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Nonequilibrium phase transitions in perturbed particle systems

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Introduction

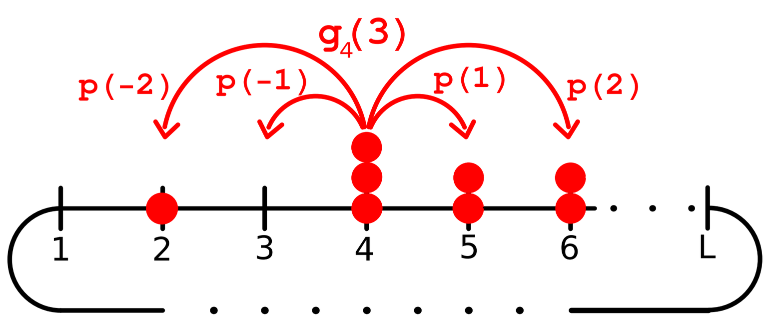
Emergent phenomena are ubiquitous in systems with a large number of interacting components. Such systems arise across physical and social sciences. Examples include traffic behaviour and ribosomes moving along messenger RNA in protein synthesis. These systems can be studied mathematically in the context of nonequilibrium statistical mechanics applied to interacting particle systems [1,2].

Figure: Phase transitions in nonequilibrium systems can be inconvenient.

This work focuses on one particular system, the zero-range process (ZRP), in which particles move on a lattice with jump rates specified by local interactions. The ZRP has been widely applied as a model for nonequilibrium phenomena and as an effective model for the dynamics of phase boundaries such as in exclusion processes [3]. Although the steady state is given exactly by a factorised form it still displays non-trivial properties such as the possibility of a condensation transition. Previous studies of the ZRP have assumed the interactions between particles are spatially homogeneous [4]. This is not typically the case for real systems. We consider the effect of a random perturbation of the jump rates and show that a small perturbation can have a significant effect on the behaviour of the system [5].

The Zero Range Process

The ZRP is a continuous time Markov Chain with state space $X_L = \mathbb{N}^{\Lambda_L}$ in which particles jump on a lattice $\Lambda_L = \{1, \dots, L\}$ at rates which depend only the departure site and the number of particles there. Configurations are given by $\boldsymbol{\eta} = (\eta_x)_{x\in\Lambda}$ where η_x is the occupation number of site x. Particles leave site x at rate $g_x(\eta_x)$ and jump to site $x+y$ with probability $p(y)$.



Generator:

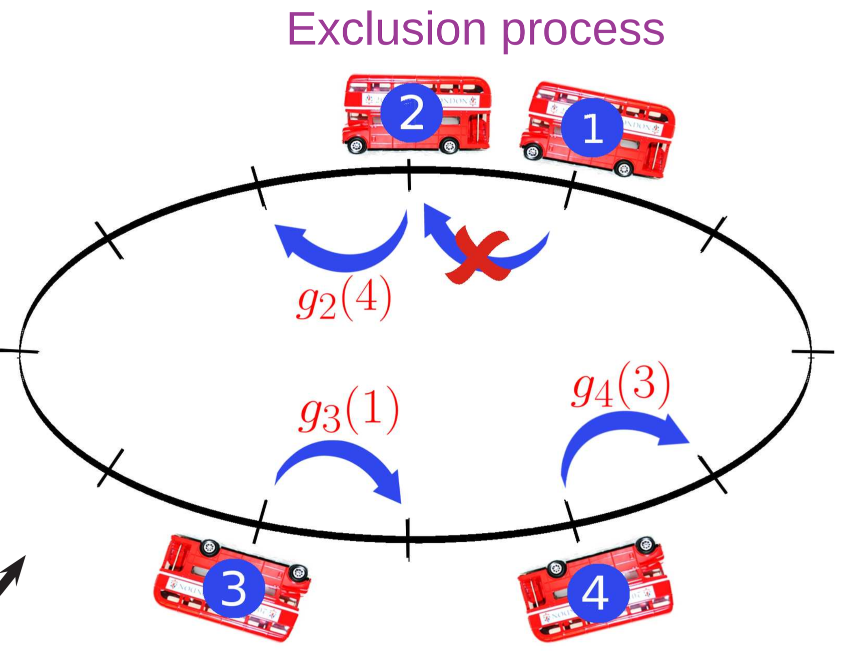
$$(\mathcal{L}f)(\boldsymbol{\eta}) = \sum_{x,y\in\Lambda_L} g(\eta_x)p(y)\left[f(\boldsymbol{\eta}^{x,y}) - f(\boldsymbol{\eta})\right]$$

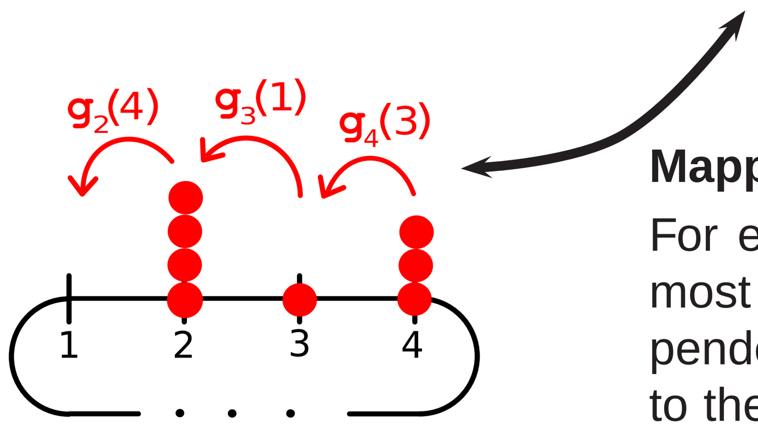
Motivation

The Bus route model

A totally asymmetric exclusion process is an example of a coarse-grained description, internal structures are integrated out leaving effective dynamics depending only on the distance between particles.

Exclusion process





ZRP

Mapping to the ZRP

For exclusion processes each site contains at most one particle and they move at rates dependent on the particle and the number of sites to the next particle. The mapping between this and the zero range process is illustrated.

Steady states

Canonical ensemble: Fixed number of particles $\sum_{x=1}^{L}\eta_x = N$,

$$\mu^{N,L}(\boldsymbol{\eta}) = \frac{1}{Z(N,L)}\prod_{x=1}^{L} w_x(\eta_x)\delta\left(\Sigma_x\eta_x - N\right),$$

where the stationary weights are given by $w_x(n) = \prod_{k=1}^{n} g_x(k)^{-1}$.

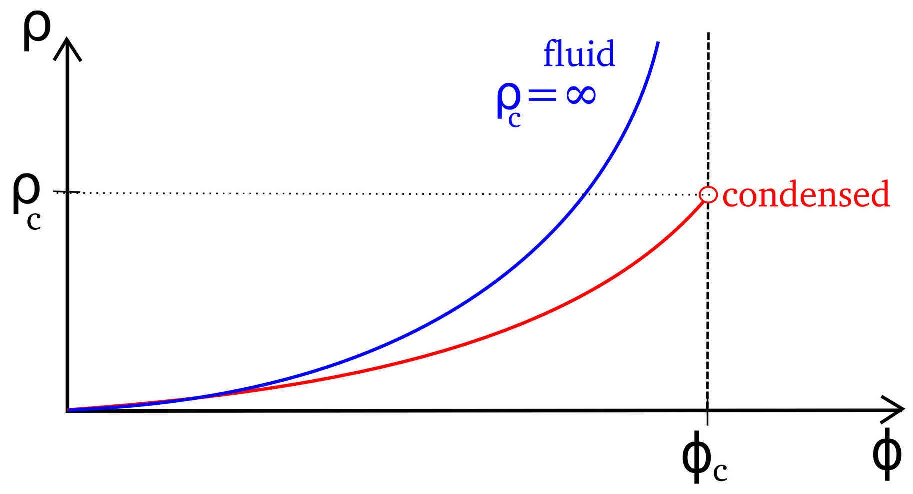
Grand canonical ensemble: Expected number of particles is fixed by the fugacity ϕ. Steady state ν_ϕ^L is given by product of one site marginals,

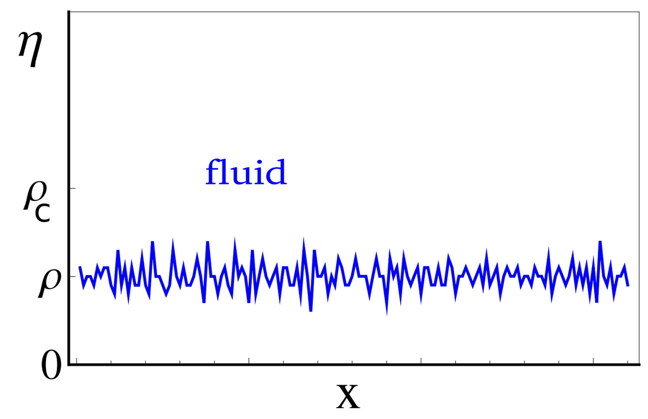
$$\nu_{x,\phi}(\eta_x) = \frac{1}{z_x(\phi)} w_x(\eta_x)\phi^{\eta_x},$$

where $z_x(\phi) = \sum_{n=0}^{\infty} w_x(n)\phi^n$ converges for $\phi \in [0, \phi_c)$.
The local particle density is $\langle\eta_x\rangle = \rho^x(\phi)$. Total density $\rho(\phi) = \frac{1}{L}\Sigma_x\rho^x(\phi)$ is invertible on $[0, \phi_c)$.

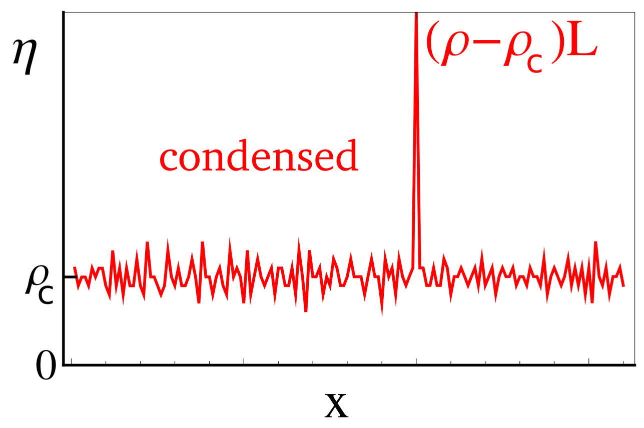
Condensation

If the jump rates $g_x(k)$ decrease with k then there can exist a critical density $\rho_c = \rho(\phi_c) < \infty$.
In the thermodynamic limit $N, L \to \infty$ with $\rho = N/L$ fixed. For $\rho > \rho_c$ excess particles condense on a single site (bellow). A similar result has also been proven rigorously for L fixed as $N \to \infty$ for a homogeneous system [6].





$\rho < \rho_c \Longrightarrow \mu^{N,L} \xrightarrow{w} \nu^{\infty}_{\phi(\rho)}$



$\rho \geq \rho_c \Longrightarrow \mu^{N,L} \xrightarrow{w} \nu^{\infty}_{\phi_c}$

Generic choice of **Jump rates**

Old (homogeneous):

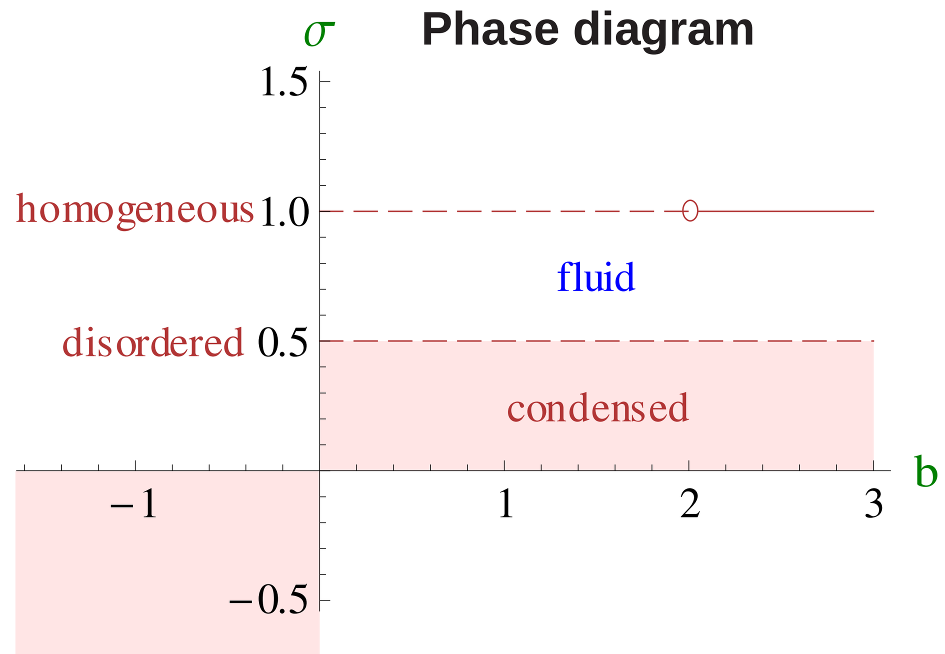
$g_x(n) = 1 + b/n^\sigma$,
$\forall n \geq 1, \quad g_x(0) = 0.$

New (disorder):

$g_x(n) = \exp\left(e_x(n) + b/n^\sigma\right)$,
$\forall n \geq 1, \quad g_x(0) = 0,$

$e_x(n)$ are iid rvs with $\mathbb{E}(e_x(n)) = 0$, variance $\delta^2 > 0$.

Phase diagram



Research Goals

- Examine stationary properties of the system in the canonical and grand-canonical ensemble from numerical studies of the current and the total entropy, $1/L \log Z(L,N)$.
- Explain finite size behaviour by studying the contribution of the condensate to the total entropy.
- √ Support theoretical results on the change of the stationary behaviour using Monte Carlo simulations [5] .

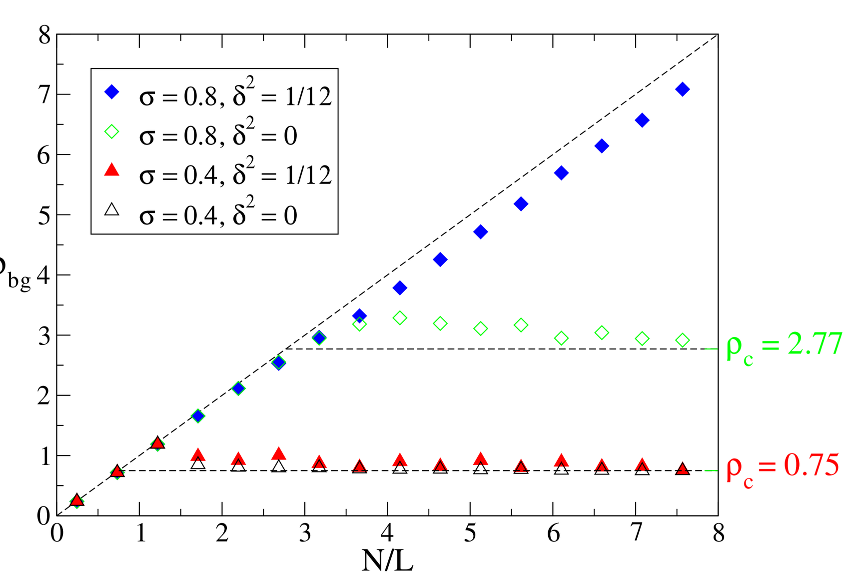
Preliminary results

Monte Carlo (MC)

Analysis of a system with fixed disorder. For MC results $L = 1024$. For $N/L > \rho_c$ $\langle\eta_x\rangle_{\mu^{N,L}} = \rho_c^x$ except for the"slowest" site, this is expected to contain the condensate. For y the slowest site we define,

$$\rho_{bg} := \frac{1}{L}\left(N - \langle\eta_y\rangle_{\mu^{N,L}}\right),$$

as $N \to \infty$ we expect $\rho_{bg} \to \rho_c$, in accordance with [6].



Numerics

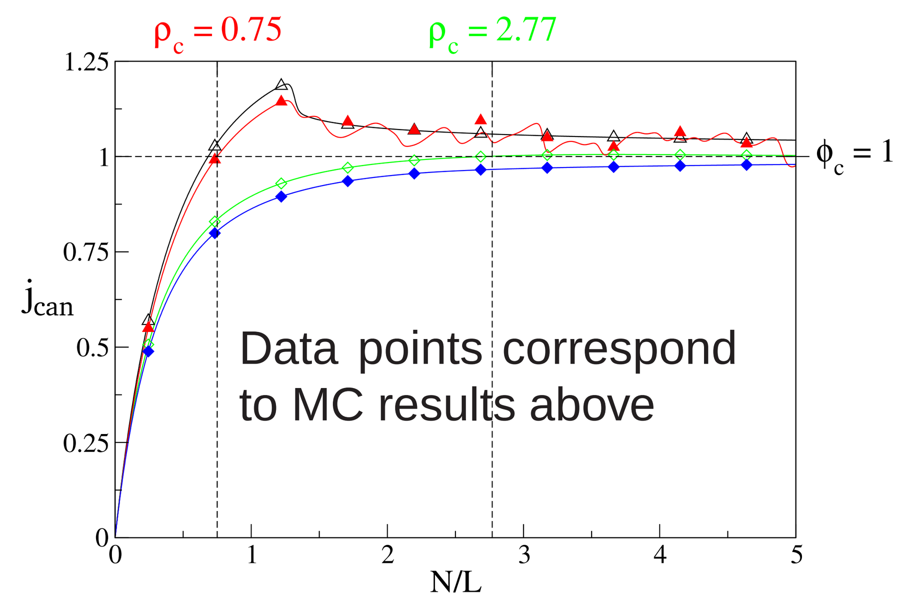
The stationary current $j = \langle g_x(\eta_x)\rangle$ is independent of the site x.
Canonical:

$$j_{can}(N/L) = \frac{Z(L, N-1)}{Z(L,N)}$$

Grand canonical:

$$j_{gcan}(\rho) = \phi(\rho) \quad \text{inverse of} \quad \rho(\phi)$$

$$\Longrightarrow \quad j_{gcan}(\rho) \leq \phi_c = \phi(\rho_c).$$



References

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[5] S. Großkinsky, P. Chleboun and G.M. Schütz. (Pre-print), Available from: http://arxiv.org/abs/0805.2748v1.
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