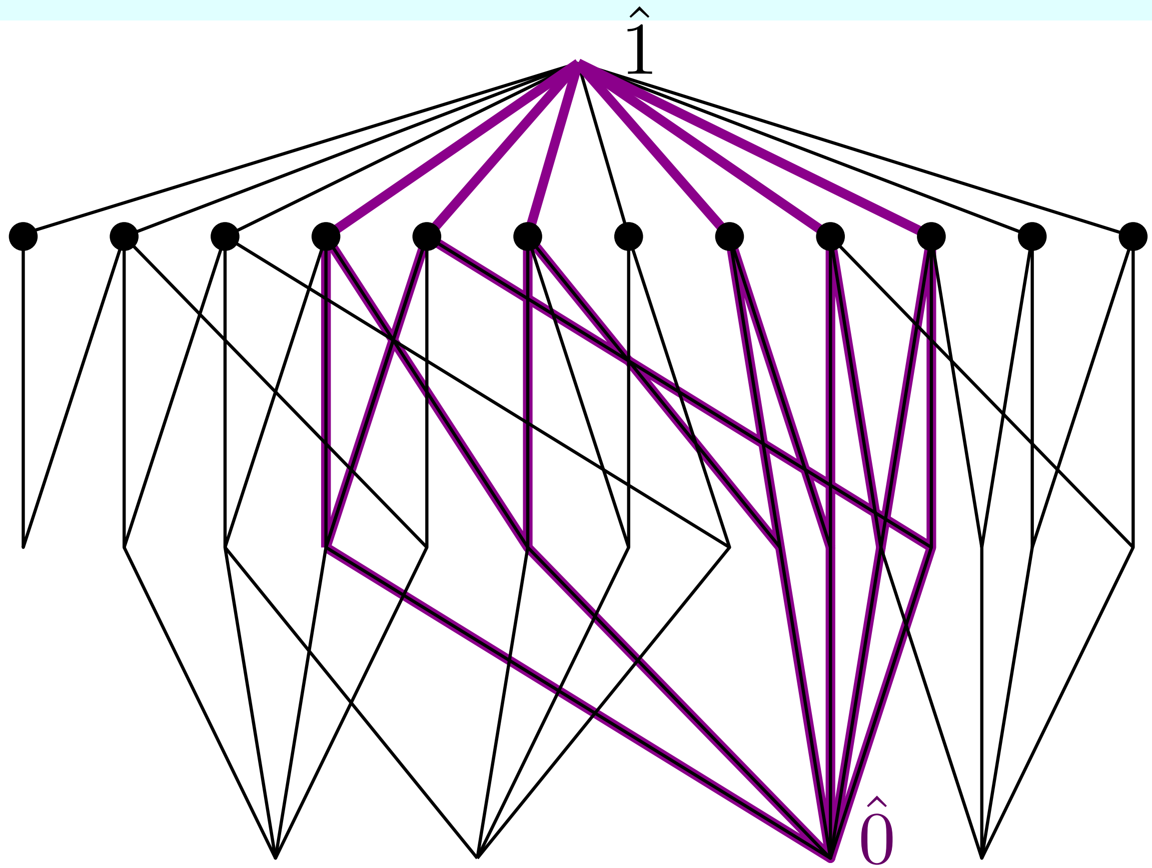
$X, Y \in \mathcal{L}, X^0 = Y^0$

$X, X_1, \ldots, X_k, Y$





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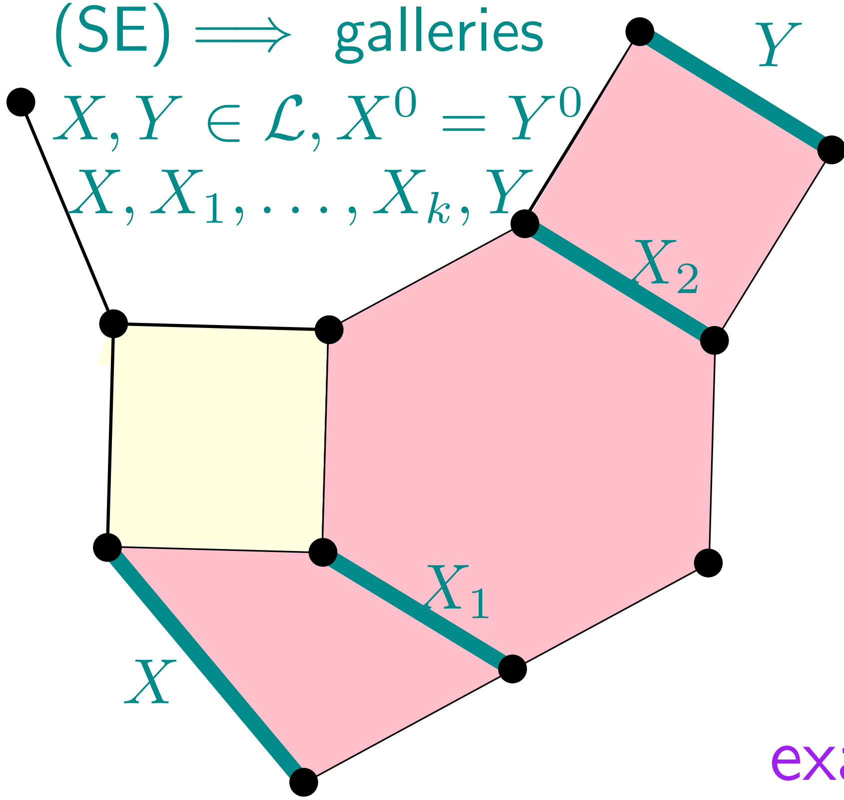
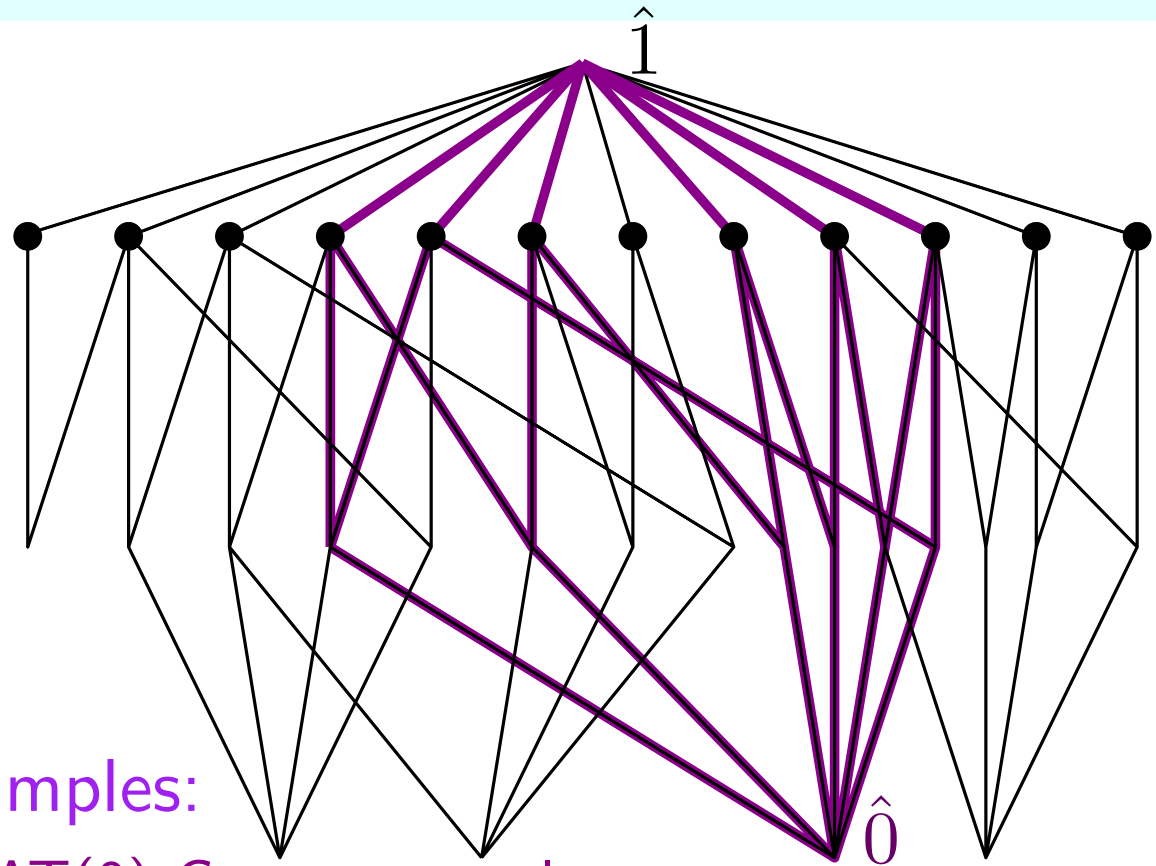
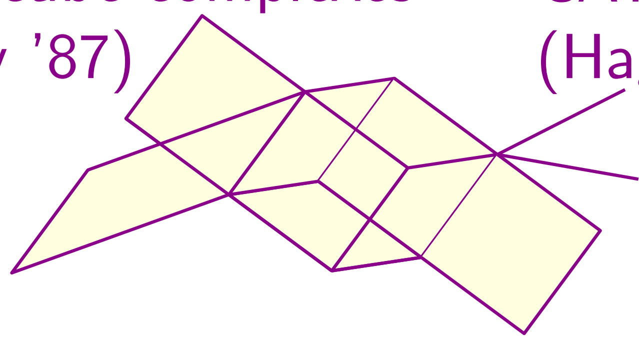




examples:

CAT(0) cube complexes
(Gromov '87)



CAT(0) Coxeter complexes
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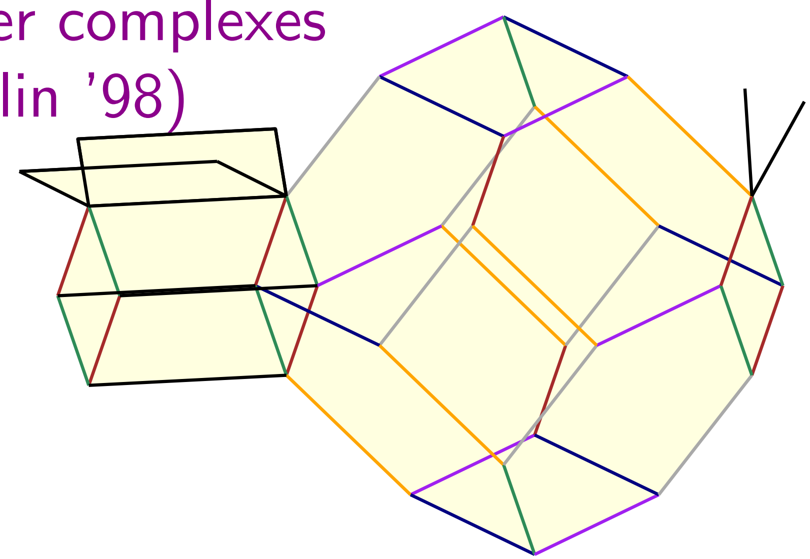

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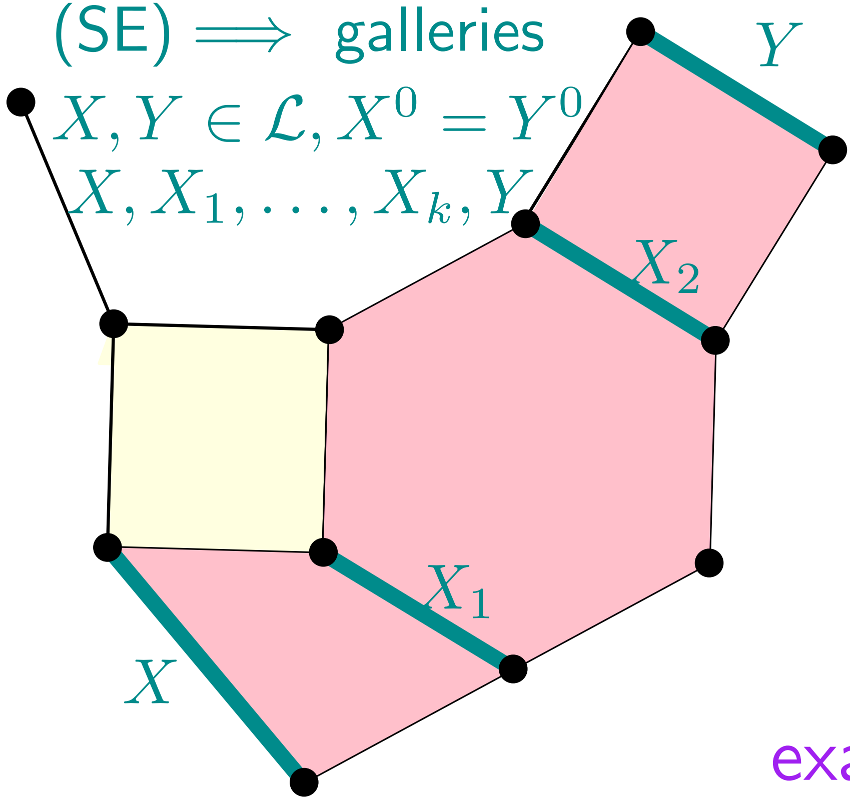

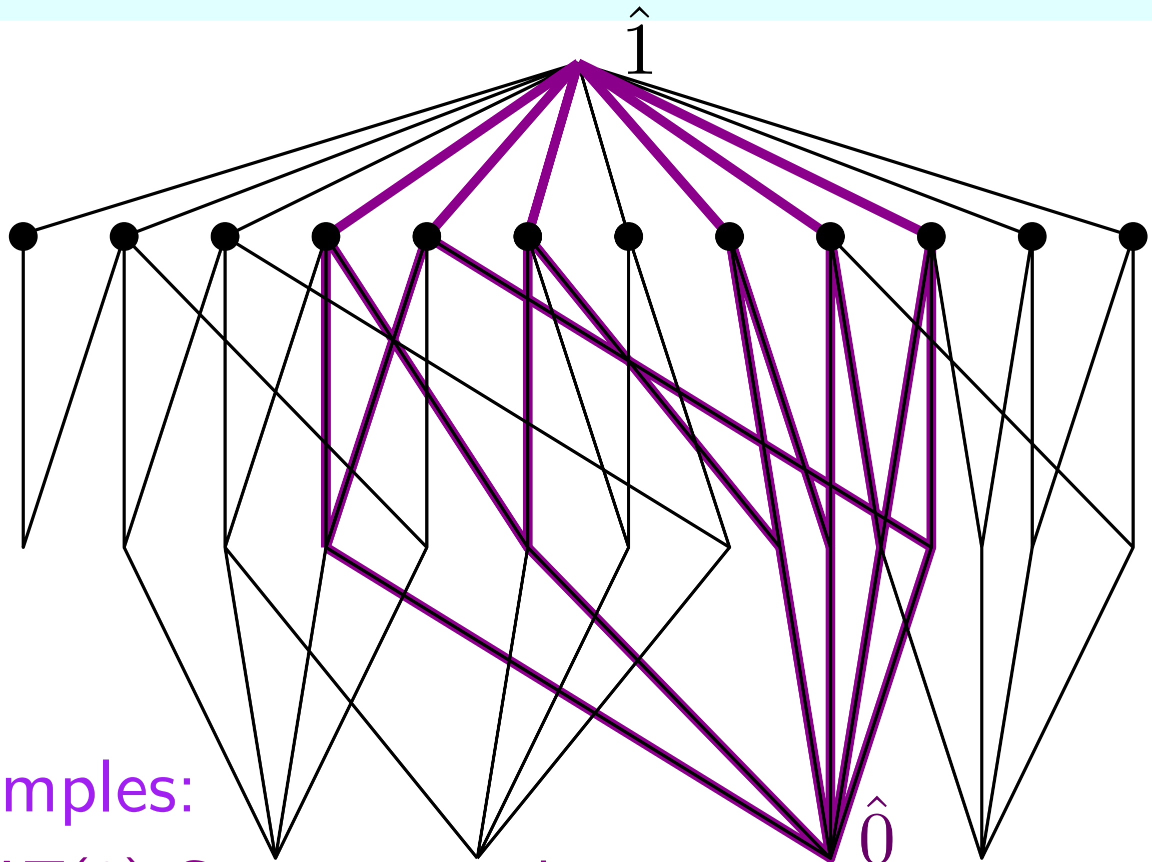

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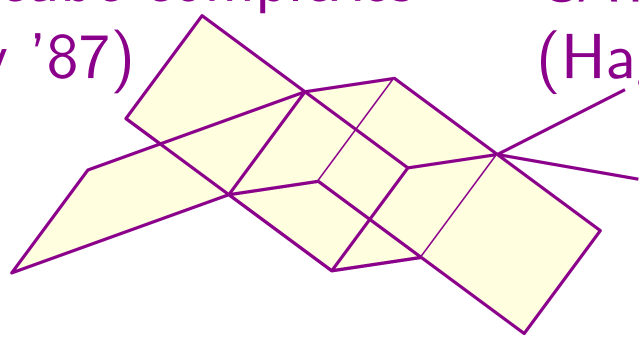

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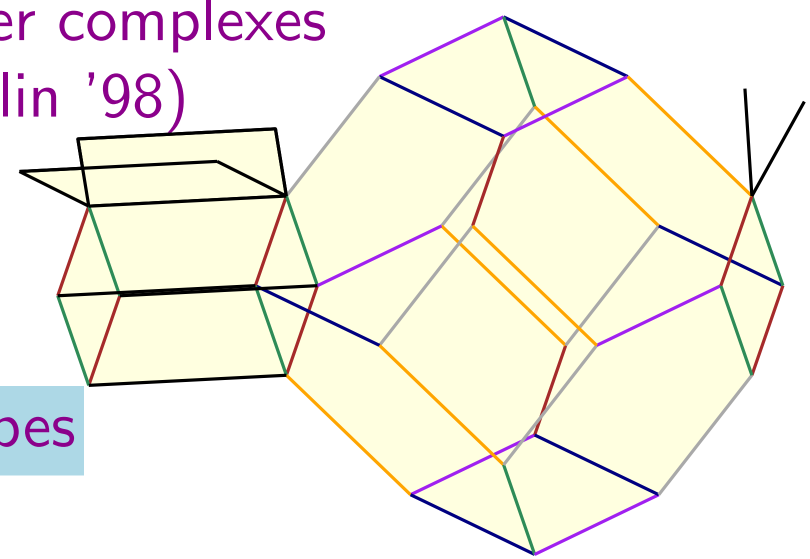

AMPs are those COMs whose faces are cubes

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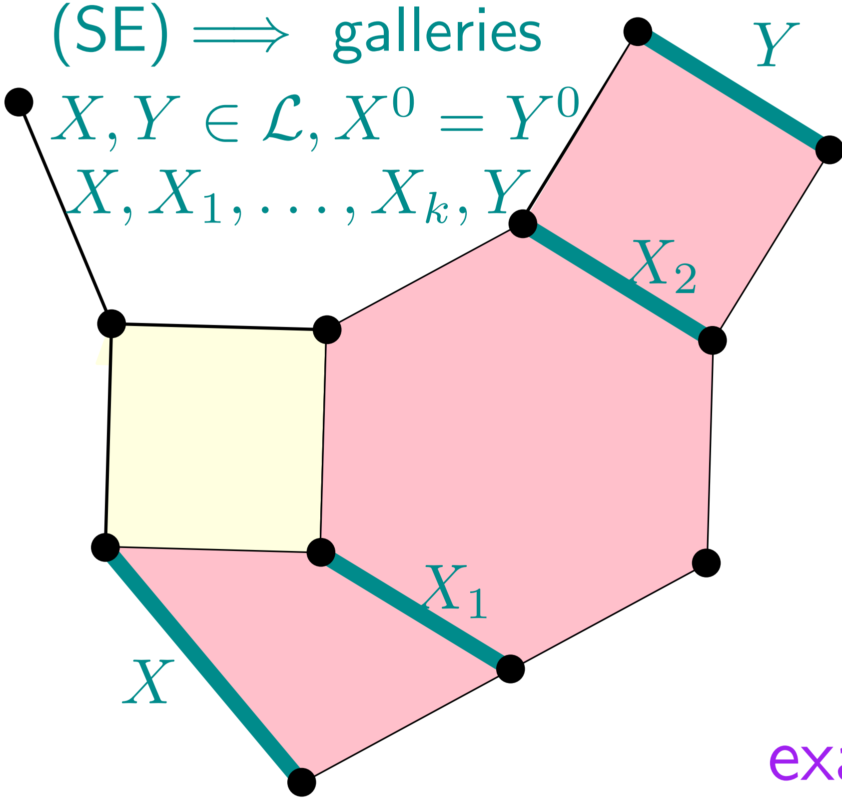

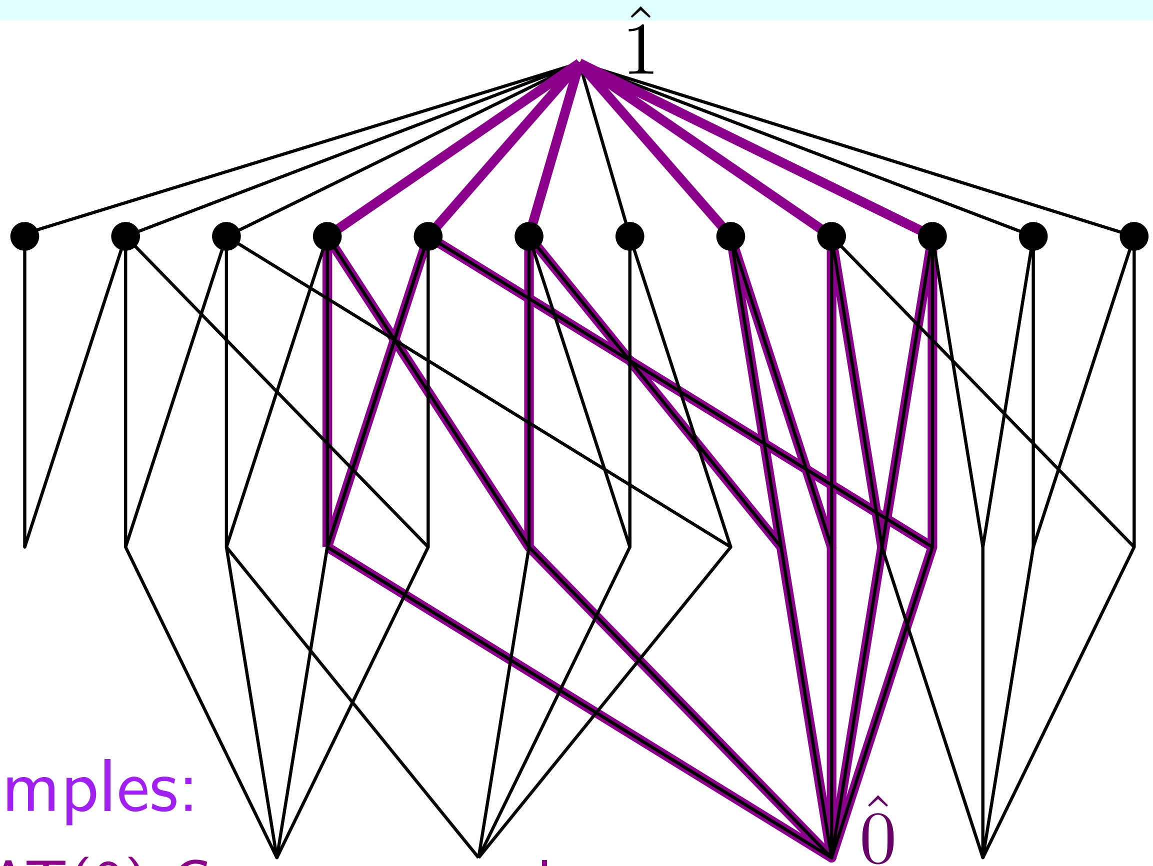

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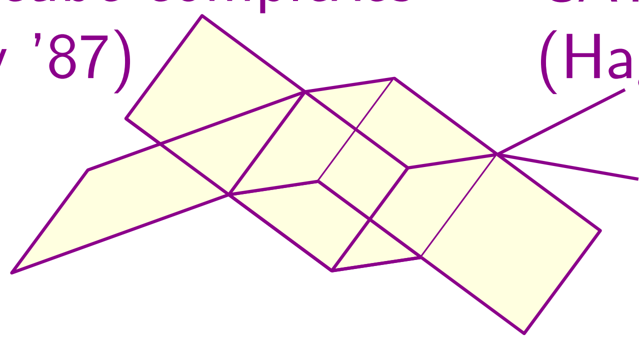

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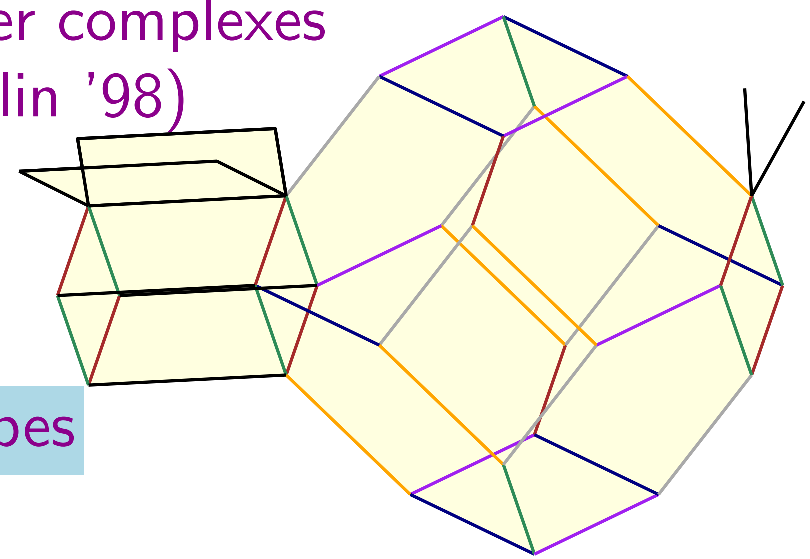

AMPs are those COMs whose faces are cubes

rank of $\mathcal{M} = \max$ rank among faces

tope graphs

◦ Covector axioms: $(E, \mathcal{L})$ COM iff

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(A) *something lengthy*

Covector axioms: $(E, \mathcal{L})$ ample set:

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(A) *someth*

Cov $,\mathcal{L})$ ample set:

(

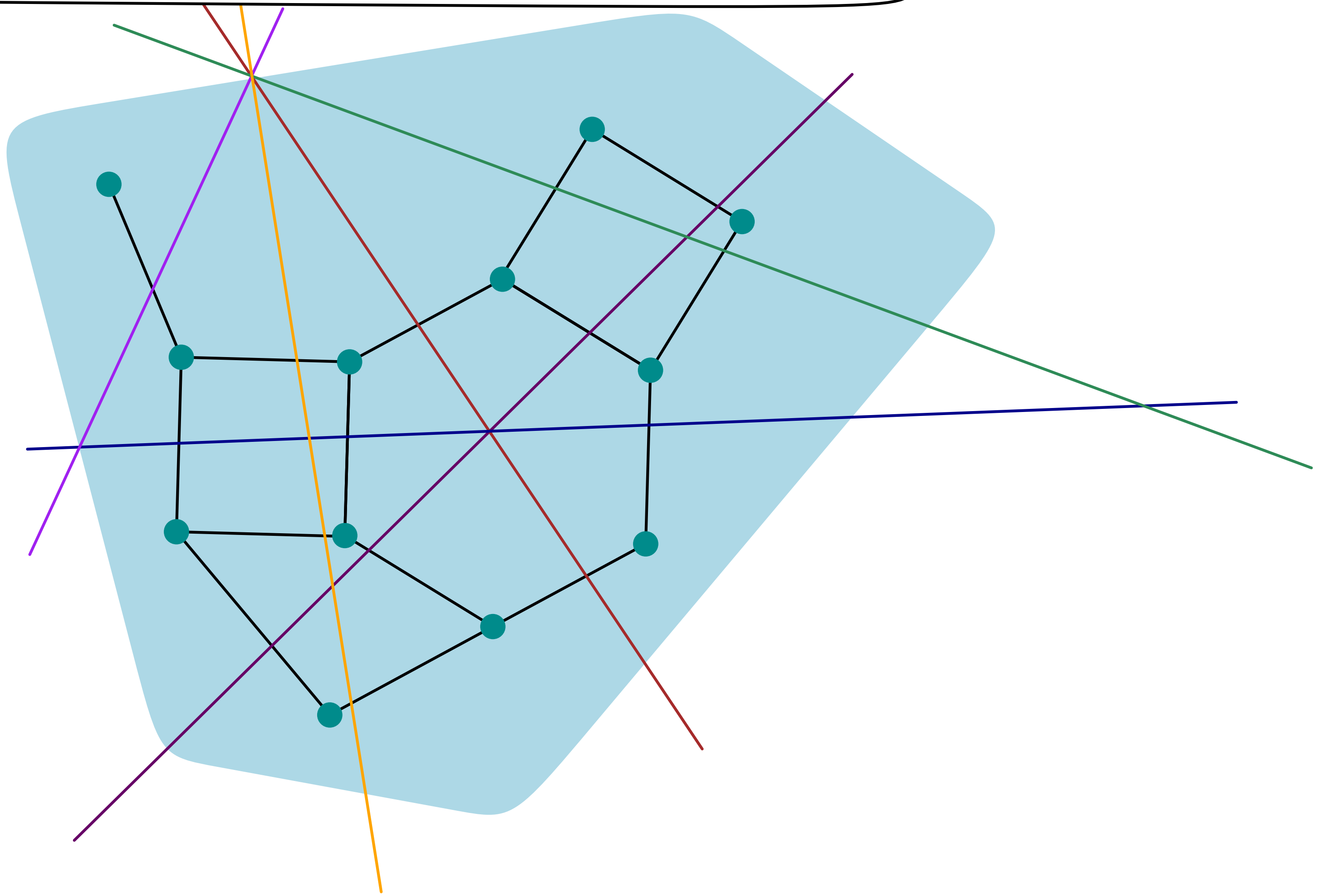
(I) $\{\pm 1\}^E = \mathcal{L}$

tope graphs are partial cubes and determine $\mathcal{L}$

tope graphs are partial cubes

G *partial cube* $:\Leftrightarrow$ G isometric subgraph of hypercube

$G \subseteq Q_n$ such that $d_G(v,w) = d_{Q_n}(v,w) \forall v,w \in G$



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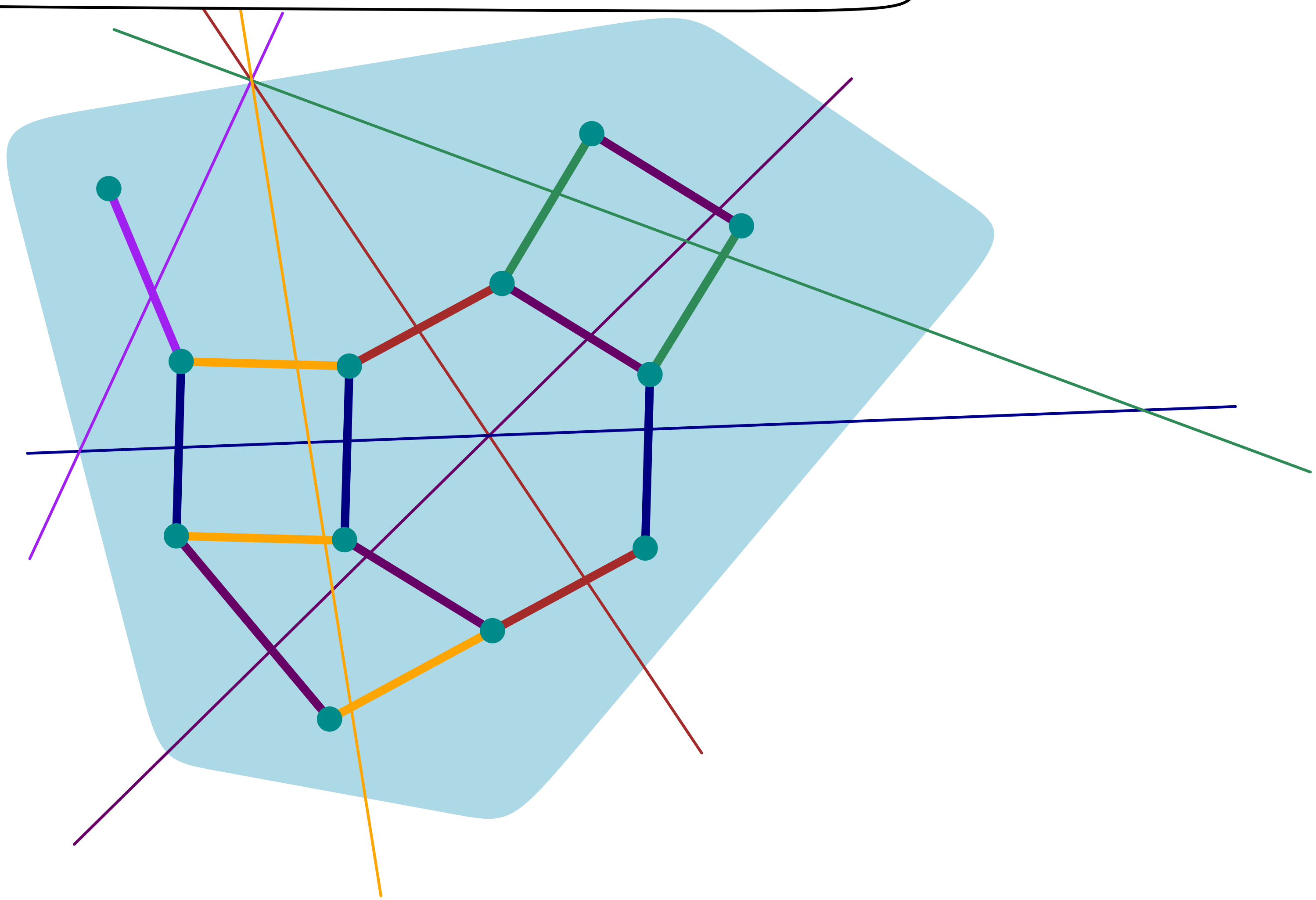

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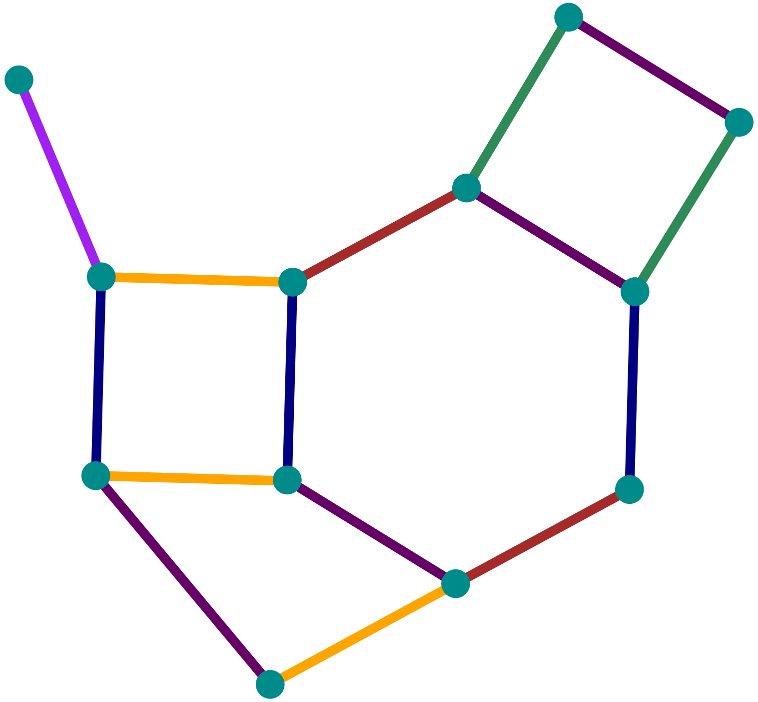

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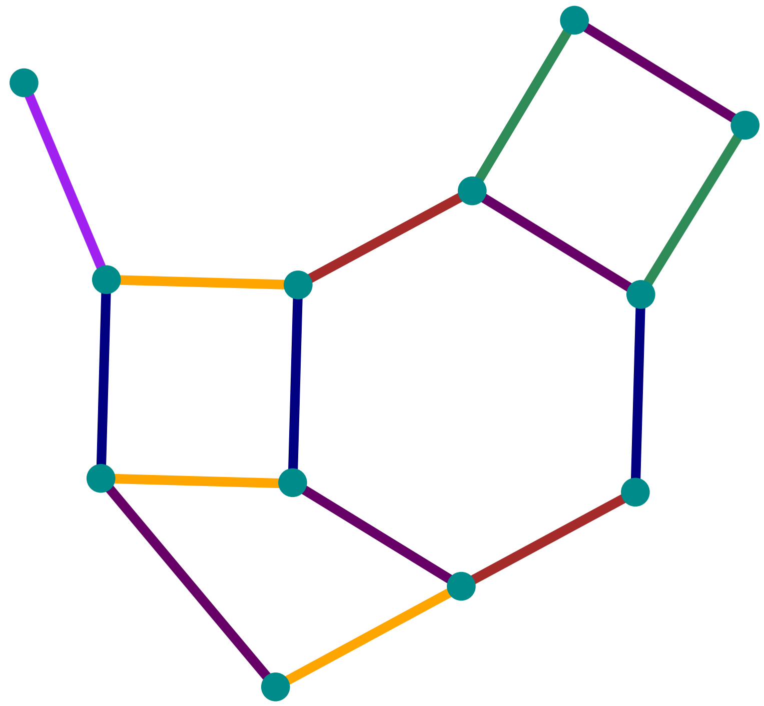

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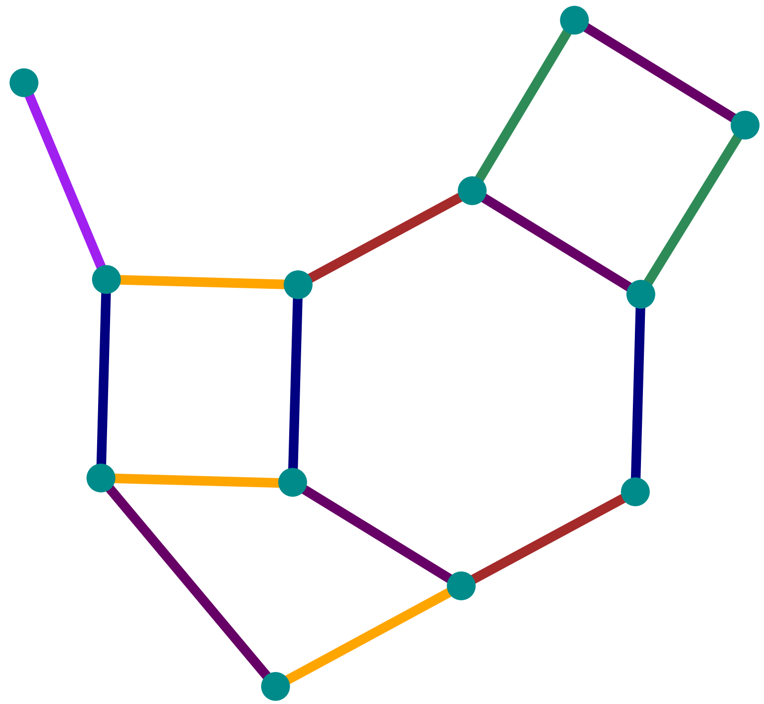

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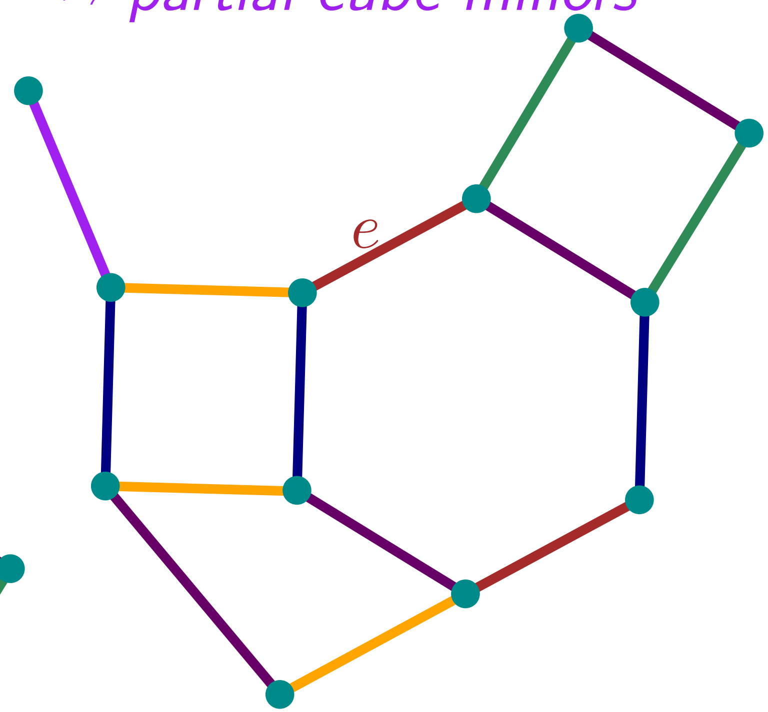
$\rightsquigarrow$ *partial-cube-minors*

Θ-class



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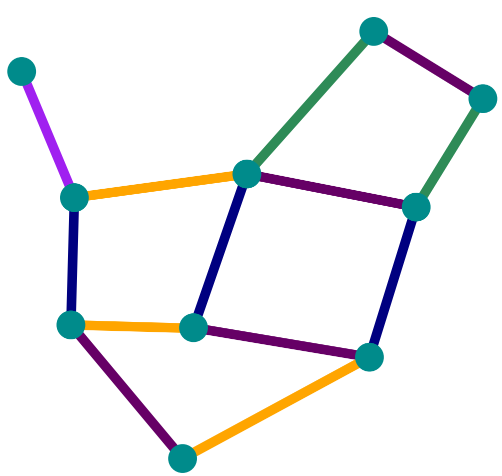
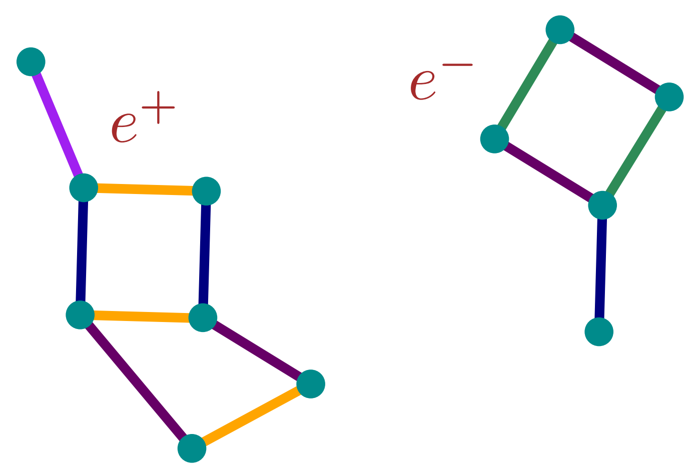


pc-contraction of e



pc-restrictions wrt e

tope graphs are partial cubes

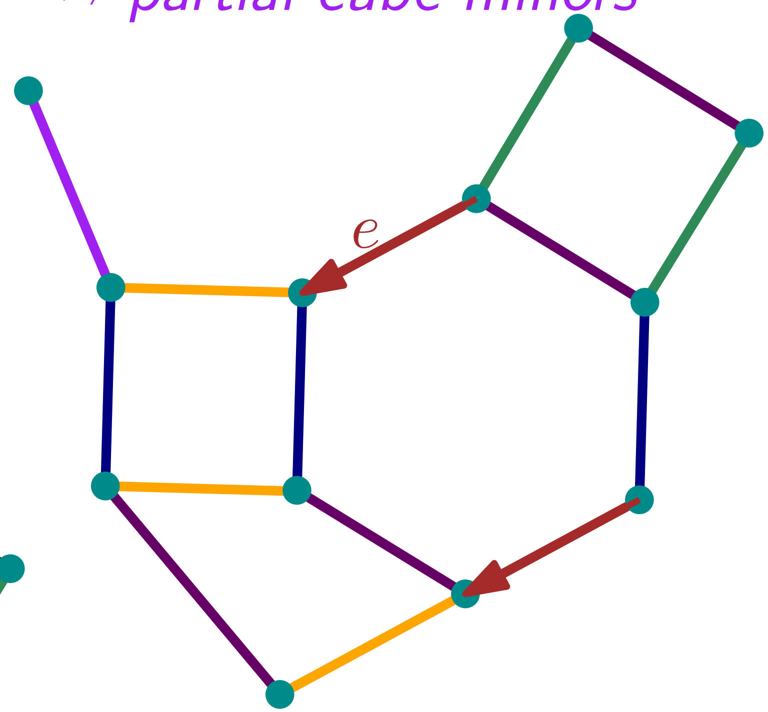
G *partial cube* $:\Leftrightarrow$ G isometric subgraph of hypercube

$G \subseteq Q_n$ such that $d_G(v,w) = d_{Q_n}(v,w) \forall v,w \in G$

isometric $\Leftrightarrow$ shortest paths use each color at most once

$\rightsquigarrow$ *partial-cube-minors*

Θ-class



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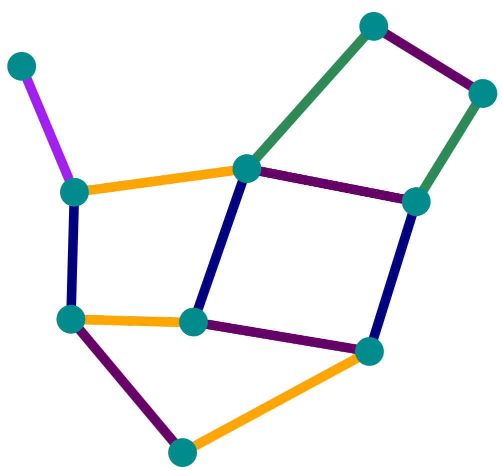

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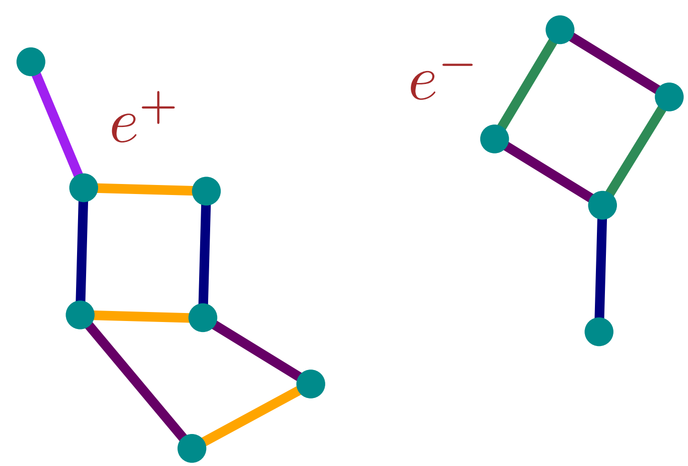

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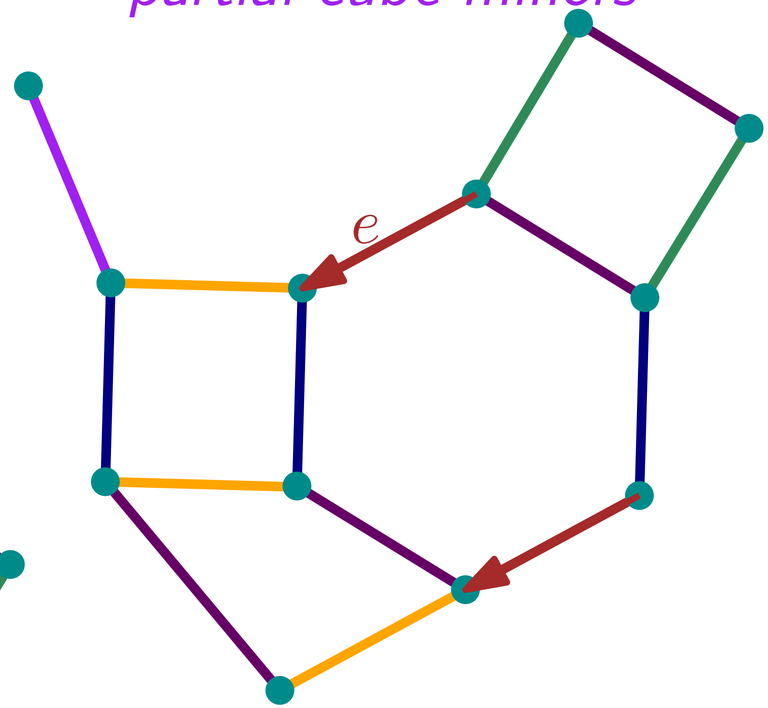

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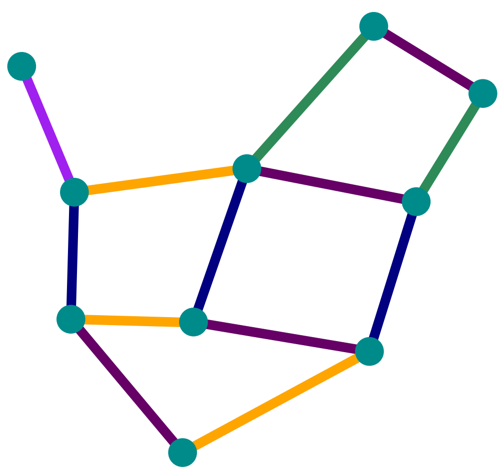
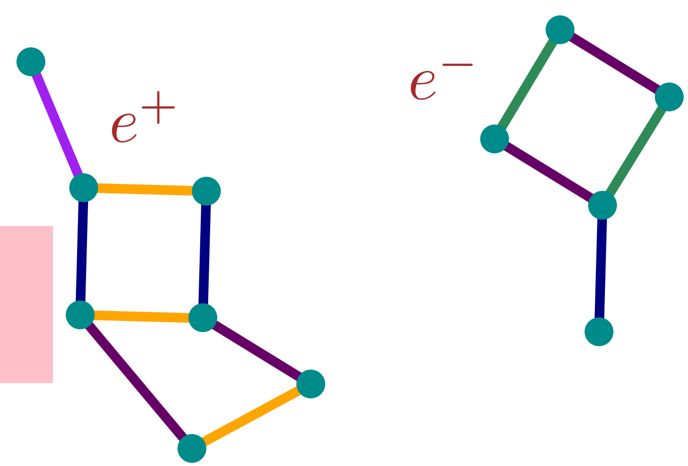


pc-contraction of e

graphs of COMs and AMPs closed under pc-minors



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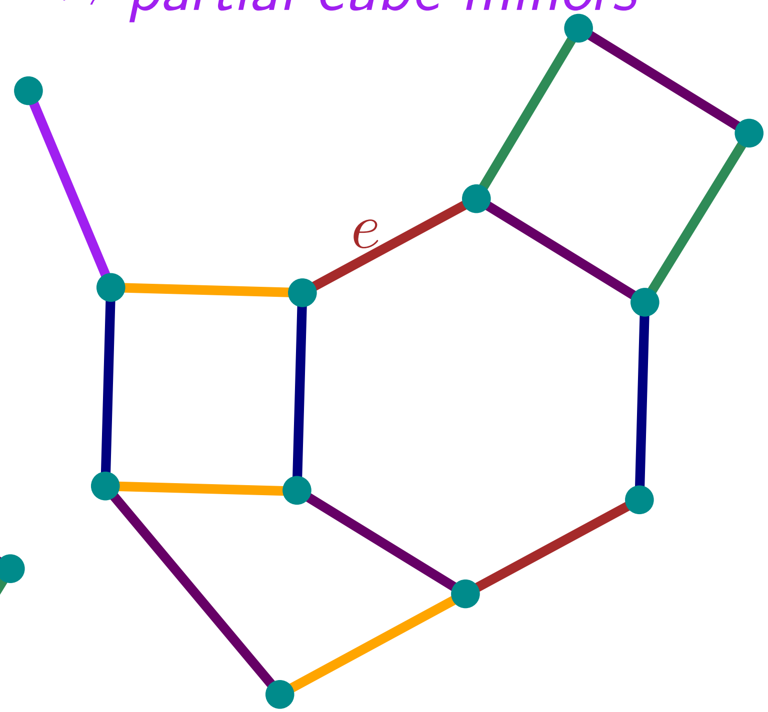

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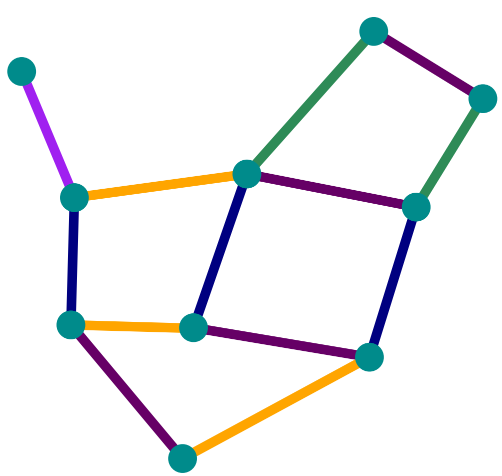

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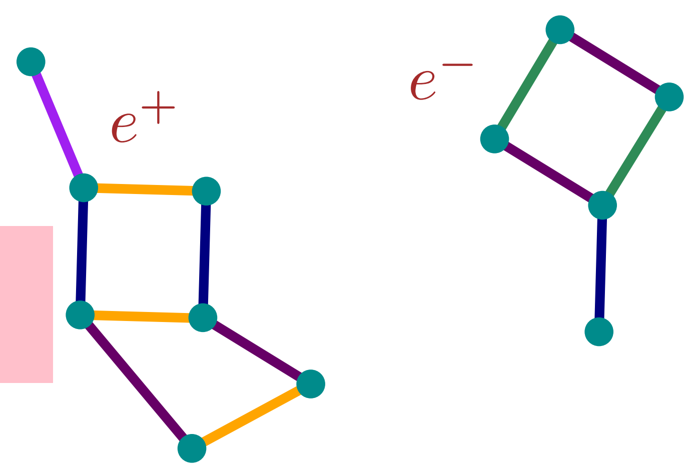
graphs of COMs and AMPs closed under pc-minors

rank of $G =$ $\max Q_r$ pc-minor



pc-restrictions wrt e

convex subgraphs and sign vectors

if G partial cube, then $G' \subset G$ **convex** $\iff$ G' restriction of G

shortest paths between vertices of G' stay in G'

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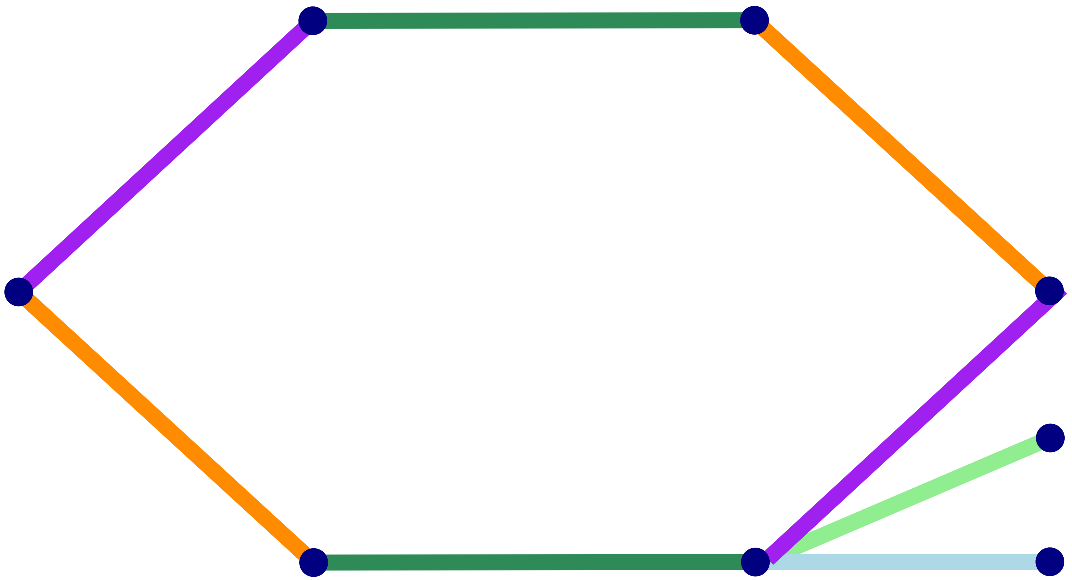

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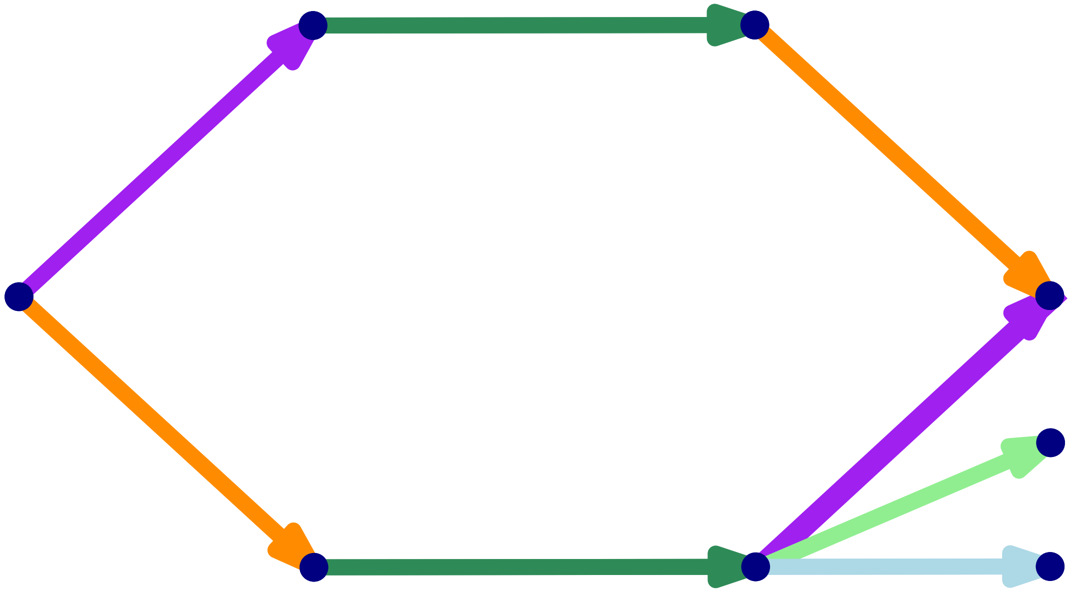

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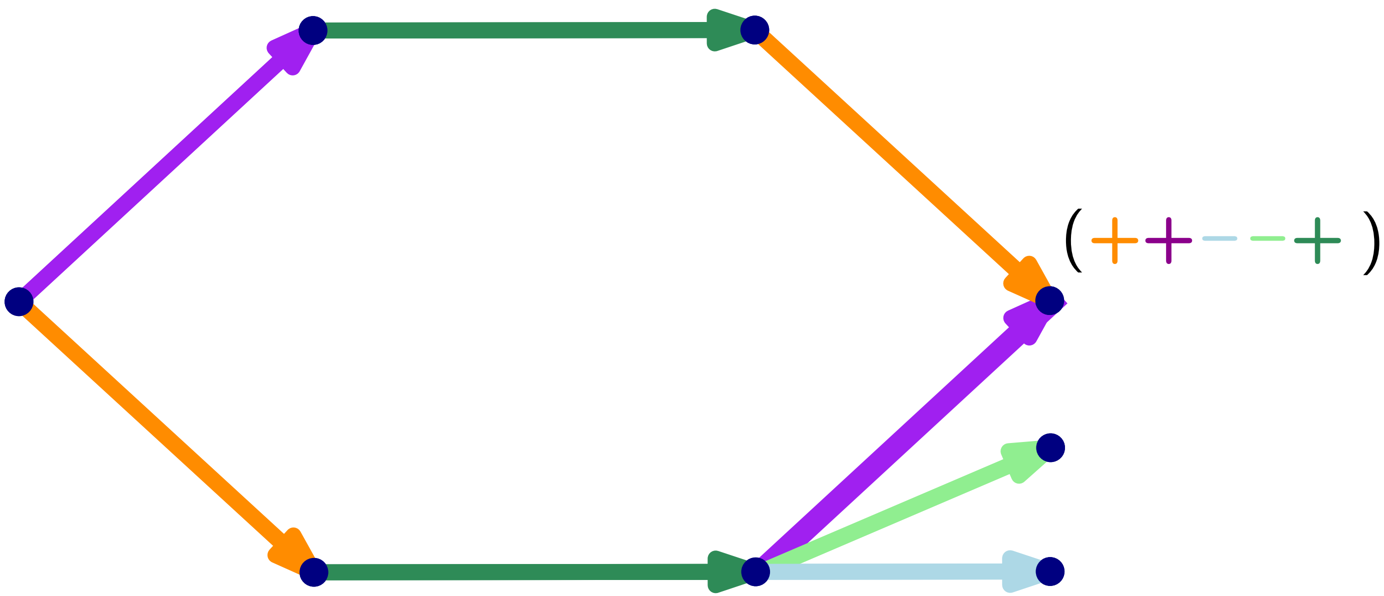

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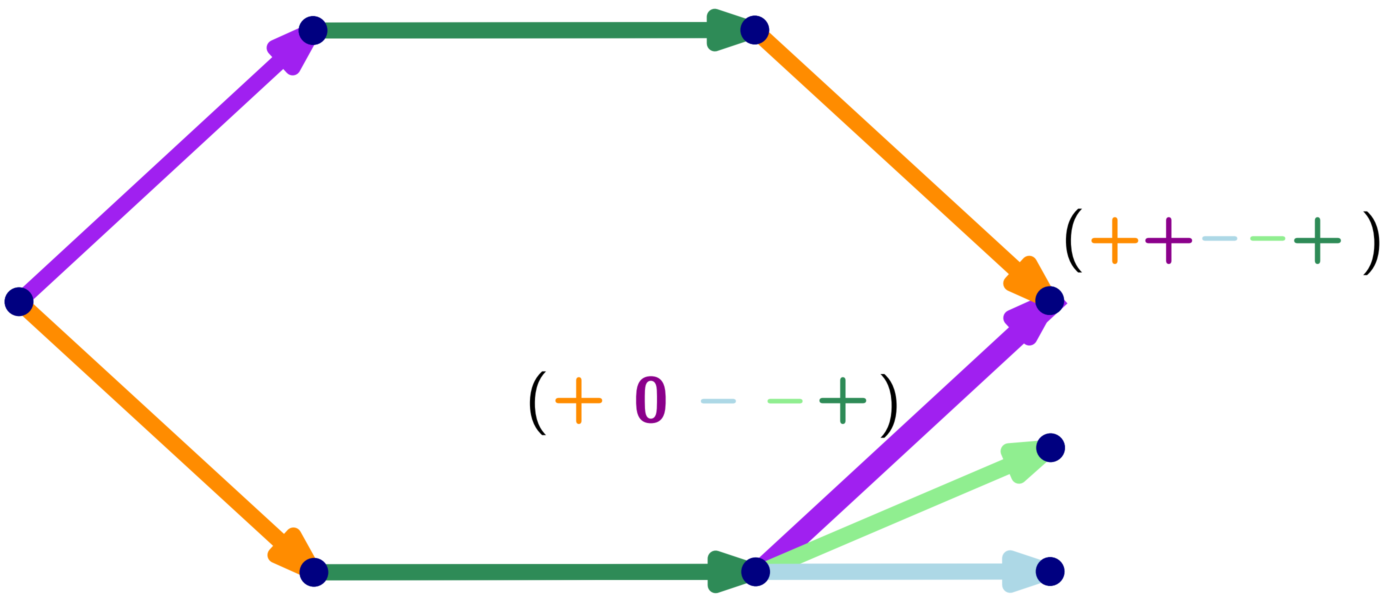

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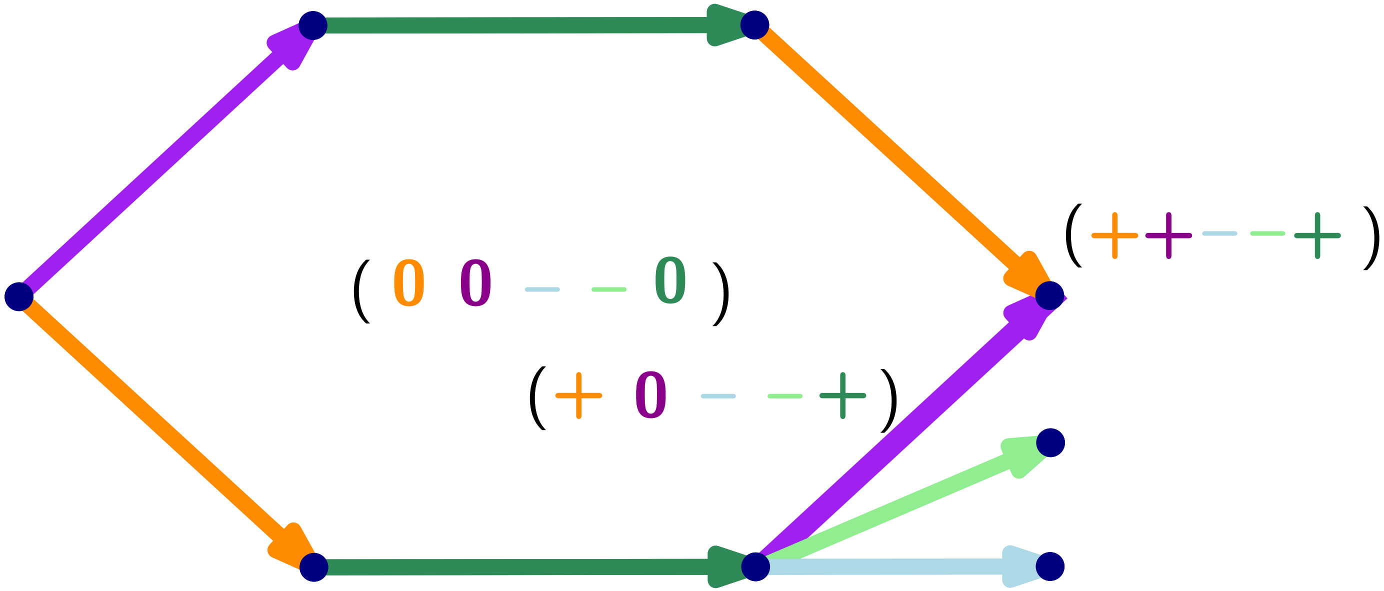

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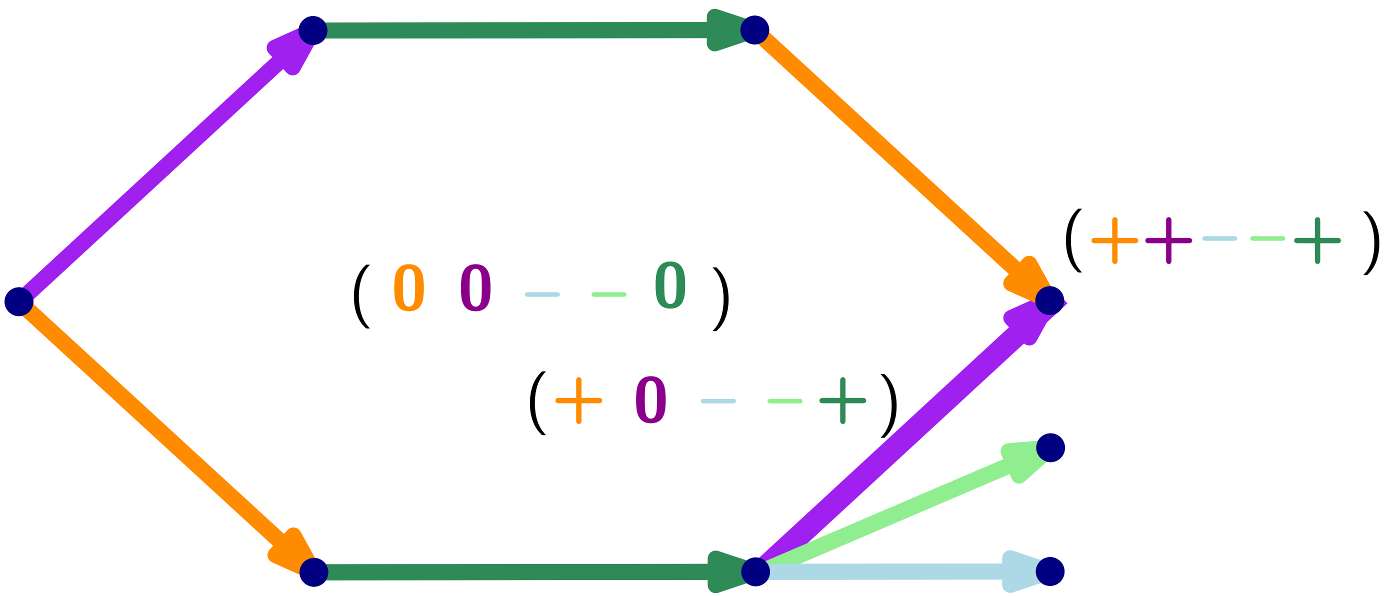

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$w \in G'$ iff there is shortest $(v, \overline{v})$-path through w

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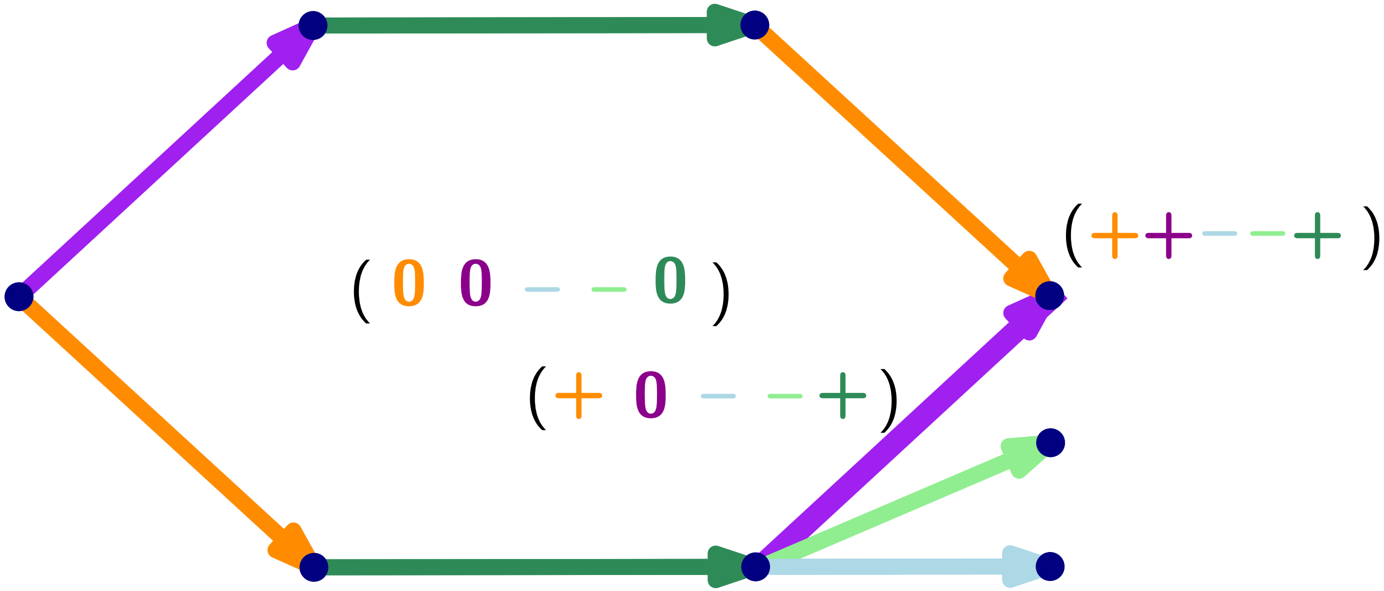

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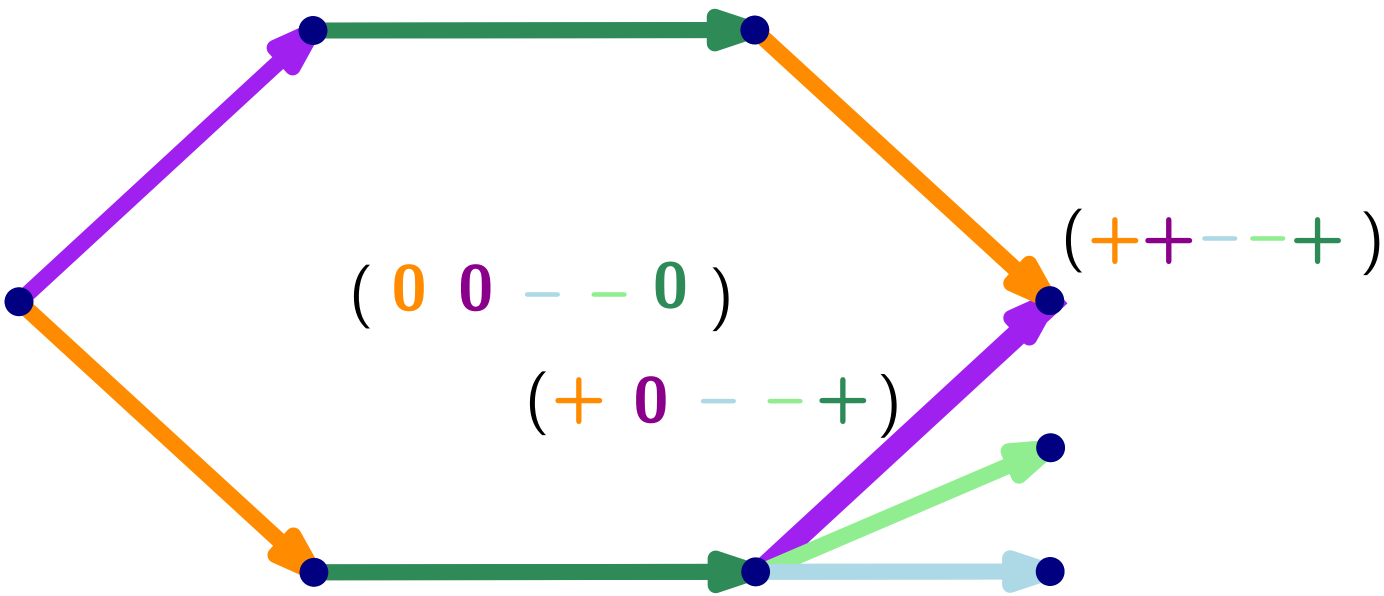

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Cor: G tope graph of AMP $\Leftrightarrow$ all antipodal subgraphs cubes

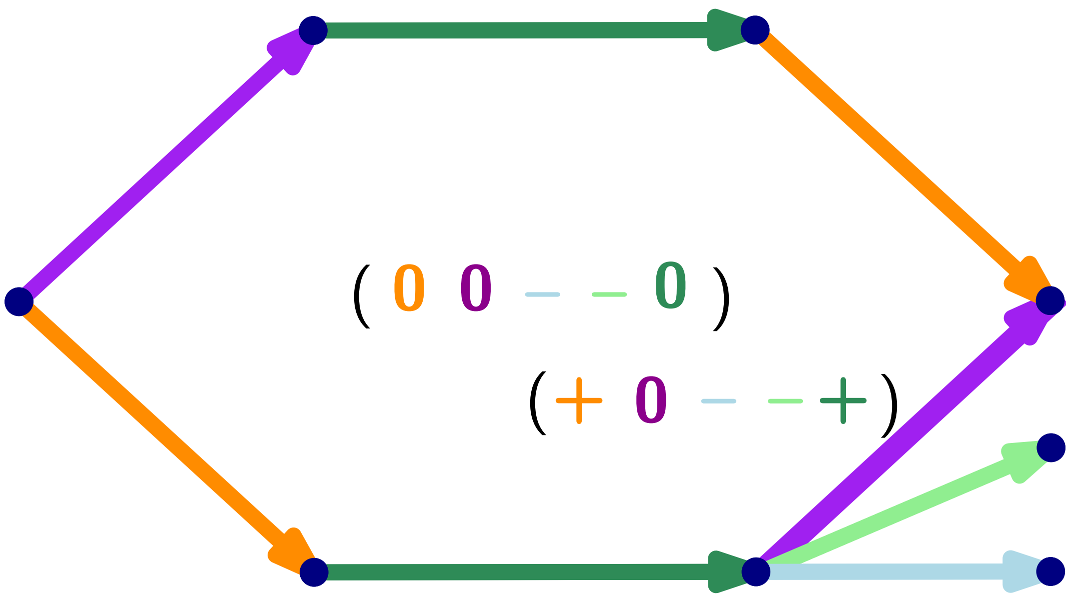
convex subgraphs and sign vectors

if G partial cube, then $G' \subset G$ **convex** $\iff$ G' restriction of G

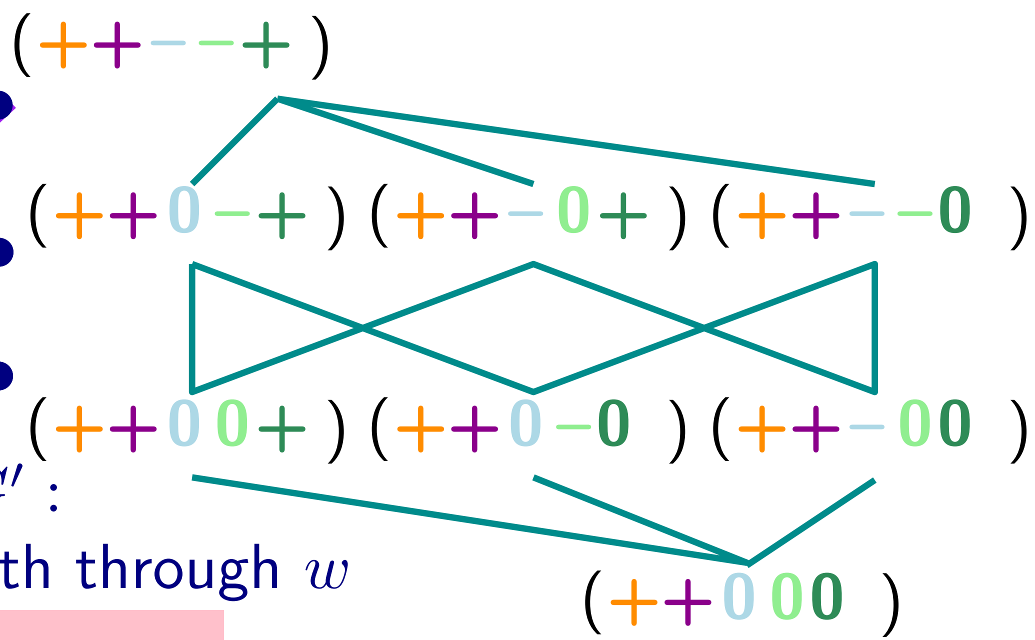
shortest paths between vertices of G' stay in G' (↑ convex)

intersection of halfspaces $X(G')$ containing G' (↑ restriction of G)

associate sign vector $X(G')$ to convex subgraph G'



actually: Boolean lattice of sign vectors for the same convex G'



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labelled sample compression

concepts $\mathcal{C} \subseteq \{\pm\}^U$ 

set system

labelled sample compression

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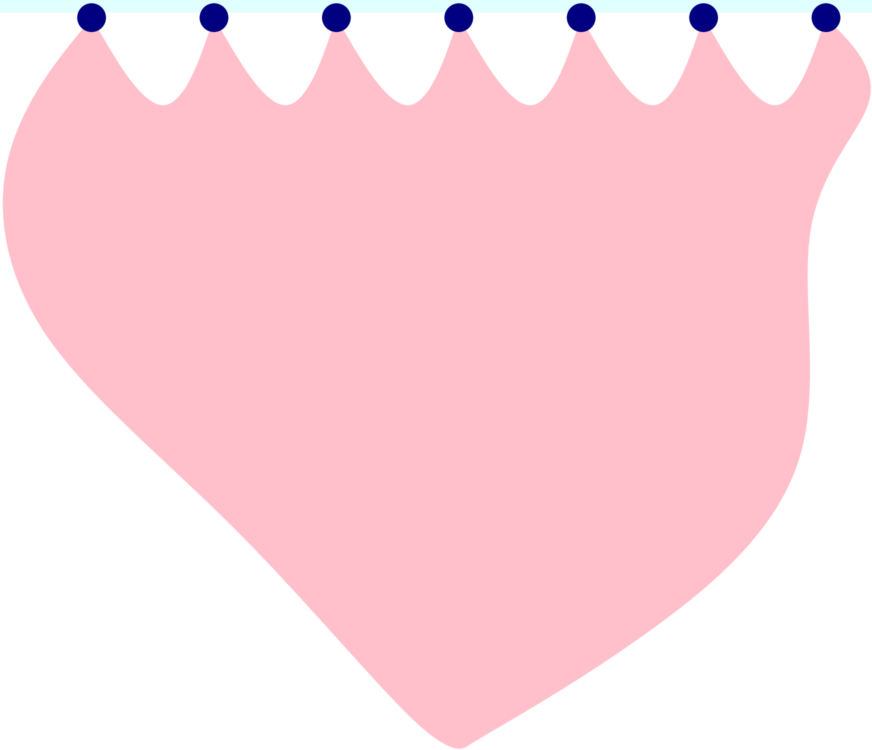
set system



realizable samples

$\downarrow \mathcal{C} := \{S \in \{\pm, 0\}^U \mid \exists T \in \mathcal{C} : S \leq T\}$

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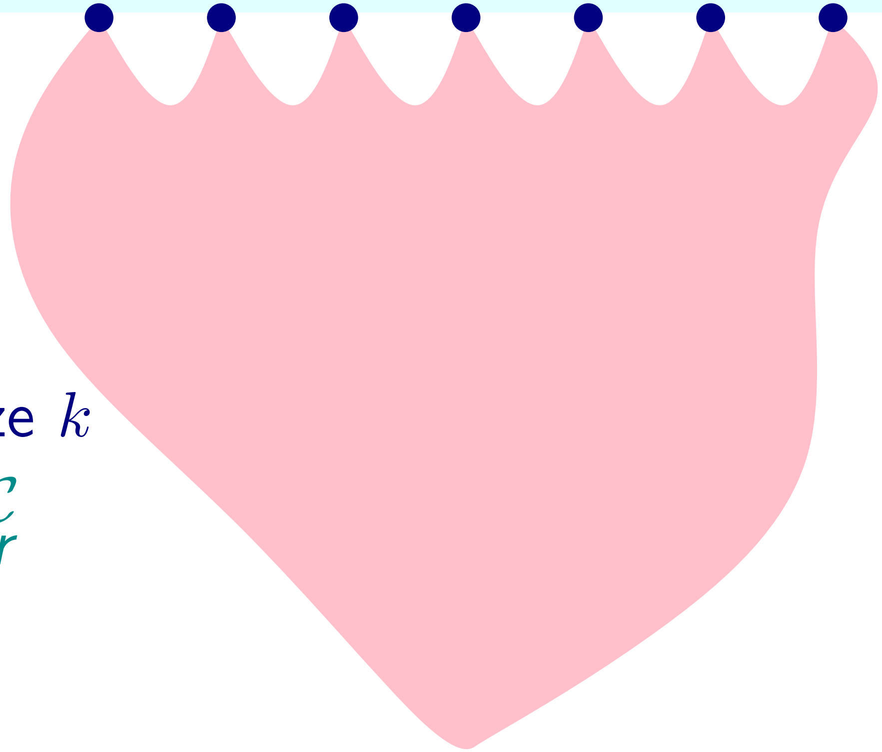

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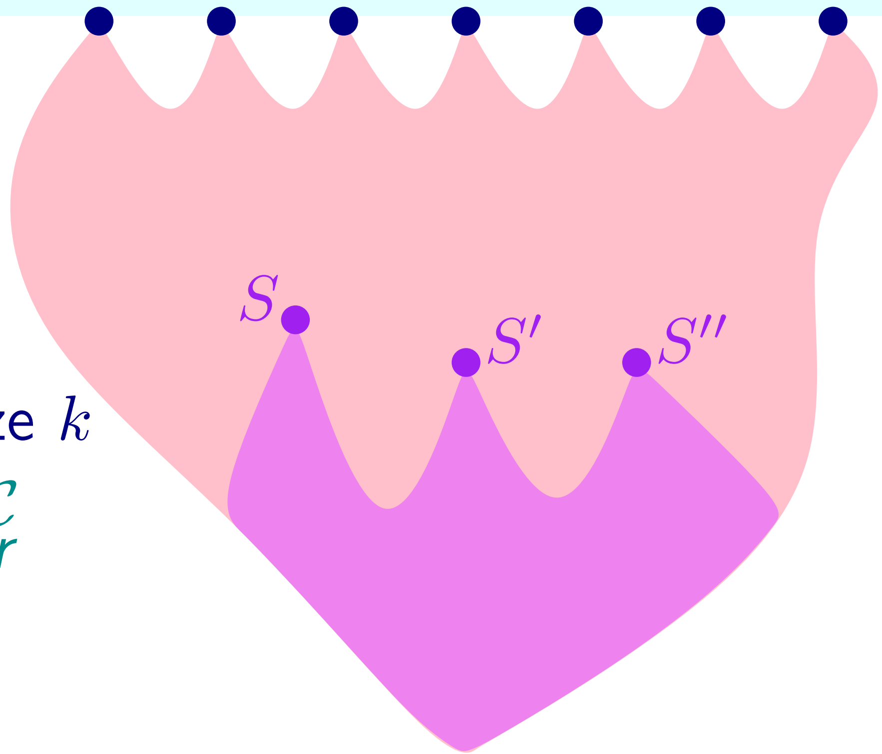

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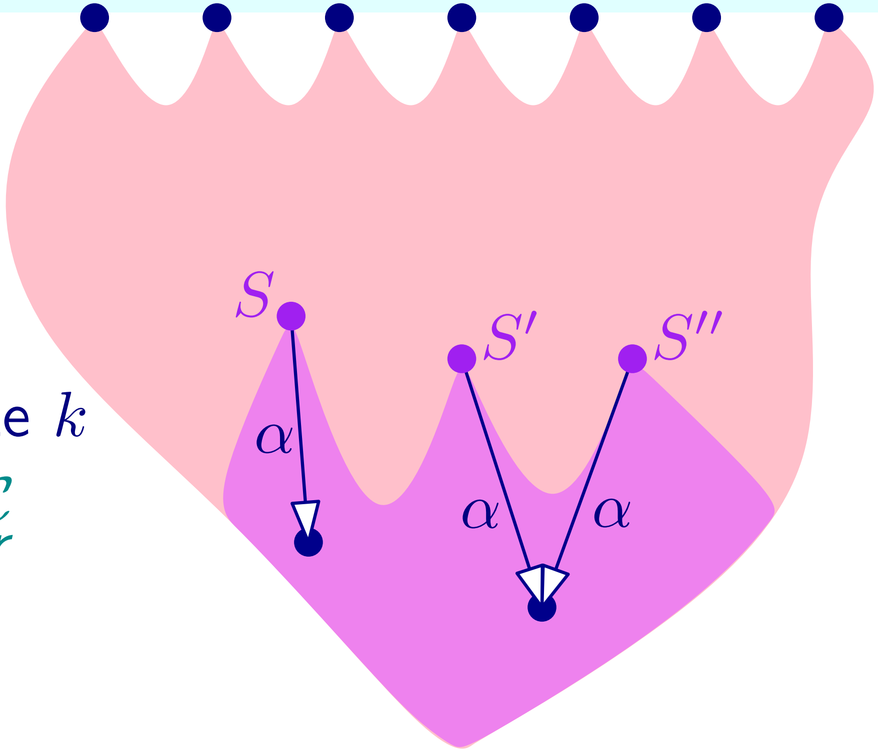

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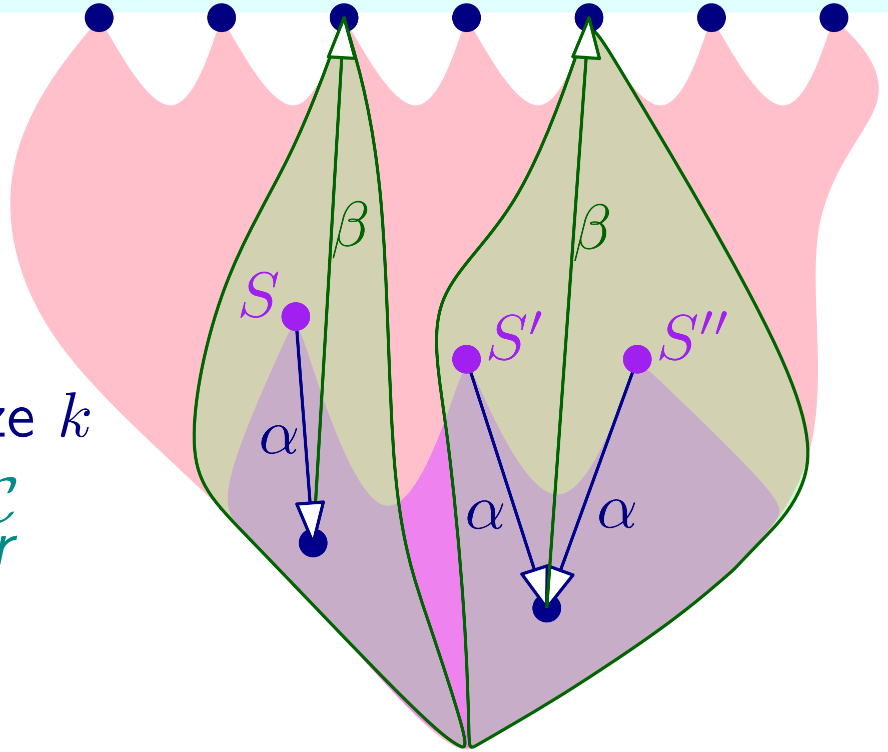

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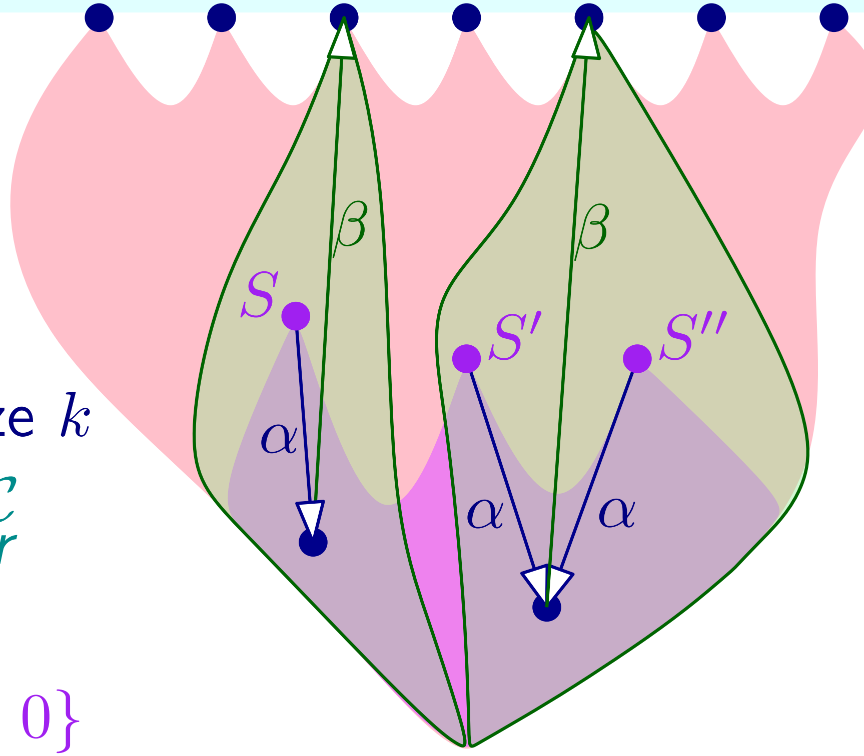

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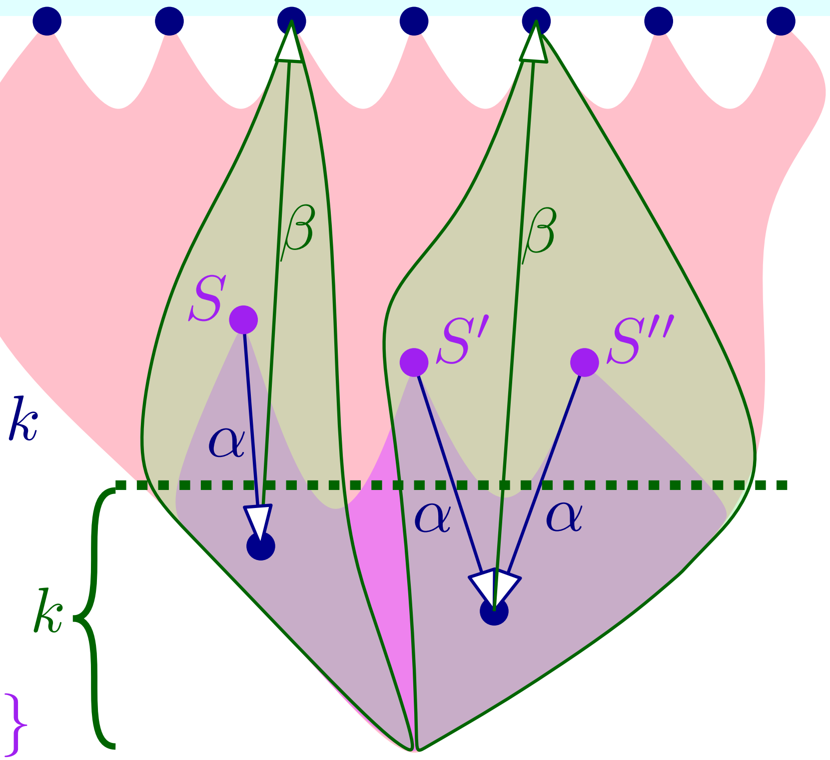

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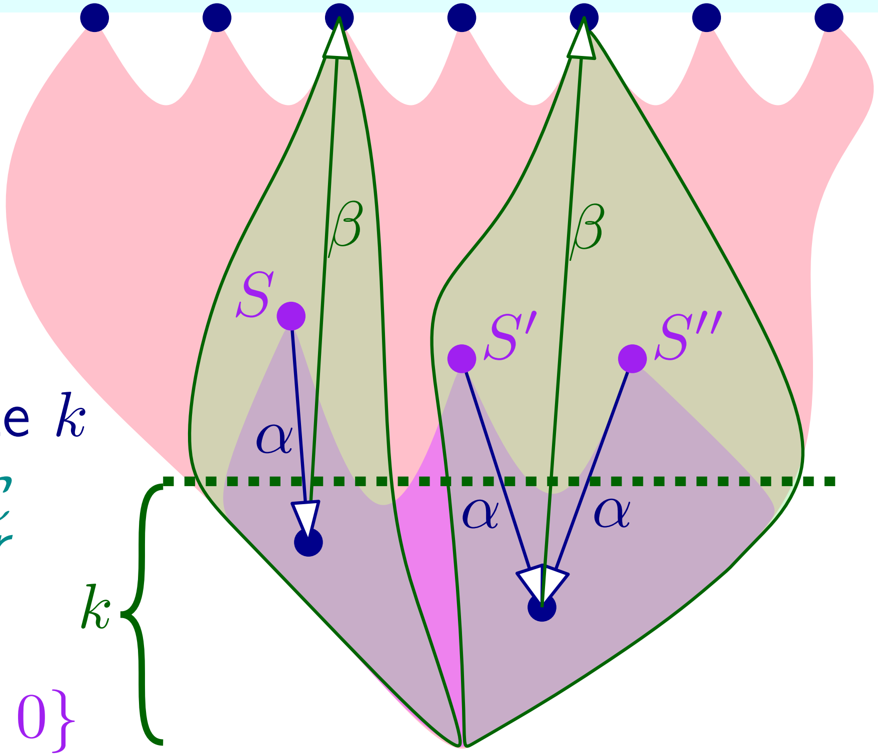

Conj[Floyd, Warmuth '95]:
concept class $\mathcal{C}$ of VC-dim d admits sample compression scheme of size $O(d)$

labelled sample compression

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subgraph of cube ⟷ set system

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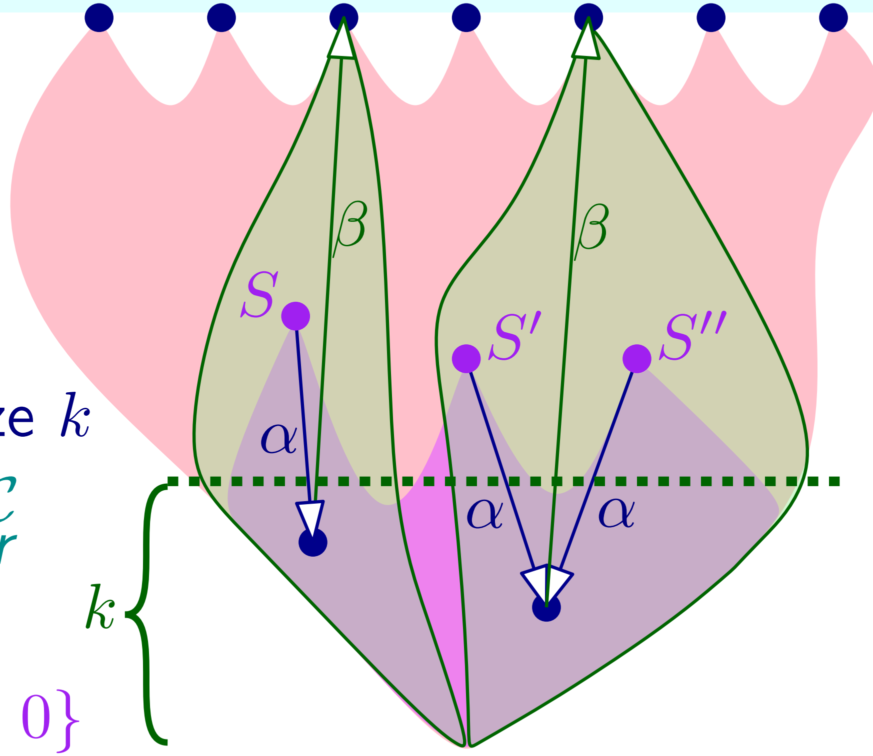

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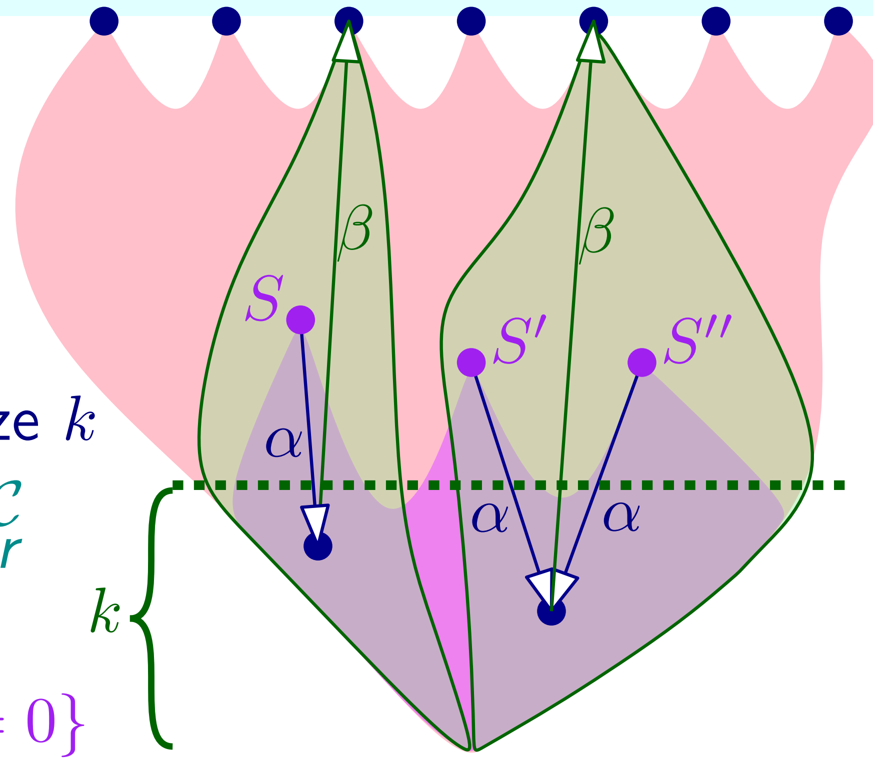

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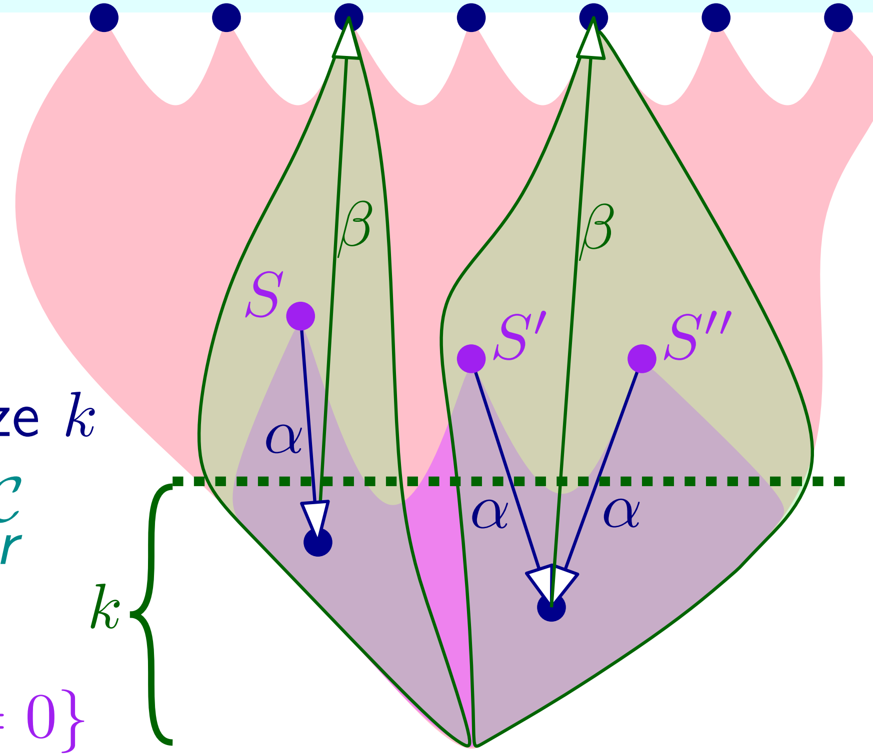

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Conj[Floyd, Warmuth '9[illegible]]:

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idea: try to complete $\mathcal{C}$ to AMP of same rank and then use MW

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subgraph of cube ⟷ set system

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improper labelled compression scheme of size k

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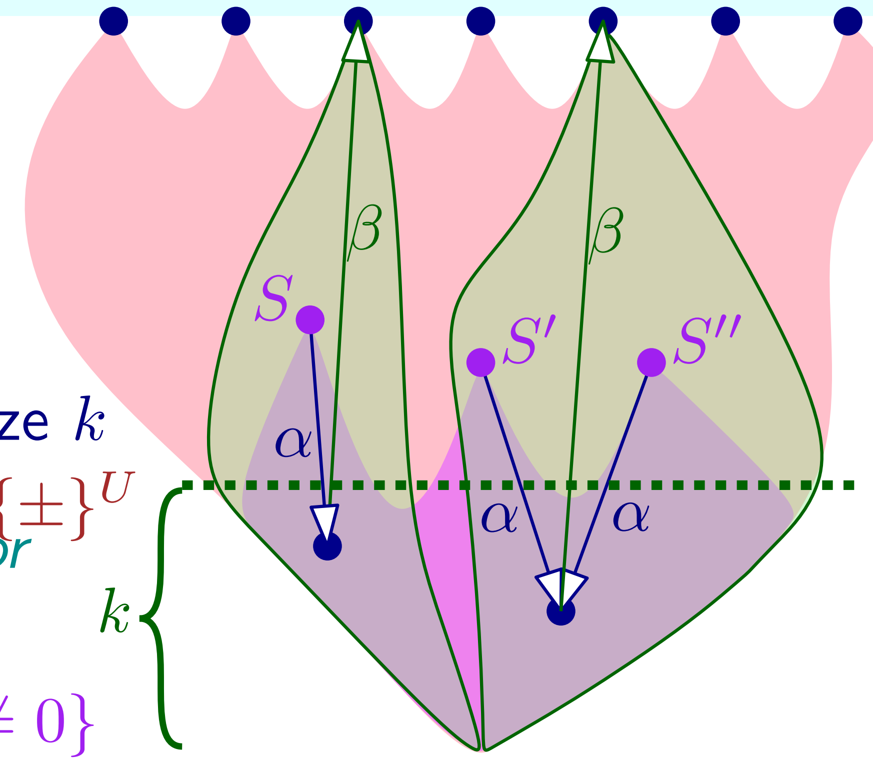

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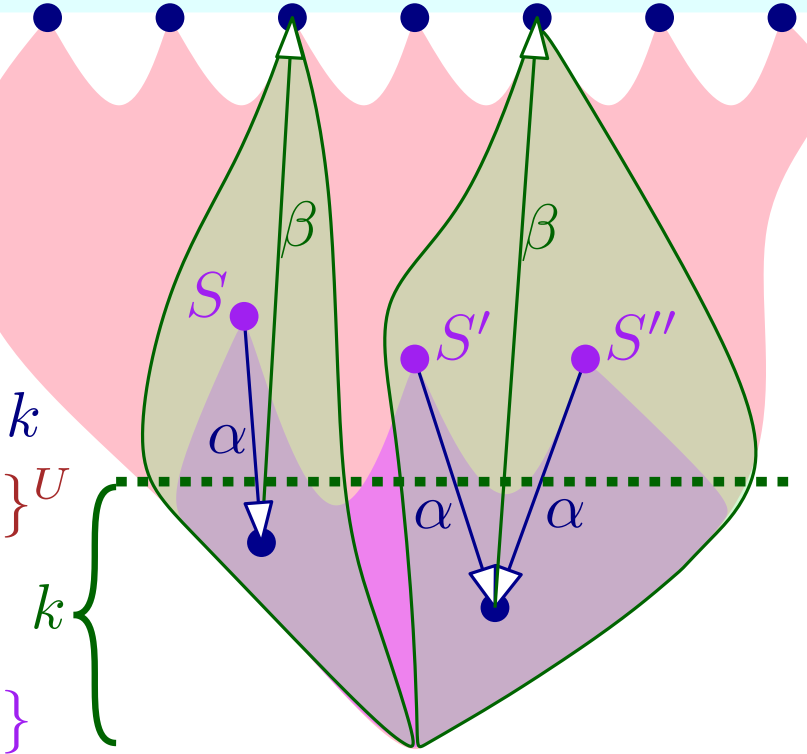

Conj[Floyd, Warmuth '[illegible]]:
concept class $\mathcal{C}$ of rank dim d admits sample compression scheme of size $O(d)$

known of size d for $\mathcal{C}$ (tope graphs of):
- realizable AOM (Ben-David, Litmann '89)
- AMP (Moran, Warmuth '16)

idea: try to complete $\mathcal{C}$ to AMP of same rank and then use MW
- rank 2 partial cubes (Chepoi, K, Philibert '20)
- OMs and CUOMs (Chepoi, K, Philibert '21)

labelled sample compression

concepts $\mathcal{C} \subseteq \{\pm\}^U$

subgraph of cube ⟷ set system

realizable samples

$\downarrow\mathcal{C} := \{S \in \{\pm, 0\}^U \mid \exists T \in \mathcal{C} : S \leq T\}$

improper labelled compression scheme of size k

$\alpha : \downarrow\mathcal{C} \to \downarrow\mathcal{C}$ *compressor*

$\beta : \alpha(\downarrow\mathcal{C}) \to \{\pm\}^U$ *reconstructor*

$\alpha(S) \leq S \leq \beta(\alpha(S)) \quad \forall S \in \downarrow\mathcal{C}$

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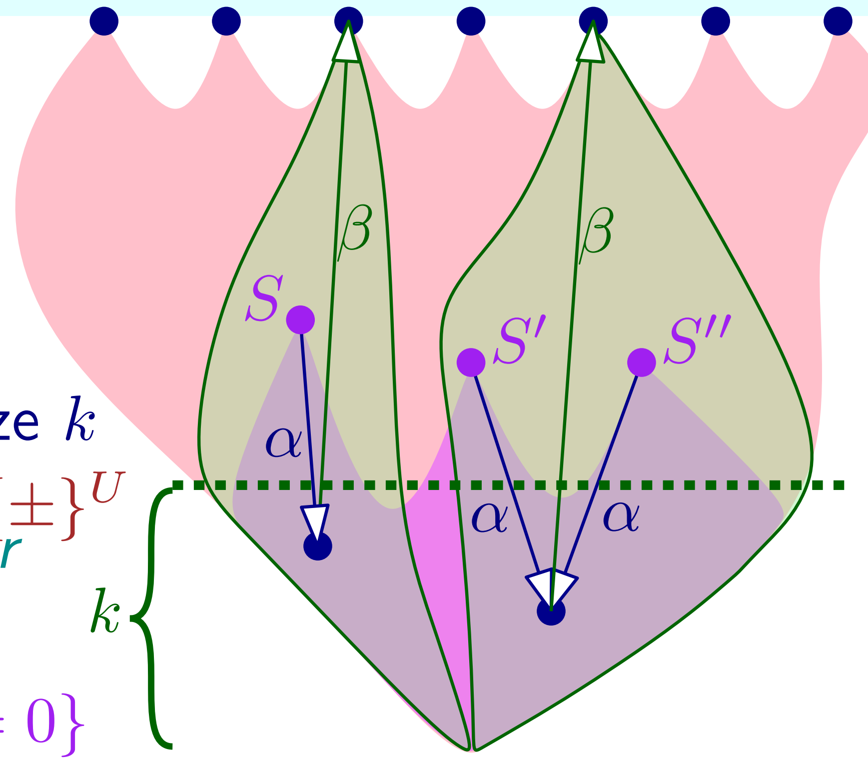

Conj[Floyd, Warmuth '9]:

concept class $\mathcal{C}$ of rank dim d admits sample compression scheme of size $O(d)$

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Conj[Chepoi, K, Philibert '21]: COMs admit AMP completion of same rank

labelled sample compression

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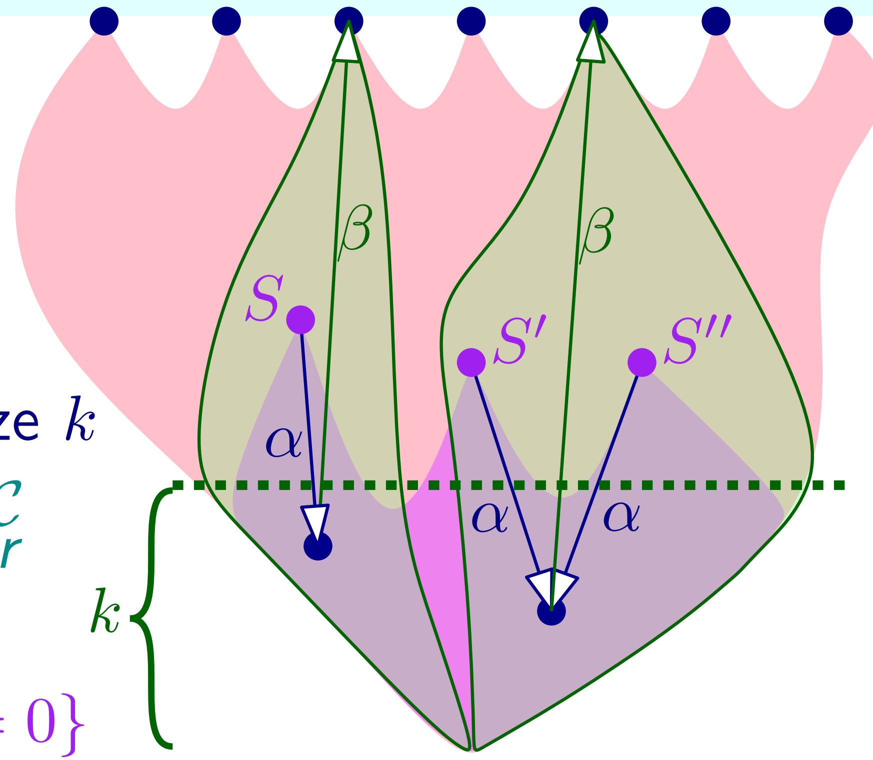

proper labelled compression scheme of size k

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Conj[Floyd, Warmuth '9]:

concept class $\mathcal{C}$ of dim d admits sample compression scheme of size $O(d)$

rank

known of size d for $\mathcal{C}$ (tope graphs of):

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Thm[Chepoi, K, Philibert '21$^+$]:

COMs of rank d admit proper labelled sample compression scheme of size d

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concepts $\mathcal{C}$ → tope graph $G \subseteq \{\pm\}^E$

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realizable samples $\downarrow \mathcal{C}$ → convex subgraphs

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$\beta : S' \to v \in S$

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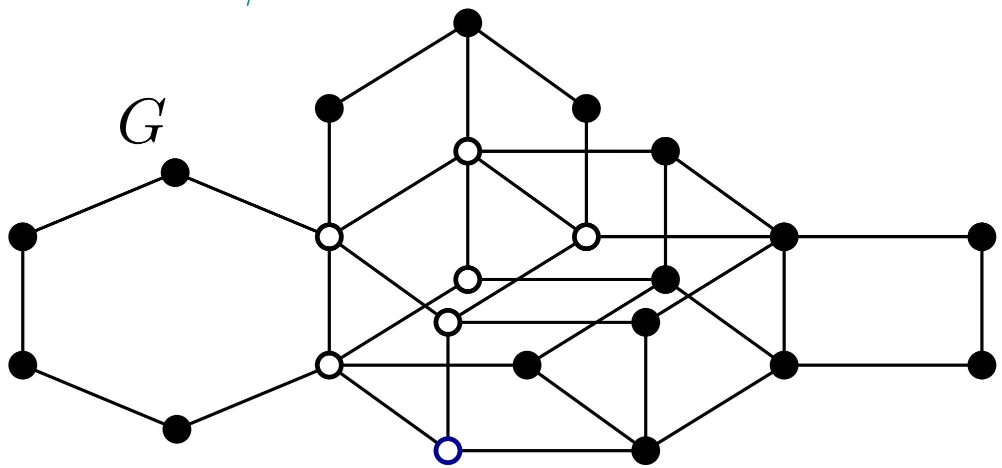

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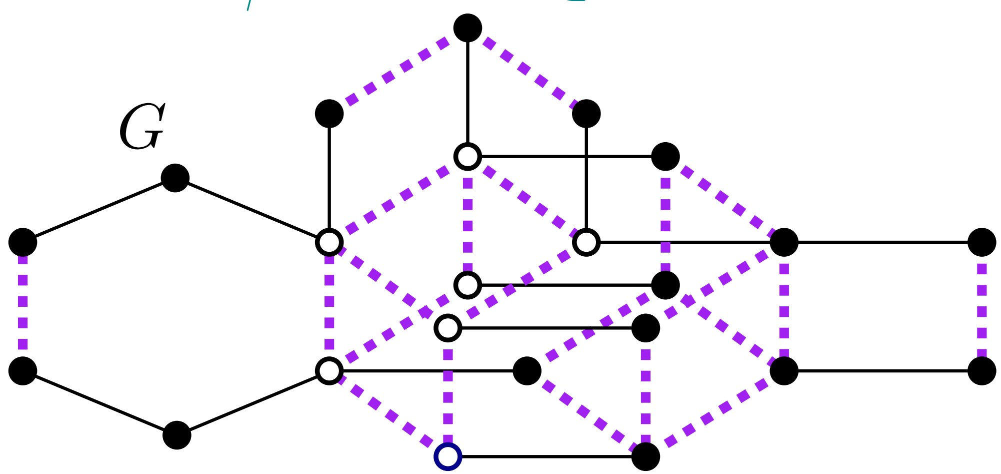

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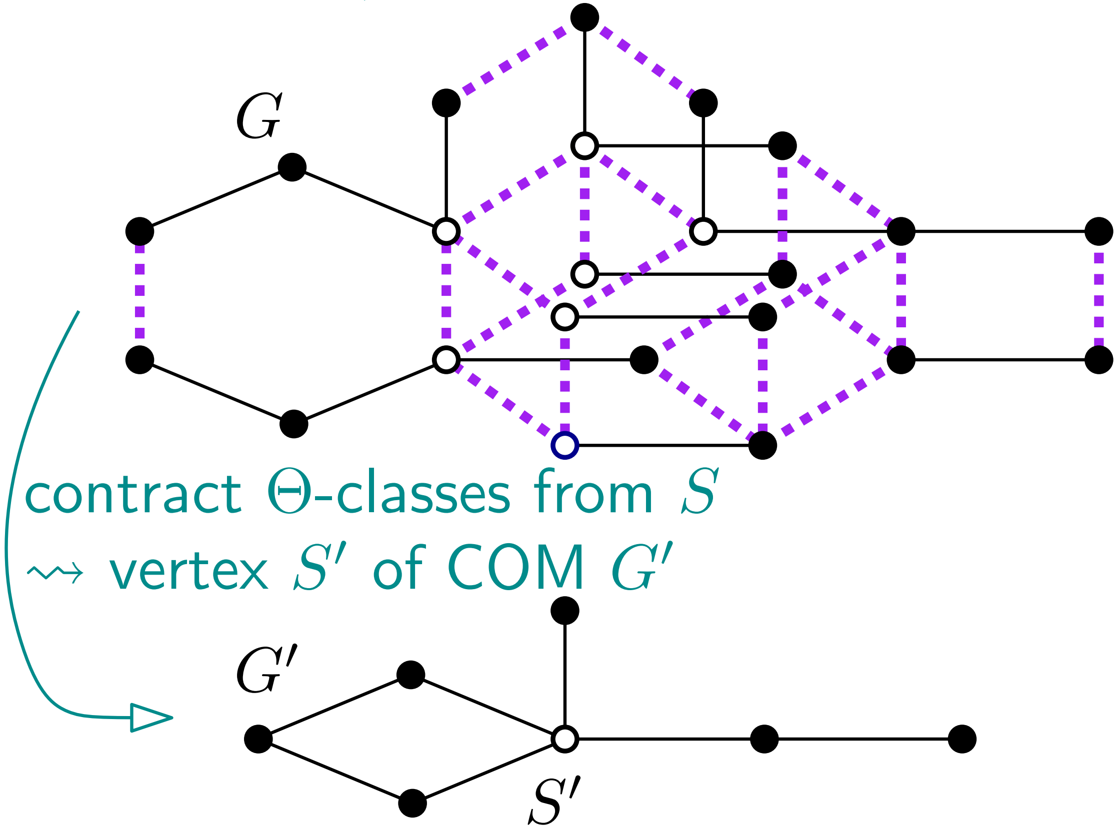

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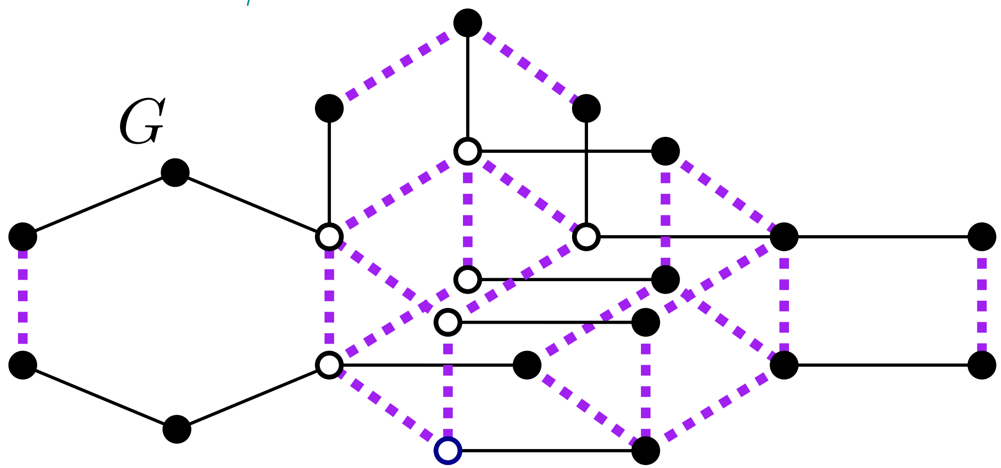
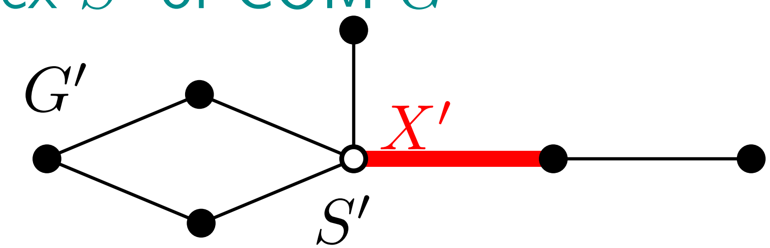


contract Θ-classes from S
$\rightsquigarrow$ vertex S' of COM G'

pick an OM-face X' containing S'



Thm[Chepoi, K, Philibert '21$^+$]:
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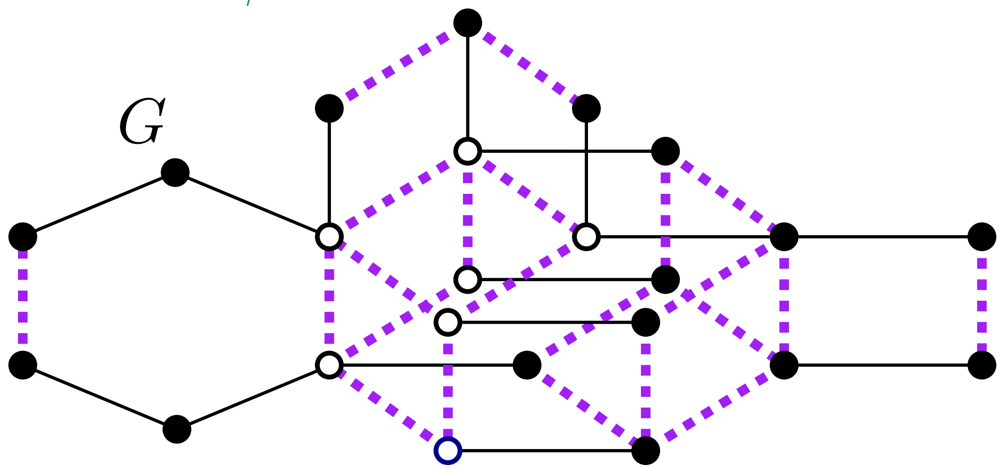

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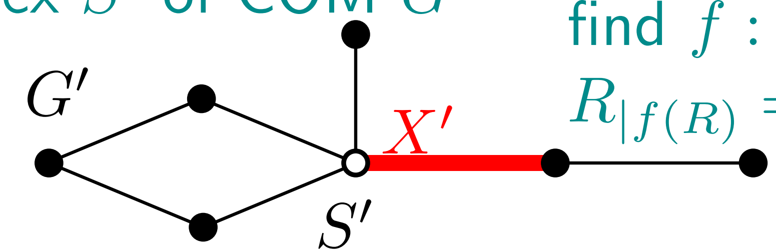

pick an OM-face X' containing S'

find $f : X' \to \binom{\Theta\text{-classes}}{d}$, such that

$R_{|f(R)} = T_{|f(T)} \Rightarrow R = T \qquad \forall R, T \in X'$

$\rightsquigarrow D := f(S'), \alpha(S) := S'_{|D}$

Thm[Chepoi, K, Philibert '21$^+$]:
COMs of rank d admit proper labelled sample compression scheme of size d

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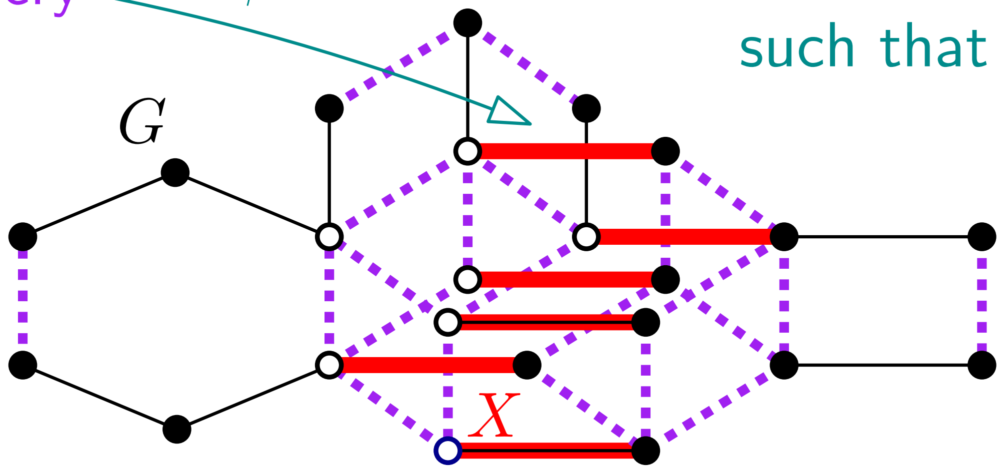
take minimal face X in G, crossed by D, such that contracting all other yields cube

gallery



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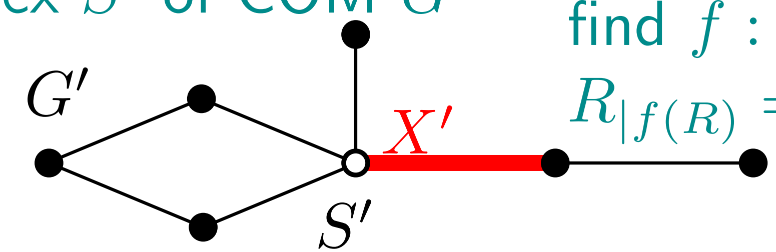

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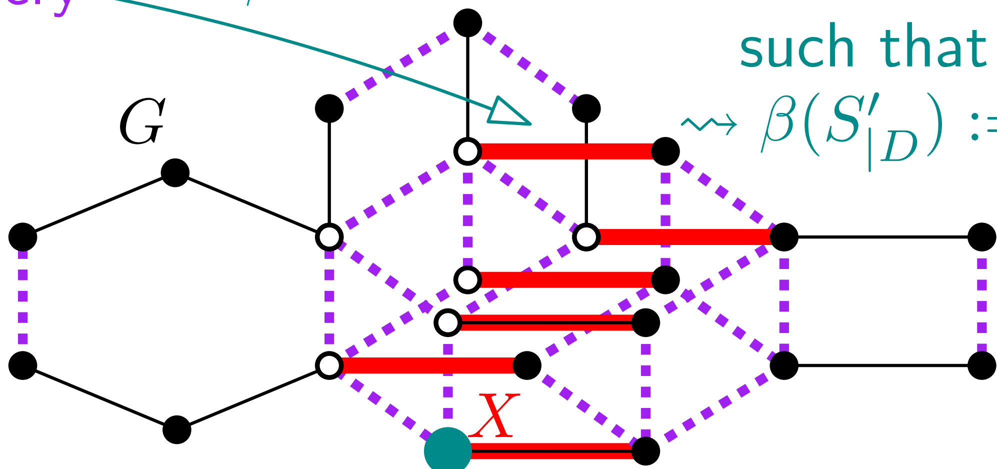
$\rightsquigarrow \beta(S'_{|D}) := T \in X$ such that $T_{|f(T)} = S'_{|D}$

gallery



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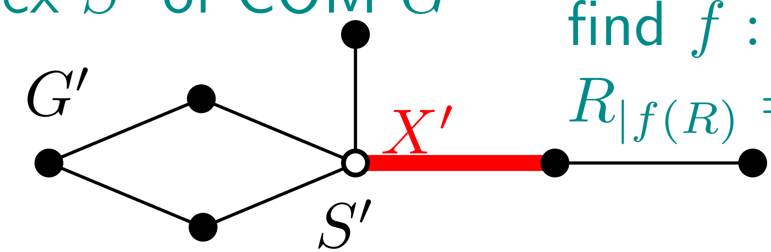

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gallery

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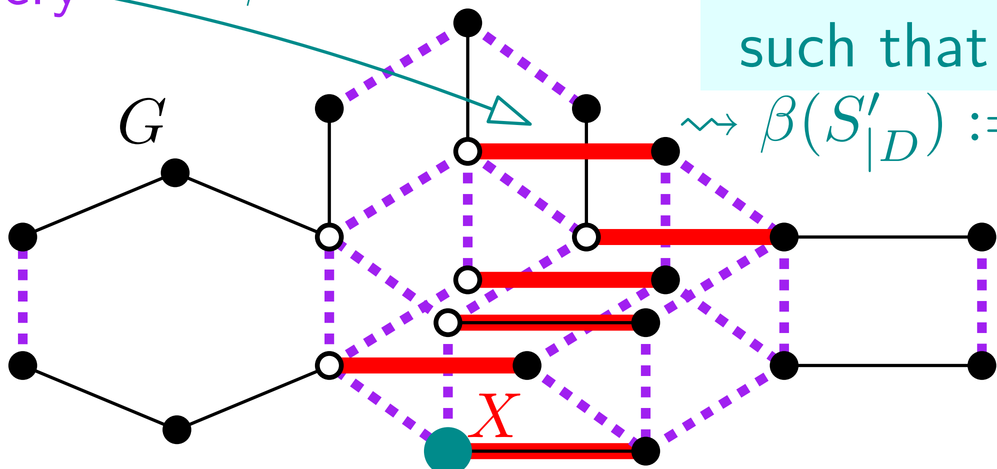

much easier if AMP, because X, X' cubes

contract Θ-classes from S $\rightsquigarrow$ vertex S' of COM G'

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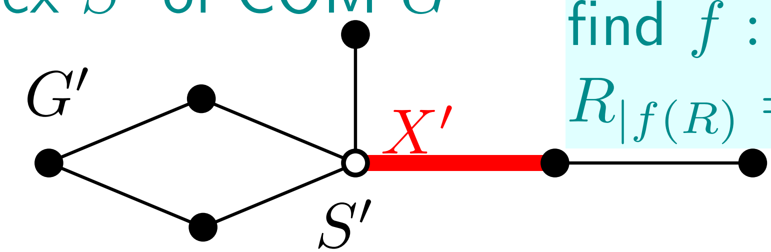

$\rightsquigarrow D := f(S'), \alpha(S) := S'_{|D}$

corners and unlabeled sample compression

computational learing theory

Conj[Kuzmin, Warmuth '04]: Every LOP has a *corner peeling*.

corner peelings yield proper **unlabeled** compression

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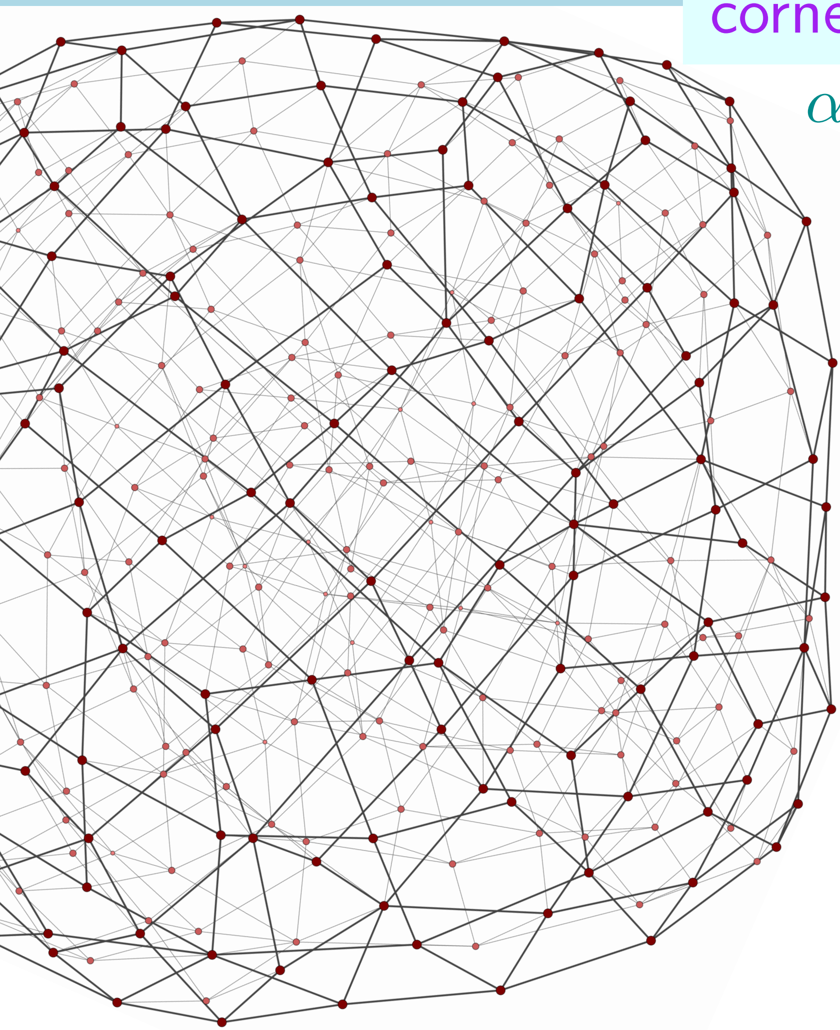

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Thm[Chalopin, Chepoi, Moran, Warmuth '18]:
$\exists$ AMP without corner peeling

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$\rightsquigarrow$ generalize corner peelings to COMs

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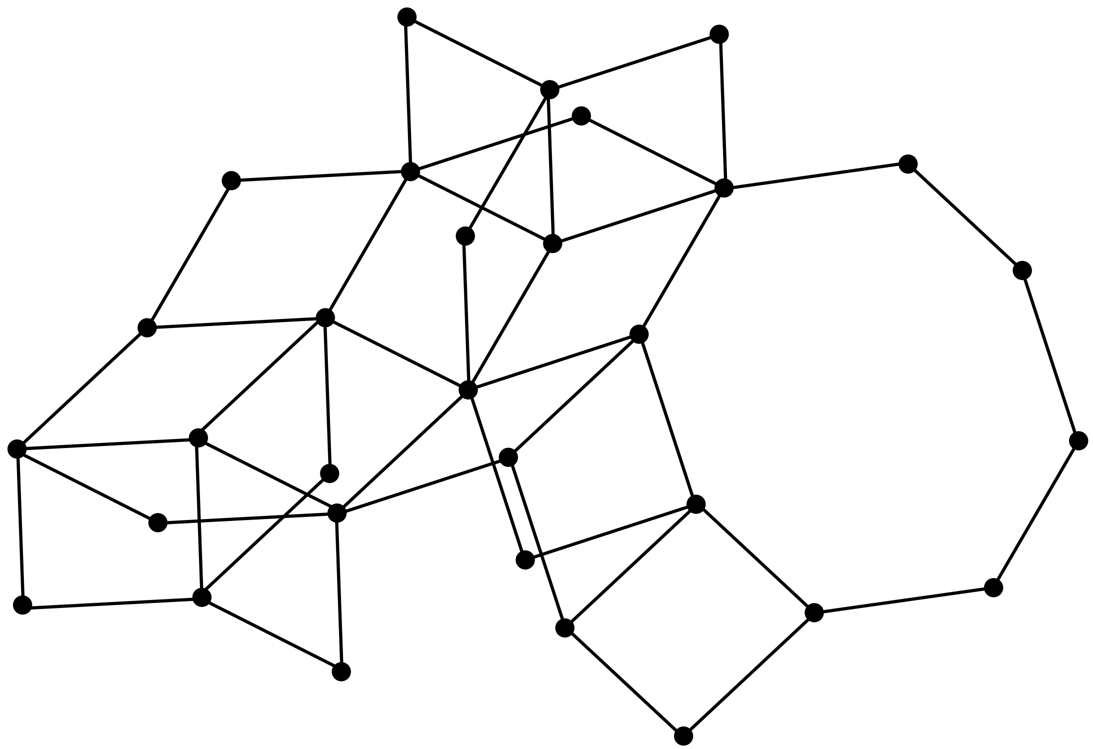

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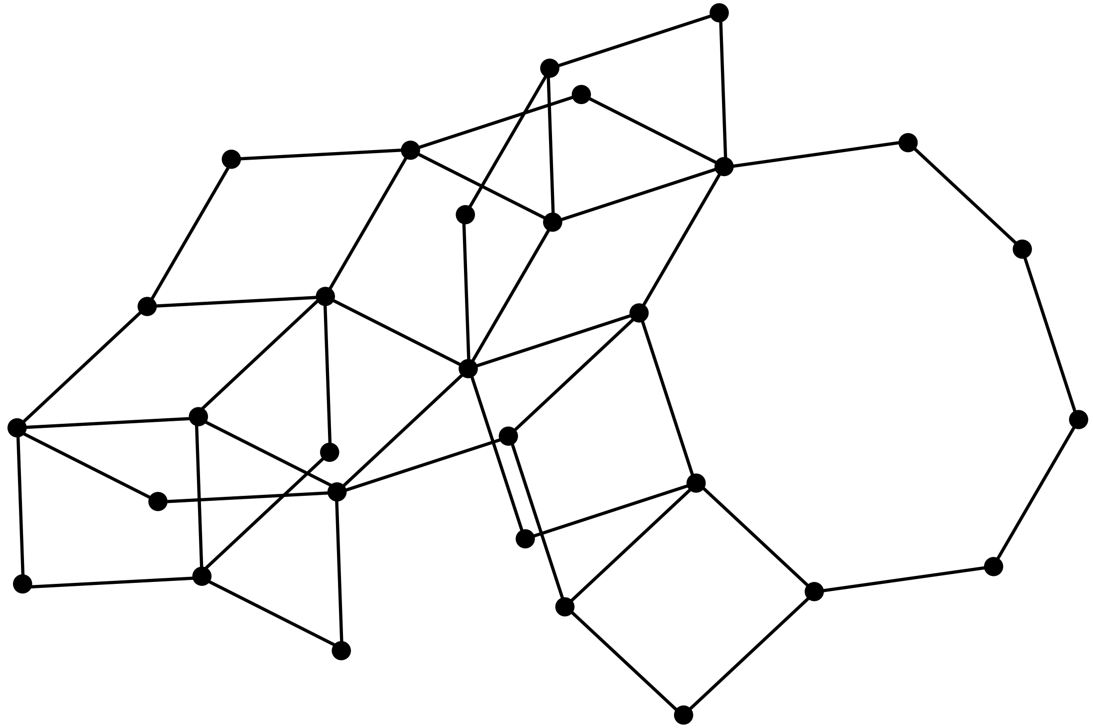

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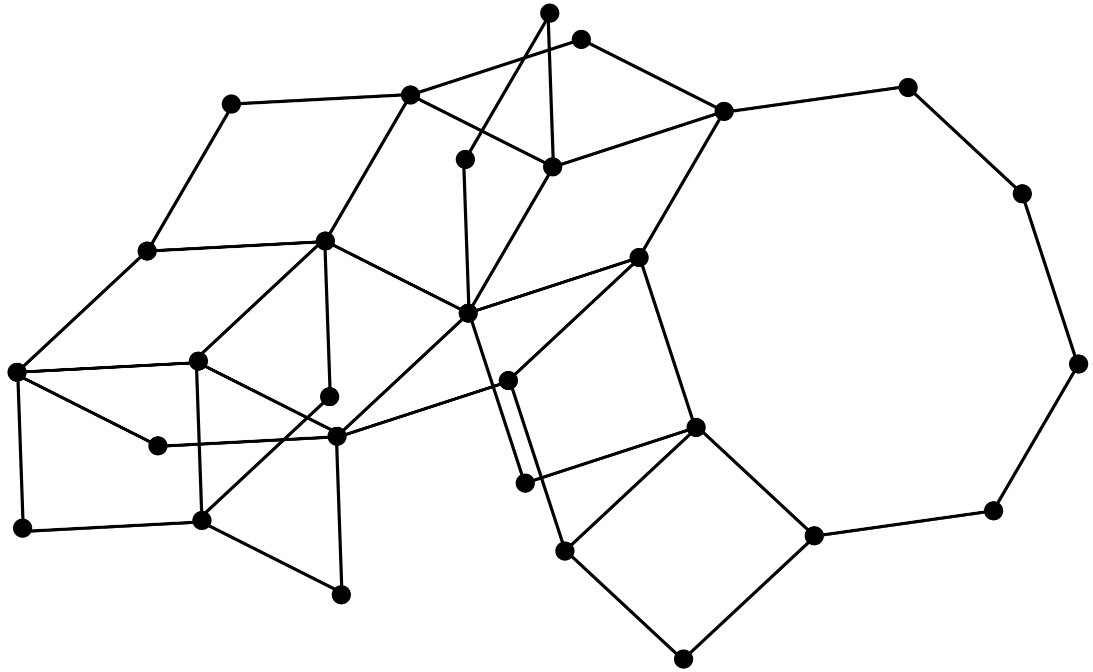

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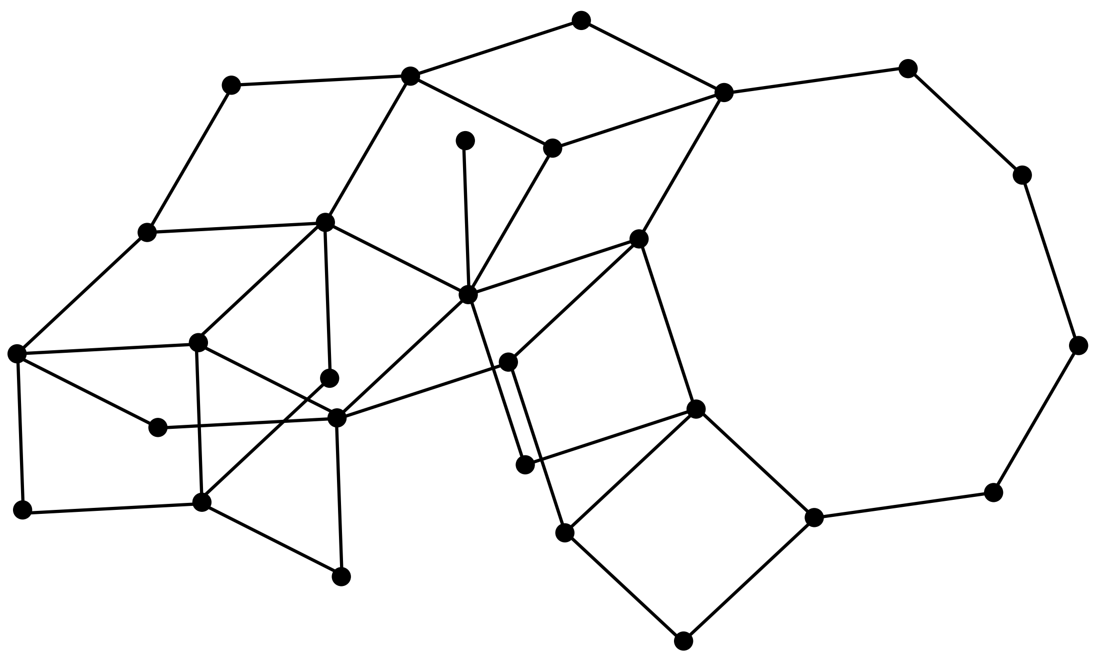

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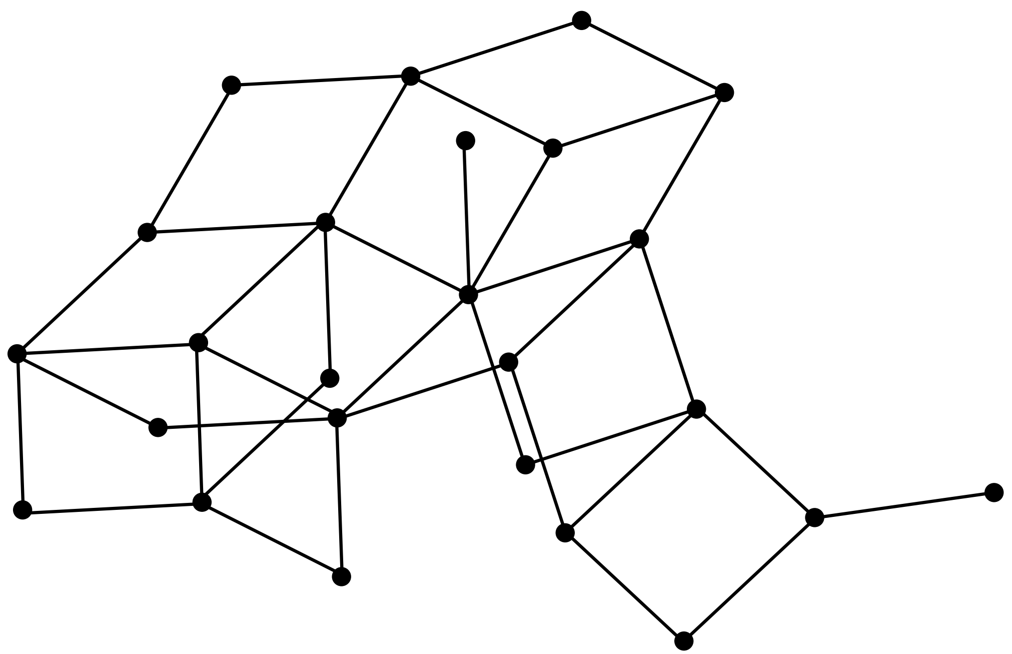

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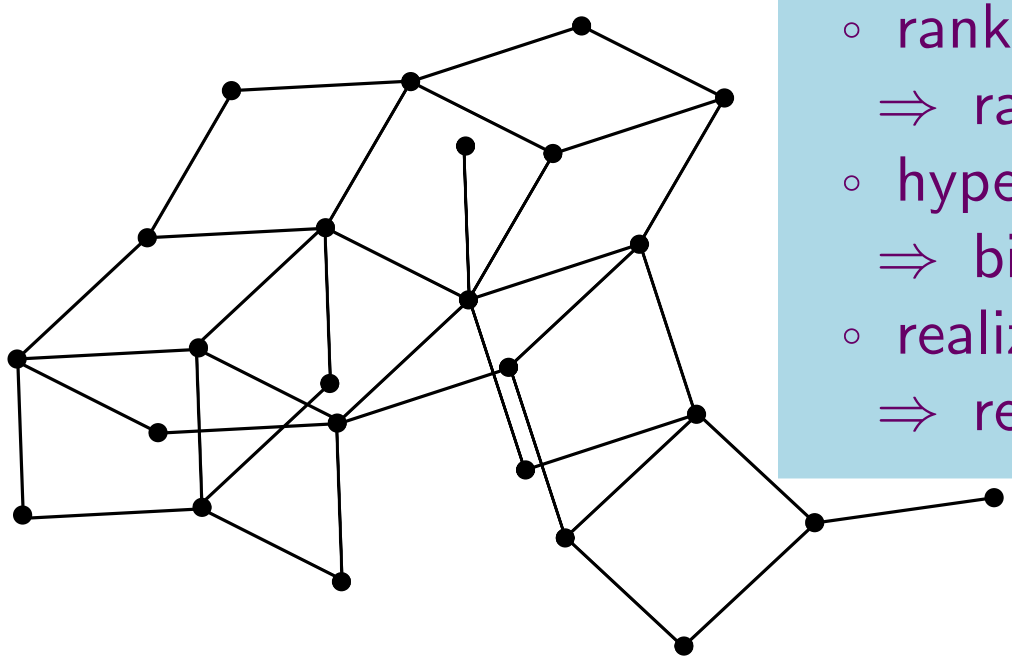

Thm[K, Marc '20]: corner peelings for:

- rank 2 COMs
  $\Rightarrow$ rank 2 AMPs [Chalopin et al '18]
- hypercellular graphs
  $\Rightarrow$ bip. cellular graphs [Bandelt, Chepoi '96]
- realizable COMs
  $\Rightarrow$ realizable AMPs [Tracy Hall '04]

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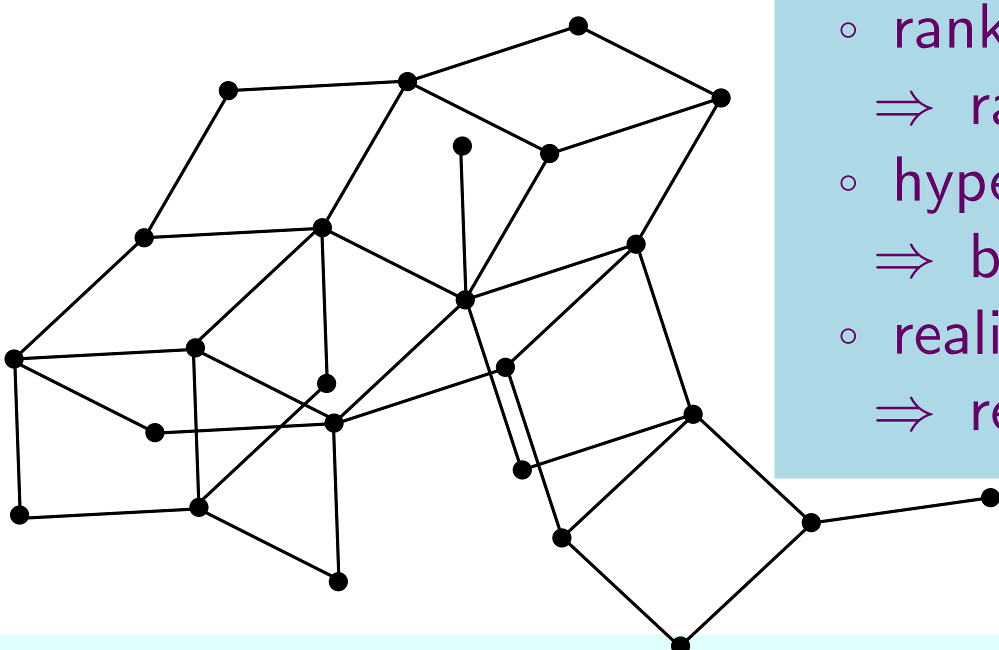

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do corner peelings of COMs yield *unlabeled compression schemes* of COMs?

last slide

proper labelled sample compression

- partial cubes
- OM-polyhedra (Bland '74)
- bouquets of oriented matroids (Deza, Fukuda '86)
- CW-left-regular bands (Margolis, Saliola, Steinberg '18)

improper labelled sample compression by completion

$$\text{set system} \overset{?}{\rightsquigarrow} \text{partial cube} \overset{?}{\rightsquigarrow} \text{COM} \overset{?}{\rightsquigarrow} \text{AMP}$$

corners

$$\text{corner peelings of COMs} \overset{?}{\Longrightarrow} \text{unlabeled compression schemes}$$

last slide

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thank you