Motility of active fluid drops on surfaces

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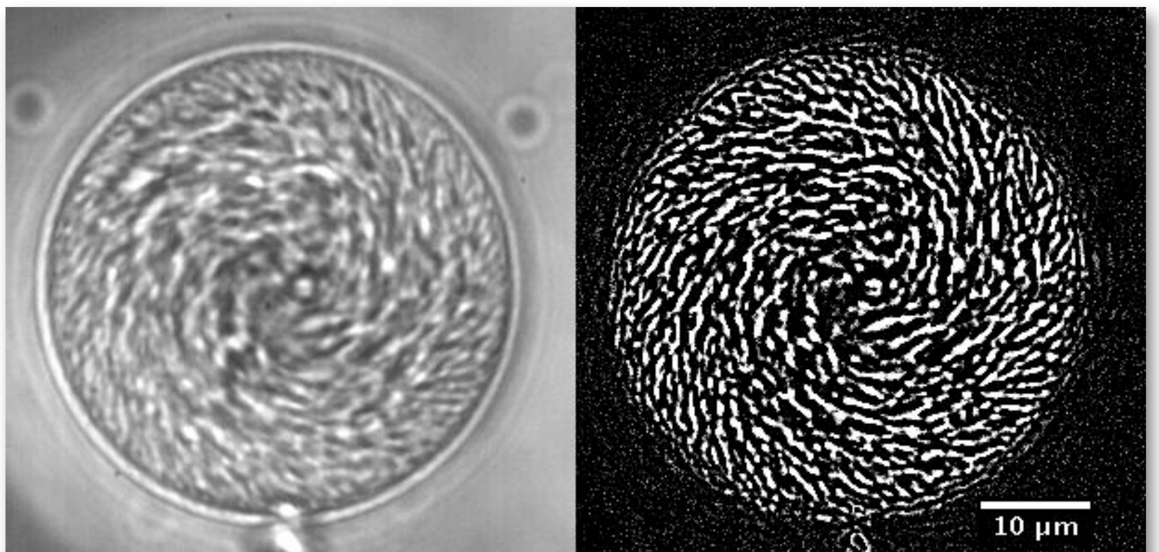
Active fluids confined to drops

Active fluids confined to droplets exhibit many biomimetic behaviours:

complex large-scale flows



[H. Wioland et al., PRL 2013]

self-driven motility

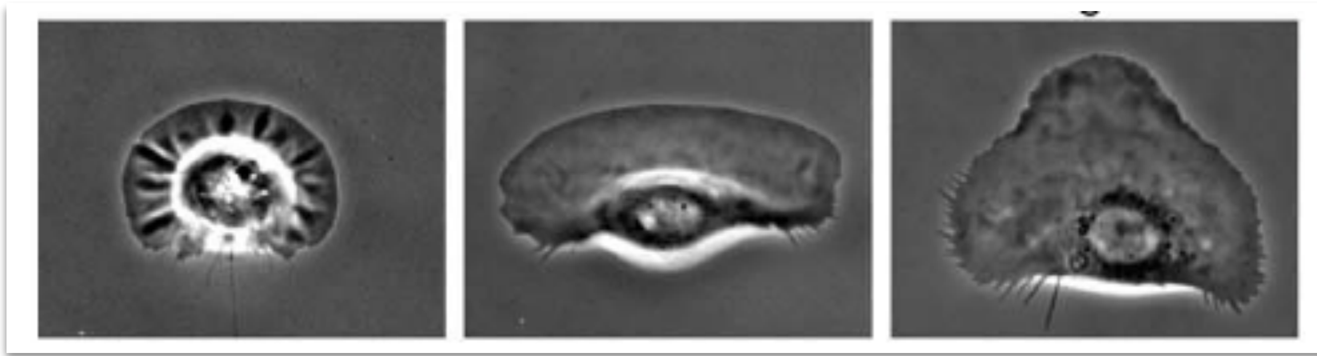
Active surface nematic
drives water droplet motility

50μm bar

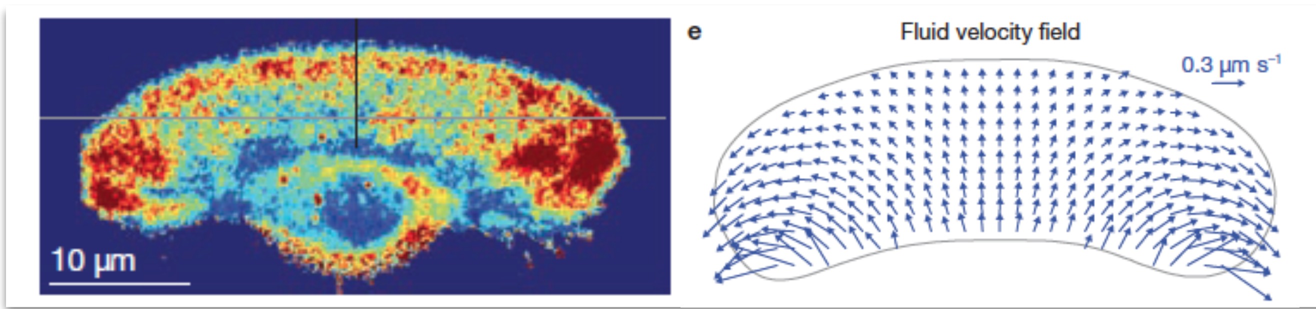
[T. Sanchez et al., Nature 2012]

How does activity interact with geometrical confinement to create these behaviours, which are not seen in bulk active systems?

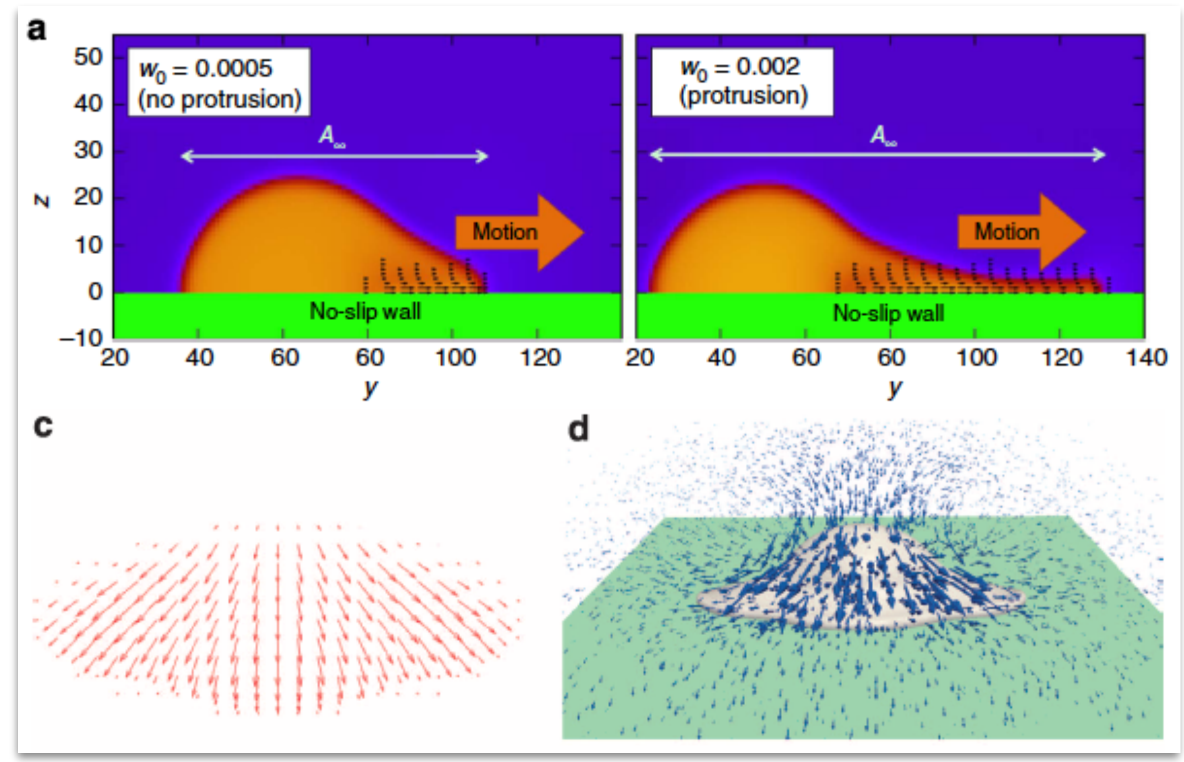
Active fluid drops on substrates



[E.L. Barnhart et al., PLOS Biol. 2011]



[K. Keren et al., Nat. Cell Biol. 2009]

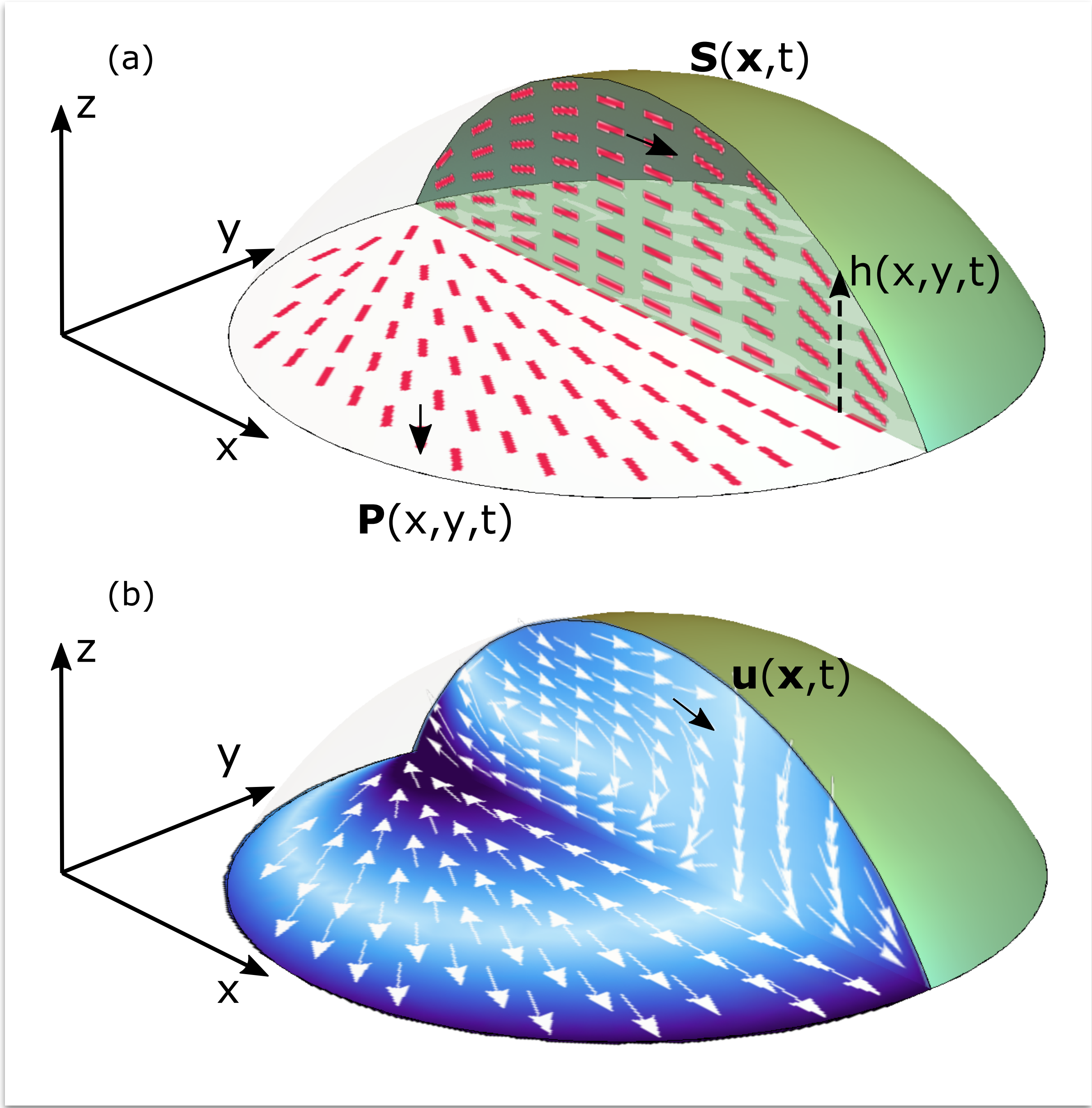


[E. Tjhung et al., Nat. Comm. 2015]

For a three-dimensional active drop on a planar substrate so far only symmetrical spreading and stationary shapes have been investigated [Joanny and Ramaswamy, J. Fluid. Mech. 2012].

We study theoretically how such drops can become motile as a result of imposed orientation profiles with topological defects, and how the shape of the drop and friction at the substrate affect motility.

Model of a drop on a surface



The orientation field satisfies tangential anchoring at both bounding surfaces

$$\mathbf{S}(\mathbf{x}) \propto \begin{pmatrix} hP_x \\ hP_y \\ z(P_x \partial_x h + P_y \partial_y h) \end{pmatrix}$$

Activity drives large-scale flows in the drop through active stress tensor

$$\sigma^a_{ij} = -\sigma_0 \left(S_i S_j - \frac{\delta_{ij}}{3} \right)$$

The flow of active fluid is given by the Stokes equation and obeys continuity:

$$\begin{aligned} -\nabla p + \mu \Delta \mathbf{u} + \nabla \cdot \sigma^a &= 0 \\ \nabla \cdot \mathbf{u} &= 0 \end{aligned}$$

Thin film approximation

Assuming a thin droplet, we can expand the equations in a small parameter $\varepsilon = \frac{h_0}{L}$

$$\partial_z^2 \mathbf{u}_\perp = \nabla_\perp p - \mathbf{f}_\perp^a , \quad 0 = \partial_z p$$

- to retain effects of activity we scale $p\,,\sigma_0 \sim \frac{1}{\varepsilon}$
- surface tension is neglected, since otherwise $\gamma \sim \frac{1}{\varepsilon^3}$
- boundary conditions: $\partial_z \mathbf{u}_\perp \big|_{z=h} = \mathbf{0}$

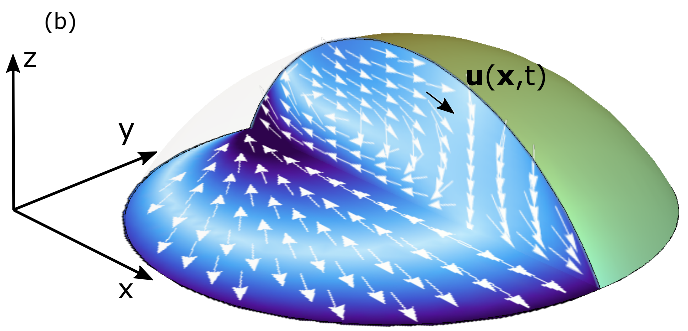
$$\mathbf{u}_\perp (z=0) = -\xi \partial_z \mathbf{u}_\perp \Big|_{z=0}$$

Solution for the flow field

The horizontal and vertical flow components, $\mathbf{u}_\perp$ and w , are mainly determined by the effective active force $\mathbf{f}^a_\perp$

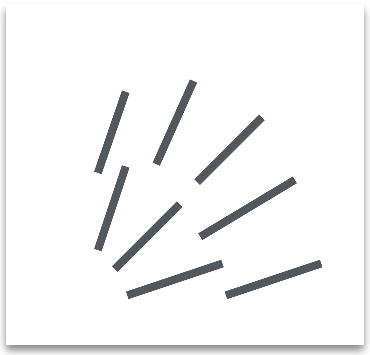
$$\mathbf{u}_\perp = -\left(\frac{z^2}{2} + h(\xi - z)\right)\mathbf{f}^a_\perp$$

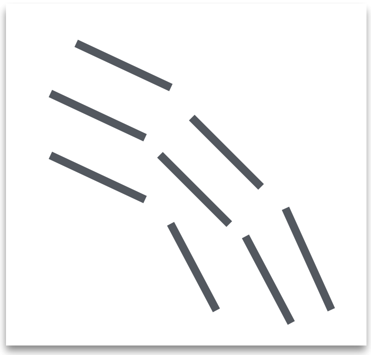
$$w = \frac{z^3}{6}\nabla_\perp \cdot \mathbf{f}^a_\perp - \left(\frac{z^2}{2} - \xi z\right)\nabla_\perp \cdot (h\mathbf{f}^a_\perp)$$



$$\mathbf{f}^a_\perp = -\sigma_0\left(\mathbf{P}\left(\nabla_\perp \cdot \mathbf{P} + \frac{1}{h}\mathbf{P}\cdot\nabla_\perp h\right) + (\mathbf{P}\cdot\nabla_\perp)\mathbf{P}\right)$$

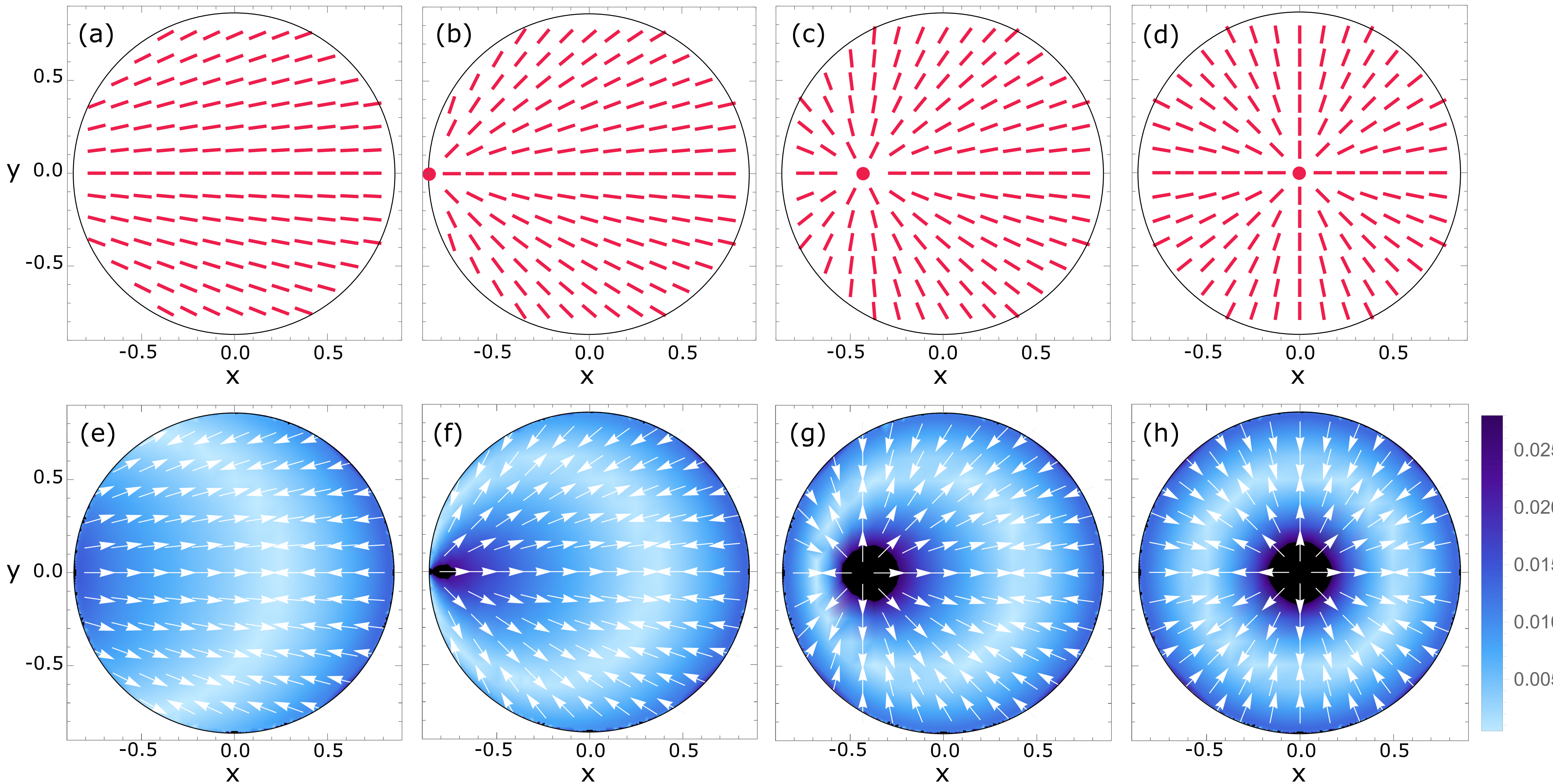
splay **shape** **bend**





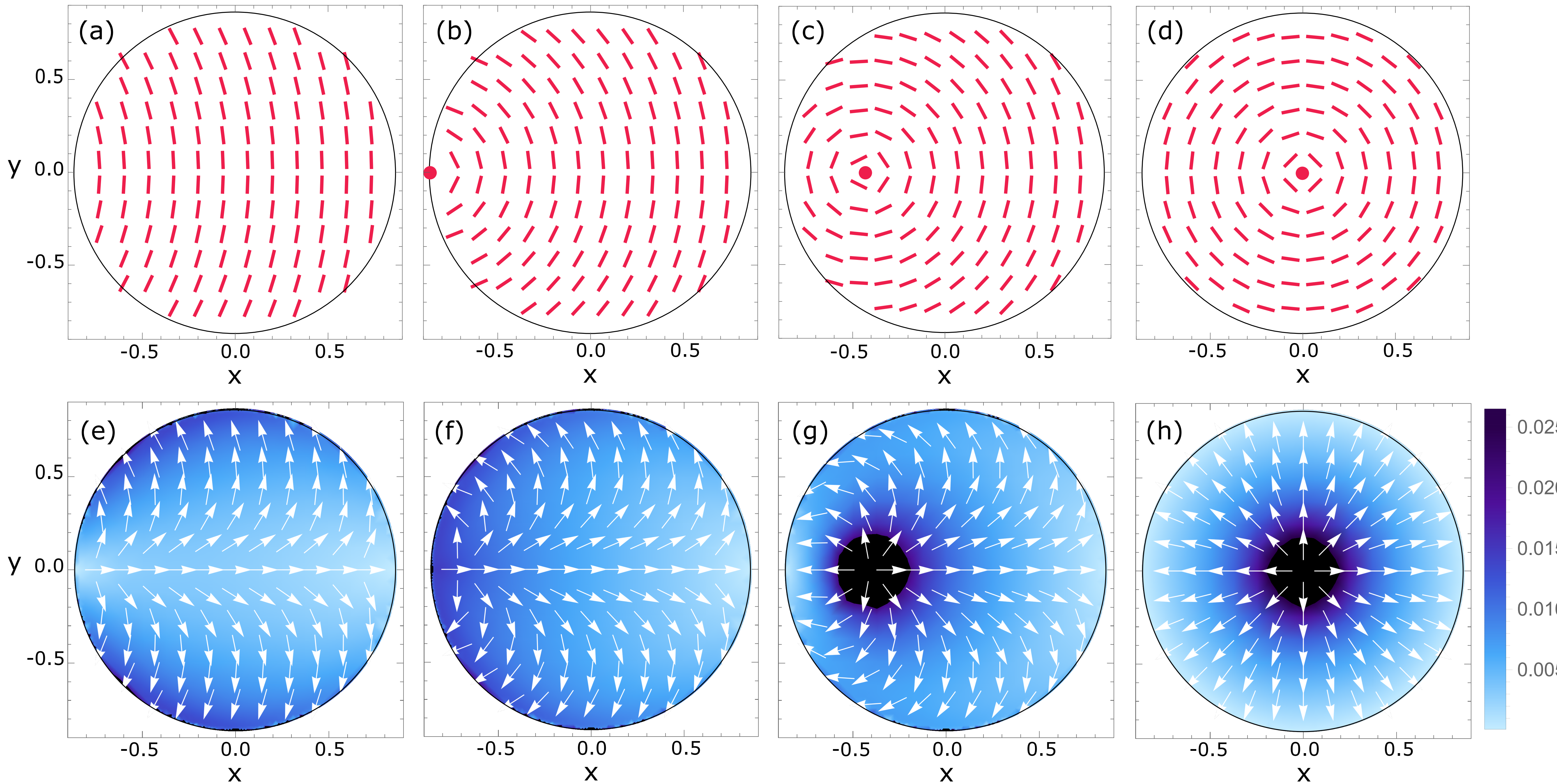
Example: aster defect

$$\mathbf{u}_\perp = \sigma_0\left(\frac{z^2}{2} + h(\xi - z)\right)\left(\mathbf{P}\left(\nabla_\perp\cdot\mathbf{P} + \frac{1}{h}\mathbf{P}\cdot\nabla_\perp h\right) + \cancel{(\mathbf{P}\cdot\nabla_\perp)\mathbf{P}}\right)$$

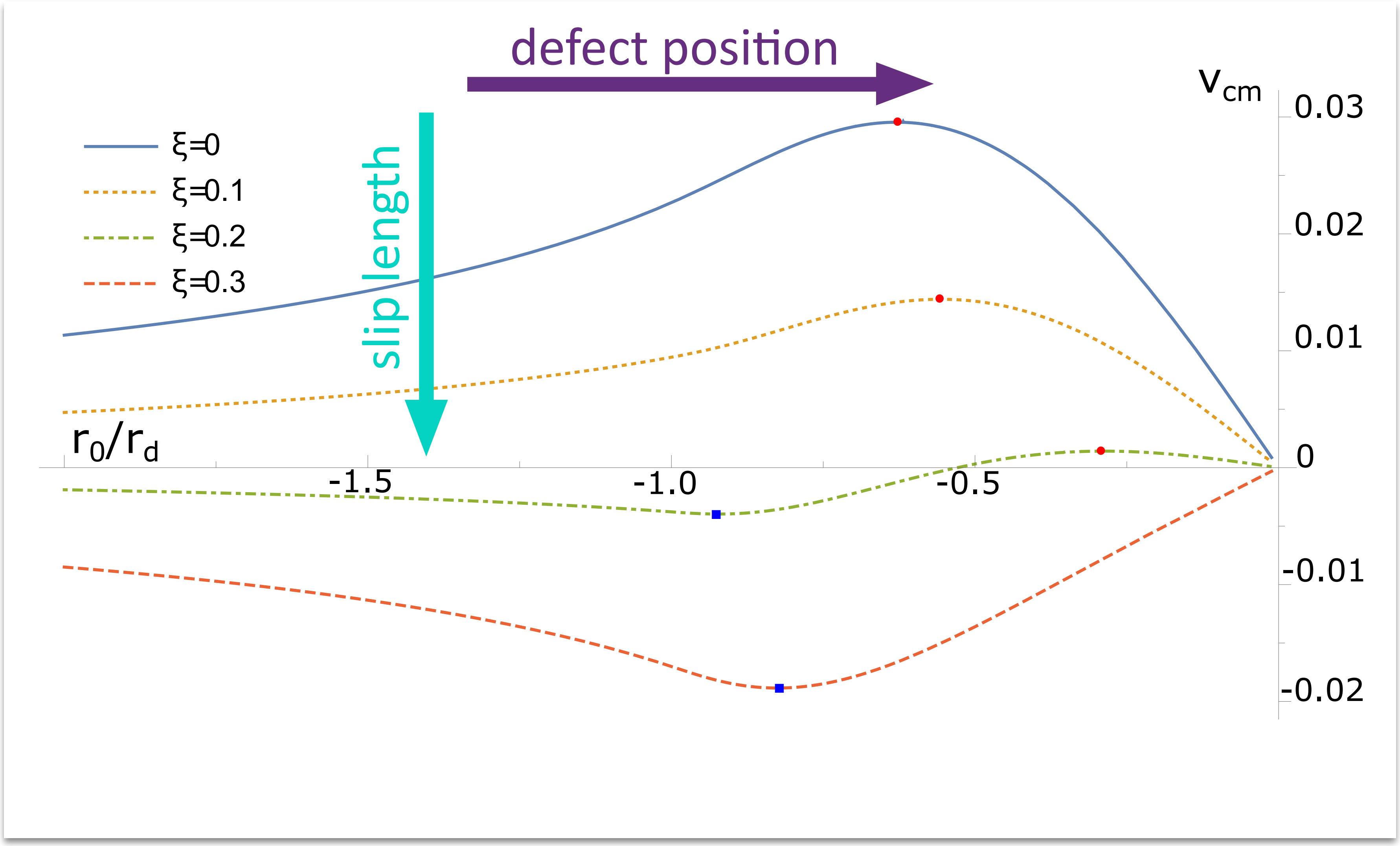


Example: vortex defect

$$\mathbf{u}_\perp = \sigma_0\left(\frac{z^2}{2} + h(\xi - z)\right)\left(\mathbf{P}\left(\cancel{\nabla_\perp \cdot \mathbf{P}} + \frac{1}{h}\,\mathbf{P}\cdot\nabla_\perp h\right) + (\mathbf{P}\cdot\nabla_\perp)\mathbf{P}\right)$$



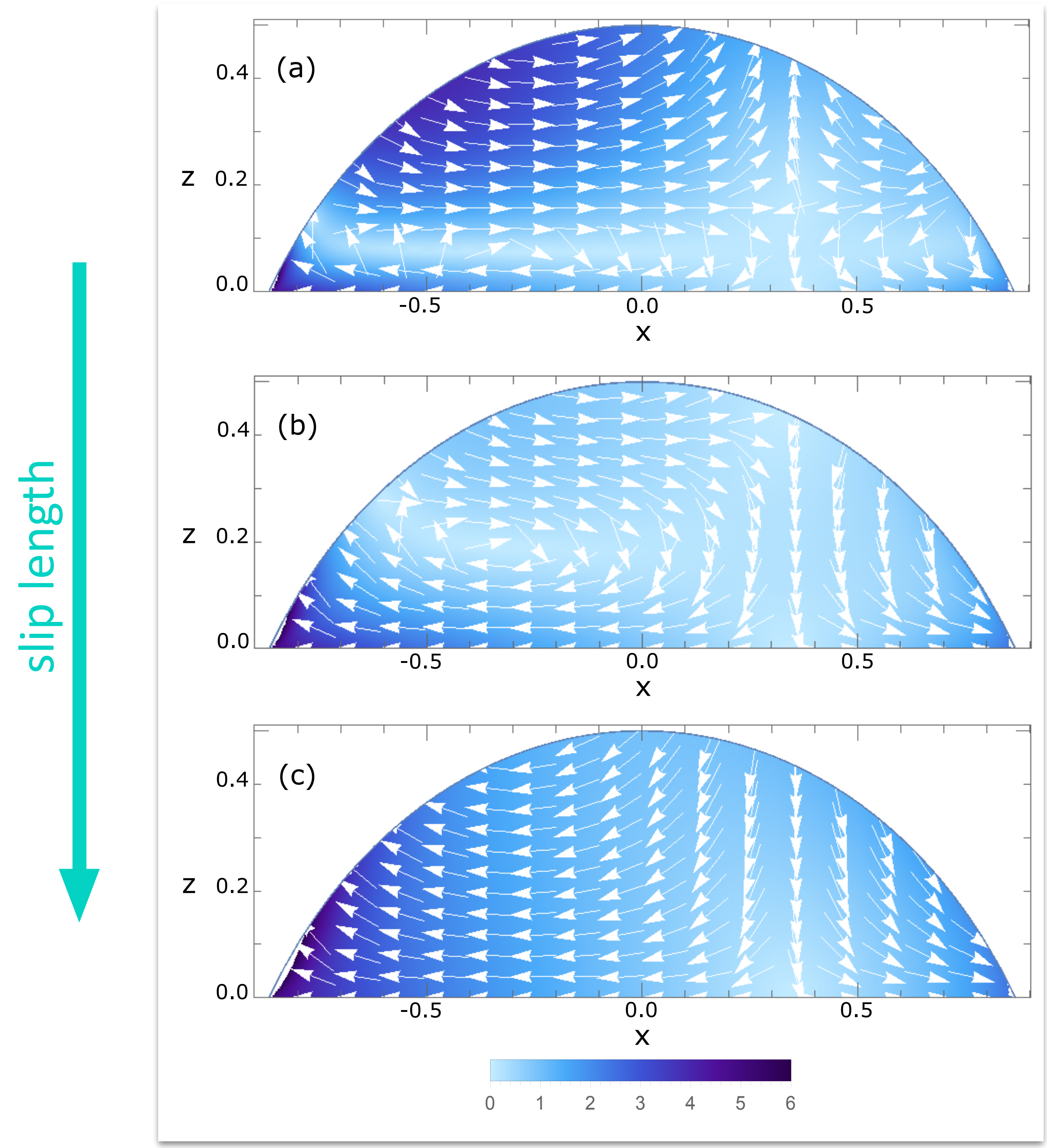
Motility of the drop



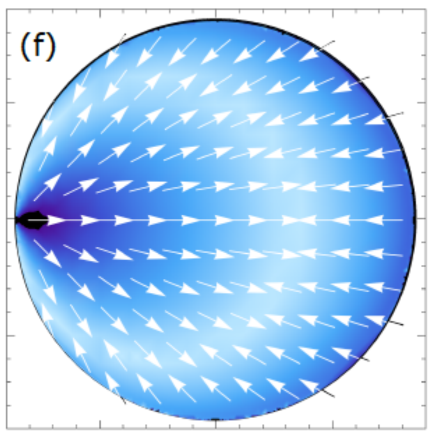
As a measure for the motion of the drop we use the centre-of-mass velocity in the x-direction

$$v_{\rm cm} = \frac{1}{V_0}\int_{\rm drop} u\,{\rm d}V$$

Effect of surface friction on motility



top view



The friction-dependent prefactor determines the amount of rotation in the flow: rolling vs. sliding

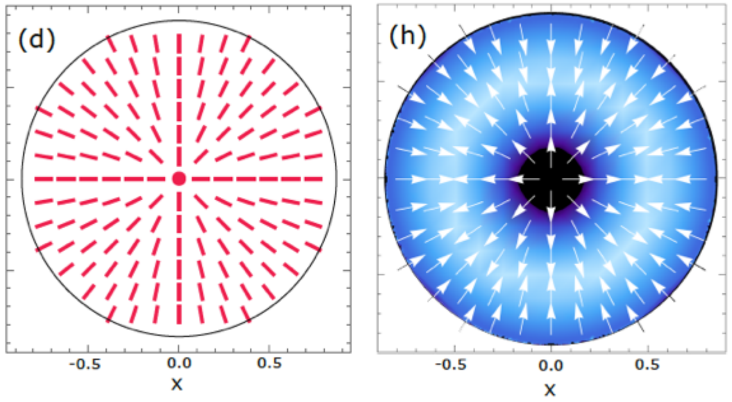
$$\mathbf{u}_\perp = -\underbrace{\left(\frac{z^2}{2} + h(\xi - z)\right)}\mathbf{f}^a_\perp$$

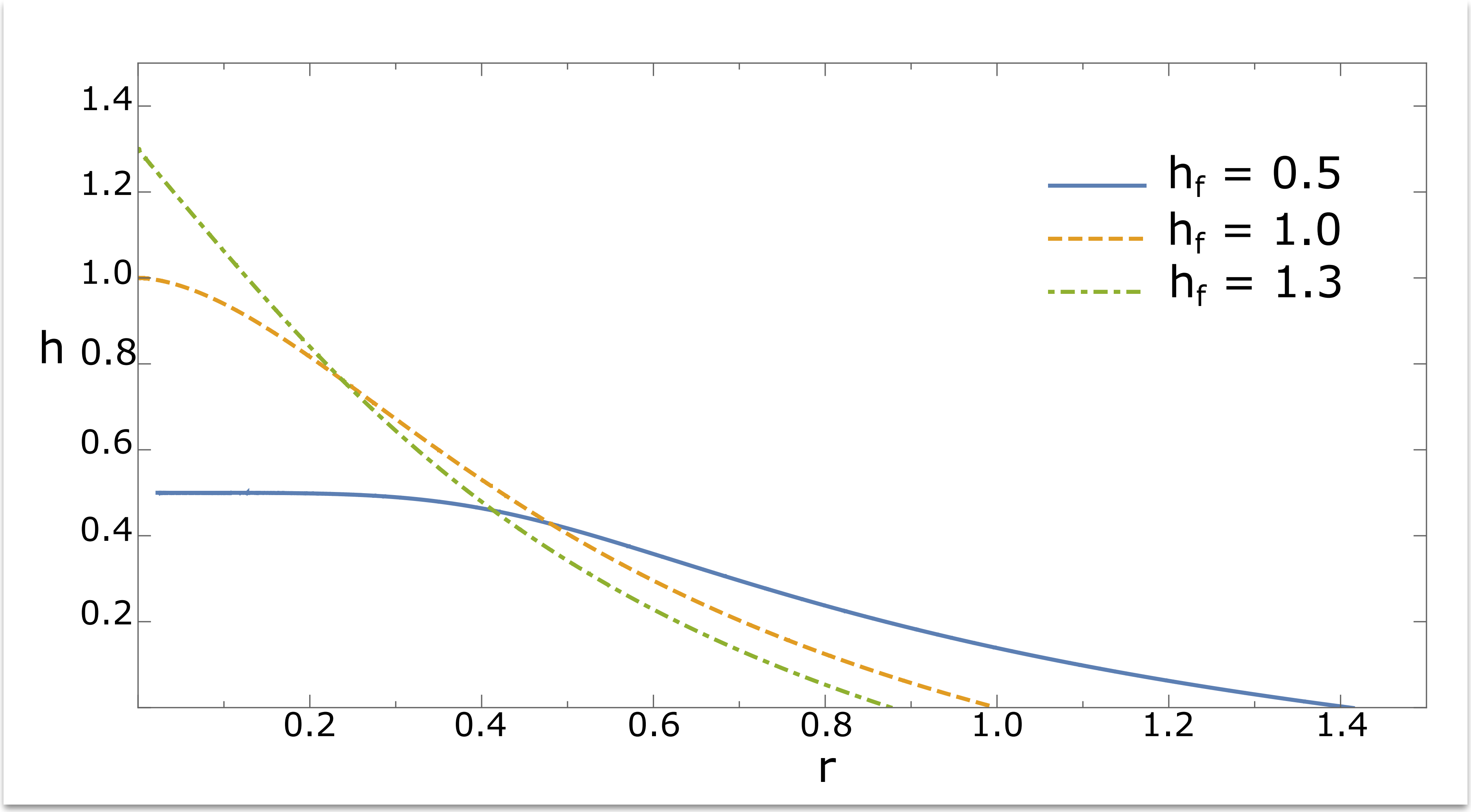
changes sign if $\xi < \dfrac{h}{2}$

Stationary shapes

For an axisymmetric drop with an aster defect we calculate the stationary shape profile analytically from

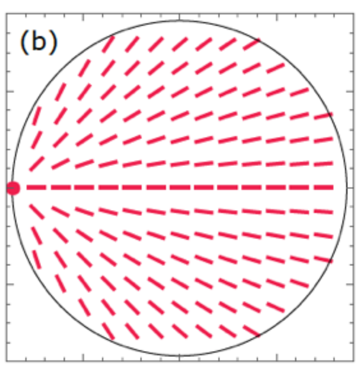
$$\partial_t h = -\nabla_\perp \cdot \left(\int_0^h \mathbf{u}_\perp \mathrm{d}z \right) = \sigma_0 \frac{1}{r} \partial_r \left(\left(\xi - \frac{h}{3} \right) h \partial_r (rh) \right)$$

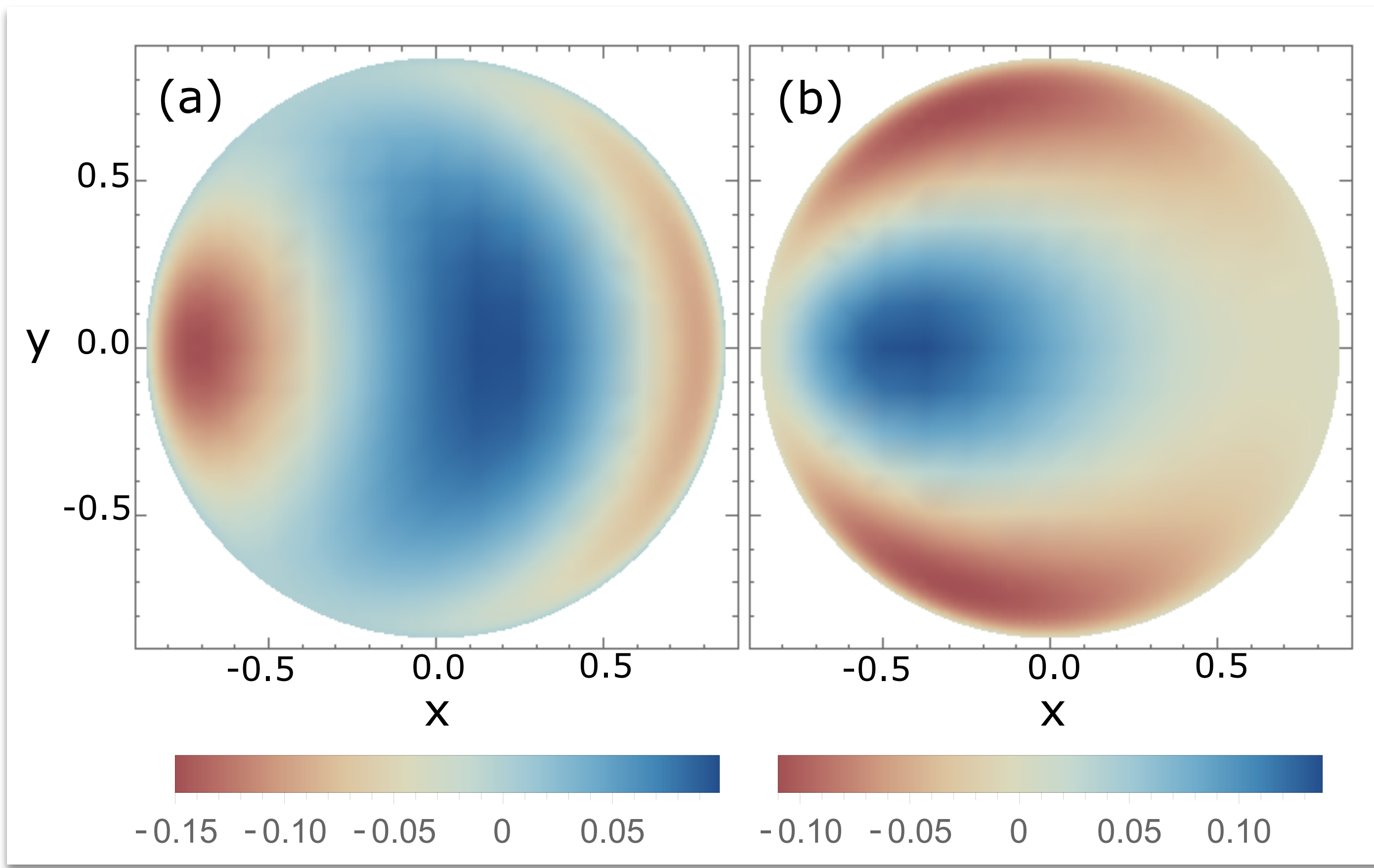


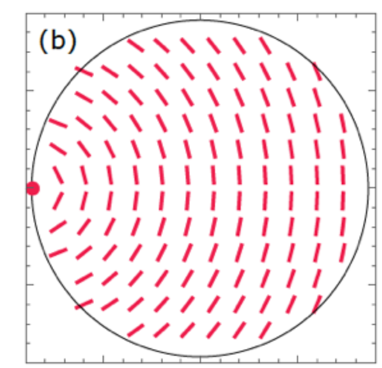


Shape deformations

For an asymmetric profile, e.g. aster or vortex defect placed at the boundary, we can look at the deviation $\Delta h = \partial_t h|_{t=t_0} \Delta t$ from the spherical cap after a small time step Δt







Conclusion and outlook

Conclusion

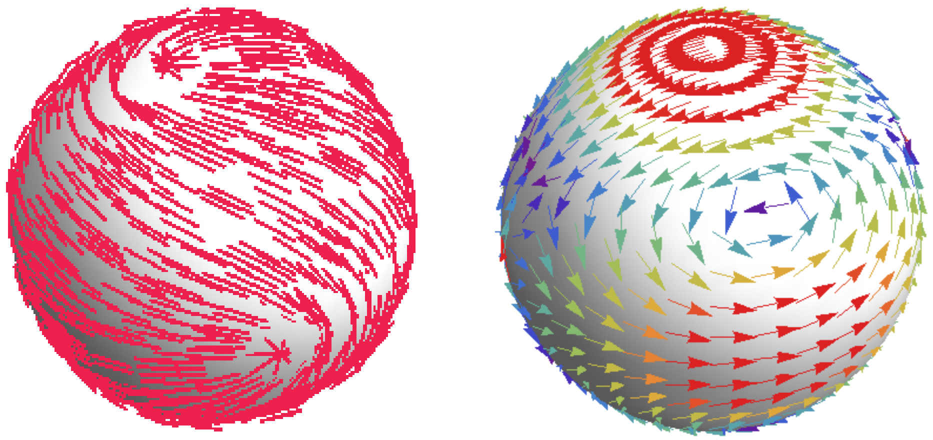
- In the scope of a thin film approximation, we derived exact expressions for the flow field in a drop of active fluid driven by a given polarisation profile

- We identified two key requirements for self-propulsion of an active drop on a planar substrate: asymmetrically bent or splayed orientation field, e.g. induced by a topological defect in the interior of the drop, and sufficient surface friction provided by the substrate

Outlook

- Investigate how micro-patterned substrates could induce drop motion

- Study active flows and shape deformations induced by topological defects on a thin spherical shell of active liquid crystal



Thank you for your attention!

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