ENDOGENIZATION OF TIME AND SPATIAL
TRAITS IN METAMIMETIC GAMES: THE
EXAMPLE OF SOCIAL DILEMMAS

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Erasmo Batta
jesus-erasmo.batta-quintero@polytechnique.fr

Supervisor:
David Chavalarias, Institut de Systèmes Complexes Paris Île de France
Summary

In the present project I worked with a model of a social system where the agents perform a game following preferences (utility functions) which are subject to change. I exemplify the emergence of a distribution of preferences based in the imitation of strategies in accordance with sociological and psychological theories where the ends are part of the perceived environment and where the individuals learn what to desire from other individuals (mimesis of desire).

Taking a spatial prisoner’s dilemma as case study, Chavalarias demonstrates that such models exist and proposes a general method for agents’ traits endogeneization in the framework of metamimetic games. This framework assumes that agent can reflect on their rules of behaviour, an thus that these latter are integral part of the agent’s strategy. Economical agents are considered as reflexive in the sense that they know the criteria or values upon which they base their choice and can take the initiative to change them if necessary. In the proposed model, reflexivity means that agents have the capacity to change their rule of behaviour if they judge that it is not the best rule to achieve their goal. We thus have a dynamics of types which depends, among other, of their spatial distribution.

My case of study presents a minimal, thus incomplete, model which nevertheless reveals several stylized facts:

- Cooperation and heterogeneity are robust phenomena. Emergent patterns with respect to agents type distribution are characteristic of the attractors.
- Agents tends to prioritized the renewal of parts of their strategy in function of their proximity to action (behaviours are updated more frequently than rules of behaviour).
- When the interactions consist in prisoner’s dilemma games, the strength of the dilemma changes types distribution in population and favours materialistic (payoffs-based) types.
- Materialistic types are the less satisfied agents and the population ”well-being” measured in terms of averaged global satisfaction decreases as the strength of the dilemma increases.
- The capability to move does not modify radically the distribution of types. Non-materialistic agents form large clusters beside which unsatisfied materialistic agents moves.

All these stylized facts can be measured by studies in psychology, sociology or economy and are as many qualitatively measurable manifestations of the predictions on types distribution.

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1Endogenization means to reach the natural values of the parameters by the dynamics of the system rather than impose an exogenous value.
Abstract

The question of modelling heterogenous socio-economic systems in microeconomics is closely related to the question of modeling heterogenous preferences and their formation. While in mainstream economics it has long been assumed that preferences are fixed, several social theories and heterodox economic approaches have challenged this view and proposed to conceptualize the complexity and endogeneity of human motivations by taking into account social influences. Many authors have identified imitation and social influence processes as key factors in the emergence of a social and economic order through the formation of agents desires and preferences [1, 2, 3, 4, 5]. In this perspective, agents motivations distribution is the output of the socio-economic dynamics rather than an a priori hypothesis needed to define agents interactions. In previous work, we demonstrated how the introduction of particular mimetic dynamics in modelisation (metamimetic games [6, 7]) makes it possible to endogenize the distribution of agents preferences. In this paper, we extend this approach to propose a way to endogenize time constants in agents strategies as well as their strategies of spatial mobility. Our investigation takes as a case study a spatial prisoners dilemma which is explored in an agent based modelling (ABM) framework. Through a sensitivity analysis of our model we highlight the entanglement between agents preference distribution, social differentiation and spatial patterns, and the strengh of the social dilemma. We found critical points in the strengh os the social dilemma wich lead to important changes in the preferences distribution and in the global game outcome.
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1 Introduction

In its Prize Lecture [8], Vernon L. Smith sketched what he considered as one of the most important challenges for future game theory:

“Technically, the issue can be posed as one of asking how most productively to model ”agent types” by extending game theory so that types are an integral part of its predictive content, rather than merely imported as an *ex-post* technical explanation of experimental results. For example, moves can signal types, and effect decision, which explains why game form matters, and why payoffs available, but foregone, can effect outcomes. These elements must be part of the internal structure of the theory such that outcomes become predictions conditional on the elementary characteristics of players who read each other’s intentions. [...] The point that needs emphasis is that it is easy to go from “types” (traditionally utility or beliefs about states) to game theoretic choice ; the hard part is to relate “types” to characteristics of the individual’s memory-sensory system.”

The emergence of agents heterogeneity from the cultural environments is not only of the competence of the economists but also of psychologists, sociologists and mathematical modelers. It is based on individuals which explore of the environment and interacts based in limited perceptive and cognitive capacities.

![Figure 1: Agents’ type are the result of the embedding of agents with particular cognitive endowment into some particular context of socio-economics interactions. Rules of behaviour and agents’ types are themselves part of the information which could be inferred from the environment and can enter into play in the definition of rules of behaviour in a more or less sophisticated manner.](image)

Chavalarias introduces a formal framework - *metamimetic games* - [6] where types distribution is endogenized and demonstrates how this makes possible to endogenize a rich variety of types attributes from the interaction of agents. The agents are characterized by the human-kind abilities as meta-cognition (cognition about cognition) and reflexivity (think of others as we think of ourselves and of ourselves from others point of view, also called sometimes specularity).

Formally, this translates into the fact that agents rules of behavior are their own meta-rules. The resulting dynamics is self-referential (as is illustrated by figure 1) and the emergence of patterns at the collective level can be understood as the selection of a particular distribution on the set of traits, rules and meta-rules of the agents and is the origin of social differentiation grounded on imitation [1]. Then types will be defined as rules of behaviour which given some information collected from the environment, the agent’s past experience and the agent’s utility function, determine an action to be taken at every decision step of the model. Utility function is one of the determinant factor in agent’s types.
In previous works Chavalarias [7] exhibited two important features of metamimetic games:

- **Social differentiation**: Agents identities and social structures are the outcome of the game,
- **Local cultures**: Metamimetic attractors are heterogeneous at all levels (preferences included). The spatial structure of the game reflects the size of agents strategies space.

In particular, this approach proposes an other interpretation of the emergence of cooperation in social dilemma situations based on social differentiation mechanisms and cultural evolution rather fitness based selection [7]. In the present work we extend this model providing an answer to two key questions:

- Which are, if exist, the natural timescales in the game when driven by imitation? Do agents use different time scales for different aspects of their decision making processes?
- When unsatisfied, how the agents would behave if they have the possibility to change their environment rather than their strategy? What kind of trade-off would we have between making efforts to adapt to the current environment and making effort to change one’s environment? Will this trade-off be specific to types?

Metamimetic framework makes possible to endogenize this new set of variables and exhibits patterns as well as their dependence on the strength of the social dilemma. We thus address in a new way the trade-off between changing an agents strategy or changing its environment while coupling it with the fact that both events could lead to a change in the agents preferences. The report is organized as follows. In section 2 I synthetically present the game that agents develop and the main features of the model. With the aim to investigate the stability of the structures in the system I introduce a intrepersonal comparison fuction named satisfaction wich is fathomed in section 3. In section 4 I show how of the metamimetic framework can be used to endogenize agent traits while providing structure to agents heterogeneity. Finally in section 5 I sumarize the results and discuss their relevance in terms of psychological theories.

## 2 The model

To illustrate the insights brought by metamimetic games to the modelling of agent heterogeneity, we will apply this framework to the modelling of the game $G$, a spatial prisoner’s dilemma [10] with the game outcomes described by table 1.

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**Table 1**: Payoff function of the game played by agents. $p$ represents the strength of dilemma and for $p \in [0, 0.5]$ the game corresponds to a Prisoner’s Dilemma.
The types will be determined by a rule which assigns to each agent $A$ an order relation $\mathcal{R}$ from the set of the possible linear orderings of the neighbourhood $\Gamma_A$, based on a valuation function (utility) $\nu$.

$$\nu_A : \Gamma_A \mapsto \mathcal{R}|\Gamma_A|, B \in \Gamma_A \mapsto \rho_B$$  \hspace{1cm} (1)

We will consider a set of four agents types defined with the following utility functions:

- **Payoffs maximization:** utility is higher when payoffs are higher,
- **Payoffs minimization:** utility is higher when payoffs are lower,
- **Conformism:** utility is higher when agent’s strategy is similar to a larger number of neighbors,
- **Anti-conformism:** utility is lower when agent’s strategy is similar to a larger number of neighbors.

Agents are defined by a unique type of behaviour and by a hierarchy of rules which control the lower level. We will call such a hierarchy a *metamimetic chain*. Given all the above, we will consider a metamimetic game where interactions take place as follows for every period of the game:

1. each agent $A$ looks at the situation of other agents in $\Gamma_A$, (payoffs, rules, behaviour),
2. for any agent $A$, if according to $A$’s utility function there are some agents in $\Gamma_A$ more successful than $A$ and if all these successful neighbours have a rule of behaviour different from $A$’s, then $A$ copy the rule of an agent taken at random among this set,
3. if according to its (eventually new) rule of behaviour and its associated utility function, $A$ is not among the most successful at the introduction of reflexivity in mimetic models makes it possible to endogenize agents types. They also give insight on how agents in $\Gamma_A$, then $A$ chooses at random one of its neighbours with the better situation and copies its behaviour ($C$ or $D$),
4. each agent plays the game $G$ with its neighbours using the same behaviour ($C$ or $D$). Then for each agent, the scores of all its pairwise games are computed and the sum is the new payoffs of the agent.

We decided to implement the metamimetic model in netlogo [18] due to the facilities to develop new agents’ traits and interactions as well for the advantages in visualisation. The code is validated with the results obtained by Chavalarias in [7]: heterogeneity in types and the effect of strength of dilemma in types distribution. All the achieved results are fully reproduced with robust simulations and are presented in extense in appendix B.

This demonstrates that the introduction of reflexivity in mimetic models makes it possible to endogenize agents types. It also gives insight on how culture can shape the social output without assuming any other particular selection process [8] and supports the idea that cooperation is the product of agents heterogeneity and not some kind of fitness optimization.

The above systems in all generality can be described as Markov processes, where the states are the spatial configurations of rules and behaviours of agents. As things stand, these Markov processes are not ergodic, which is reflected mainly by the influence
of initial conditions (e.g. initial rate of cooperation) on the attractor. However, real socio-economic systems face a large variety of perturbations and uncertainties, such that it is reasonable to assume that they should be modelled as noisy systems. To investigate the attractors of the perturbed Markov process, we consider a natural source of noise in the modelling of social systems induced by the necessary limited lifespan of the agents. To do so agents are replaced from the system according to the probability of death distribution from the 2010 US Census Bureau [22] with new agents whose rule and behavior are randomly selected among all the possible alternatives.

The introduction of ergodicity in the model has two consequences:

- The initial rate of cooperation has no more influence on the attractor,
- The proportion of payoffs-based types increases as the strength of the dilemma increases and rate of cooperation oscillates around 50%.

Types distribution does not depend any more on initial conditions and the only parameter being the strength of the social dilemma $p$. This approach is particularly interesting for modelling situations where $p$ can be interpreted as a political leverage (for example employment legislation or collective agreements could be thought as instruments to modify the employment security, trust, and effort dilemma on the job market [11]). This approach makes possible investigate how political decisions could impact on the evolution of preferences in the population [12].

3 Interpersonal comparison: Satisfaction function

The concept of equilibrium associated to this kind of game are metamimetic equilibria with are counterfactually stable states i.e. states such that no agent can find itself better when it imagines itself in the place of one of its neighbors [6]. However, since we are considering evolutionary games with potentially noisy dynamics, we will more frequently encounter stable sets of states, metamimetic attractors. In order to characterize these attractors and render them somehow comparable, we introduce a measure of satisfaction.

There exists some reluctance in social choice theory and in welfare economics to make any kind of interpersonal comparison of utility with the particular exception of that result from weighing all individuals dollars equally. Nevertheless, the definition of a social welfare function as the aggregated utility of individuals enforces the exclusion of interpersonal comparison and so leads to a dictatorship [13]. Besides, the heterogeneity of ends and its consequent derivation in different utility functions reach in the necessity of interpretative comparison.

Then the existence of an interpersonal utility function is inferred but its construction remains unclear. We introduce in our frame an absolute utility function which is strongly grounded in what the agents can perceive from their social context. It measures the self-esteem or amour-propre, a love of self that depends on comparing oneself with others [14]. Such that function only determines a perceived hierarchy which is established according the preferences and should permit comparisons between agents even if have different valuation functions. The relevance of this arises from the concept of mimesis of desire introduced by Girard [2] where the desire for certain object is provoked by the desire of a third person. If actions are driven for an imitated desire (preference) the self-esteem or satisfaction is worth to be consider as indicative of the target to imitate.

We will note $\nu_A(B, \Gamma_A)$ the utility attributed by $A$ for being in the place of $B$, given the information available in $\Gamma_A$. For example, if $A$ is a maxi agent, $\nu_A(B, \Gamma_A)$ will be
B’s payoffs. If A is conformist, \( \nu_A(B, \Gamma_A) \) will be the density of B behaviour or rule (according to what is evaluated) in \( \Gamma_A \). Consequently, an agent can compare its own situation with the one of a neighbour comparing \( \nu_A(A, \Gamma_A) \) and \( \nu_A(B, \Gamma_A) \).

So far the cardinalization of individual utilities does not achieve a comparable measure. Isbell [15] propose to assume a bounded person’s utility space based in Neumann-Morgenstern axioms on probability combinations. This is supported by some plausible arguments in connection with specialization. Shick exemplifies this method in [12]

“What exactly has Adam in mind when he believes that Eve sets a utility of y on r? It may be supposed that Adam is thinking of the absolute intensity of Eve’s feelings on r. But this is hard to get clear. Let me suggest that he is focusing in terms of the utility function assigning the value of 1 to the proposition Eve ranks highest and the value 0 to the proposition she ranks lowest, then to say that he believes her to set a utility of y on r is to say that he supposes she would locate r at a point y (some fraction) of the way up from the bottom of the scale”

On Shick’s view this normalization does not constitute an interpersonal comparison while Adam and Eve share the same utility function since there is no hypothesis on the identity of the reach of everyone’s feelings. This idea is not unrelated with Smith prospective [8] which claims

”Over 50 years ago experiments with animal behaviour demonstrated that motivation was based on relative or foregone reward - opportunity cost - and not on an absolute scale of values generated by the brain”.

We will thus assume that if \( \nu_A(A, \Gamma_A) - \nu_A(B, \Gamma_A) < \epsilon \), with \( \epsilon > 0 \), A will consider that A and B are performing equally well. On the other hand, instead of considering the absolute value \( \nu_A(A, \Gamma_A) \) and \( \nu_A(B, \Gamma_A) \) for inter-personal comparison assessment, we will consider that inter-personal comparison is based on normalized value with respect to the agent’s neighbourhood. We thus define the satisfaction \( S(A, \Gamma_A) \) of an agent A as:

\[
S(A, \Gamma_A) = \begin{cases} 
1, & \text{if } \nu(B_{\text{max}}, \Gamma_A) - \nu(B_{\text{min}}, \Gamma_A) < \epsilon \\
\frac{\nu_A(A, \Gamma_A) - \nu_A(B_{\text{min}}, \Gamma_A)}{\nu_A(B_{\text{max}}, \Gamma_A) - \nu_A(B_{\text{min}}, \Gamma_A)}, & \text{Otherwise}
\end{cases}
\]

where

\[
B_{\text{max}} = \max_{B \in \Gamma_A} \nu_A(B, \Gamma_A), \quad B_{\text{min}} = \min_{B \in \Gamma_A} \nu_A(B, \Gamma_A).
\]

\( S(A, \Gamma_A) < 1 \) means that A is not satisfied and will change its strategy (rule or behaviour) at the next decision step. Consequently, if we note \( P \) the population of agents and \( \Psi = \sum_{A \in P} S(A, \Gamma_A) \) we have \( \Psi = 1 \) at a metamimetic equilibrium and \( \Psi < 1 \) otherwise. \( \Psi \) is the average satisfaction of agent in the population and is an indicator of how unstable it is.

Even if the spatial prisoner’s dilemma studied here is too simple to allow a direct analogy with a real-word problem, it is noteworthy, as shown by figure 2 that \( \Psi \) is a decreasing function of the strength of the dilemma and that there is an heterogeneity in the satisfaction of agents type, payoffs-based types being the less satisfied. This phenomena has already been underlined by several psychological studies [16].

\[2^2\]In these work we establish \( \xi = \frac{1}{8} \).
Figure 2: Effect of the strength of the social dilemma on agent’s satisfaction at the attractor. Agents types have different satisfaction levels, payoffs-based types being the less satisfied, which has already been underlined by several psychological studies. The population satisfaction $\Psi$ is significantly higher for low strength dilemma compared to high strength dilemma.

4 Traits endogenization

4.1 Time parameters

In a real game agents can perform or change their strategies during the same period of time. However there is no reason to assume that agents follow an strict timescale. Is more likely that agents decide when interact with neighbors depending on a personal decision.

Time-scales and their hierarchies are an important issue in socio-economic systems modelling although this issue is hardly addressed. For example, it is well known that several models of spatial dilemma are critically sensitive to the synchronous or asynchronous aspect of the behaviours update.

We introduce as part of the modifiable traits of agents the probability of update each cognitive level as is exemplified by Chavalarias in [17]. Each time a trait is assumed to be part of the agents strategy and could be somehow inferred by other agents, it could be endogenized through metamimetic principles. Then the agent’s strategy is extended to include the updating probabilities $\theta_r \in [0, 1]$ and $\theta_a \in [0, 1]$ (then both traits can be imitated). We assume that instead update their strategy every time-step, agents do it stochastically. When an agent is not the most successful of its neighbourhood, we assume that it engages in a rule updating process with a probability $\theta_r$ and in a behaviour updating process with a probability $\theta_a$.

We will assume that the inference of continuous traits can be done up to a given precision by adding a noise $\xi$ to the measurement. Moreover, we will assume that agents can update a continuous traits $\alpha$ by weighting it with the traits $\alpha_{best}$ associated to the best rule: $(1 - \iota) * \alpha + \iota * (\alpha_{best} + \xi)$. The influence $\iota$ appears here as an additional parameter. In principle, it could also be endogenized, which was not the chosen option in the following computational studies. However, we did a sensibility study on $\iota$ and it
Figure 3: A - Left: Effect of the influence $\iota$ on $r_{\theta}$ averaged value at the attractor. B - Right: Scatter of $r_{\theta}$ against average satisfaction (red crosses correspond to settings where the social influence is of 100%).

Figure 4: A - Left: Effect of the influence $\iota$ averaged $\theta_r$ per type and B - Right: Effect of the influence $\iota$ on types distribution for $p = 0.4$.

reveals that all results are qualitatively similar for $\iota \in [0.1, 0.9]$. Extreme $\iota$ values reveal particular dynamics but are not realistic from the psychological perspective and are moreover associated with pathologically low levels of $\Psi$.

Computational studies performed with NetLogo [18] (for the multi-agent model) and Open Mole [19] (for the distributed processing) reveal several interesting stylized facts.

- All results presented in section 2 and appendix B are robust under time scale endogeneization for $\iota \in [0.1, 0.9]$. Thus, asynchronous update and heterogeneity in time constants do not change the dynamics,

- The means of $\theta_r$ and $\theta_a$ per type converge toward values which are lower than 1 (around 0.5 for $\theta_r$) (cf. fig. 3 -A). Conformist agents are the population with the highest update frequency, they are the most concerned with what others think (fig. 4 -A),

- The ratio $r_{\theta} = \frac{\theta_r}{\theta_a}$ is always lower than 1 (cf. fig. 3-A), indicating a clear hierarchy between action level time-scales and rule level time-scales. This is a behaviour that could be expected from rational agents (give your rule a chance before changing it), despite the fact that no rationality assumption has been introduced here.
4.2 Spatial mobility

We extend the model to allow the agents to move across the toric grid. In this case the density of agents $\rho$ becomes a tuning parameter and a new trait is defined: the likelihood to move $\mu \in [0, 1]$.

Such that parameter provides a new dimension in the social differentiation; there are agents that prefer to do a physical effort as moving in the lattice rather than perform learning and the distribution of them can be endogeneous if $\mu$ is part of the strategy. An agents decide to move to an empty cell with probability $\mu$ and if she does not move she perform the learning methods described above.

We assume that an agent necessarely performs an action every timestep, or at least do the attempt to do so. Then, if originally the agent would prefer to move but there is no available space then it inspects the metamimetic chain. In the other hand, if the agent tries to check its neighbours strategies but is alone, it will change her position. With the former point we assume that human being are social ents and so, no one could consider herself succesfull if isolated [20].

As in [7], the attractors of the system show some structure that represents social differentiation. The strength of dilemma tune the maxi and mini populations, but not with the same intensity. Is observed that the average propensivity to move of maxi type agents is always bigger than the one of mini type, and both depends on the strenght of dilemma in a similar way (Fig. 5). The endogeneous distribution of this trait shows the same behavior when the density of the system changes. Beyond the endogenous propensity of move, is seen that the the relative proportion of agents which move is considerably lower for the conformist and that in general is not dominated enterely by $\mu$. This conduces to the agglomeration of them in compact areas around which payoff-based types moves oftenly. As expected, a change in density modify the effective mobility of agents.

5 Conclusions

I tried to exemplify the emergence of a distribution of preferences based in the imitation of strategies in accordance with Tarde’s theory, where the ends are part of the perceived environment and with the mimesis of the desire proposed by Girard. Taking a spatial prisoner’s dilemma as case study, Chavalarias demonstrate that such models exist
and propose a general method for agents trait endogeneization in the framework of metamimetic games. My case of study presents a minimal, thus incomplete, model which nevertheless reveals several stylized facts:

- Cooperation and heterogeneity are robust phenomena. Emergent patterns with respect to agents type distribution are characteristic of the attractors.
- Agents tend to prioritize the renewal of parts of their strategy in function of their proximity to action (behaviours are updated more frequently than rules of behaviour);
- The strength of a social dilemma changes types distribution in population and favours payoffs-based types,
- Payoffs-based types are the less satisfied agents and the population “well-being” measured in terms of averaged global satisfaction decreases as the strength of the dilemma increases.
- The capability to move does not modify radically the distribution of types. Conformist agents form large clusters beside which unsatisfied materialistic agents moves.

All these stylized facts can be measured by studies in psychology, sociology or economy and are as many qualitatively measurable manifestations of the predictions on types distribution.

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References


A metamimetic games

Metamimetic games [6] are designed to account for utility functions. To sketch the given biological content plus the cultural shaping, we will derive these utility functions from the agents’ cognitive endowment. Agents are embedded in a social network and only can learn from agents they interact with in a lattice. Imitation occurs when an agent decides to adopt a trait observed in one of its neighbours.

A.1 Agents and interactions

The agent in the model is defined by a particular type related with its rule of behavior and in some cases with scalar parameter which conform the strategy and are subject of imitation. Functionally the rule of behavior of agent $A$ assigns an order relation $\mathcal{R}$ from the set of the possible linear orderings of the agent’s neighbourhood $\Gamma_A$, based on a valuation function (utility) $\nu$ (equation 1).

$$\nu_A : \Gamma_A \mapsto \mathcal{R}\mid \Gamma_A| B \in \Gamma_A \mapsto \rho_B$$

We consider a minimal model which mixes materialistic and non-materialistic individuals (non comparable utilities) playing a game $G$ with two possible moves $C$ or $D$ (which will stand for cooperation of defection in our case study). In all generality, agents are embedded in a population, with a set of neighbours with which they play the game $G$ pairwise.

We assume minimal processing capacities on these two types of information. Concerning ordinal values (e.g. payoffs), agents are able to compare two figures and take the bigger or the lower. They are consequently able to process the minimal and maximal payoffs in their neighbourhood. Concerning cardinal quantities (e.g. the distributions of moves in their neighbourhood), agents are able to process frequencies to convert this information into ordinal quantities. They are thus able to know what is the majority of behaviours for example.

These hypothesis on agents’ cognitive endowment generate four possibilities for the utility functions:

- **Payoffs maximization**: utility is higher when payoffs are higher,
- **Payoffs minimization**: utility is higher when payoffs are lower,
- **Conformism**: utility is higher when agent’s strategy is similar to a larger number of neighbors,
- **Anti-conformism**: utility is lower when agent’s strategy is similar to a larger number of neighbors.

“To go from “types” to game theoretic choice”, we will consider simple mimetic agents [10]. Agent’s types will be named after their underlying utility function, with two payoffs based (or materialistic) types, $maxi, mini$ and two non-materialistic types $conformist, non-conformist$.

Agents are defined by a unique type of behaviour ($C$ or $D$) and by a hierarchy of rules which control the lower level. We will call such a hierarchy a metamimetic chain. In these simulations, the metamimetic chain only have two levels.
Figure 6: Rule acting as its own meta-rule. At time $t$, an agent $A$ described by the strategy (rule=maxi, behaviour= D) has a conformist neighbour $B$ which is strictly more successful than all other neighbours. If $A$ ascribes the success of $B$ to its conformist rule, it might adopt this rule replacing its original maxi-rule. Thereafter, it might be that according to this new rule, the current behaviour is not the best one, and has to be changed.

A.2 Imitation

To introduce elements to our framework to make the distribution of types an "integral part of its predictive content” we adopt the framework of metamimetic games [6] which assumes that agent can reflect on their rules of behaviour, an thus that these latter are integral part of the agent’s strategy. Agent are considered as reflexive in the sense that they know the criteria or values upon which they base their choice and can take the initiative to change them if necessary. Such statements about strategies are common in literature. For example, according to Smith, there exists incentives to follow a non-profit-maximizing behavior in repeated games [21]. In the proposed model, reflexivity means that agents have the capacity to change their rule of behaviour if they judge that it is not the best rule to achieve their goal 5. We thus have a dynamics of types which depends, among other, of their spatial distribution.

Given all the above, we will consider a metamimetic game where interactions take place as follows for every period of the game:

1. each agent looks at the situation of other agents in its neighbours $\Gamma_A$, (payoffs, rules, behaviour),

2. for any agent $A$, if according to $A$’s utility function there are some agents in $\Gamma_A$ more successful than $A$ and if all these successful neighbours have a rule of behaviour different from $A$’s, then $A$ copy the rule of an agent taken at random among this set,

3. if according to its (eventually new) rule of behaviour and its associated utility function, $A$ is not among the most successful agents in $\Gamma_A$, then $A$ chooses at random one of its neighbours with the better situation and copies its behaviour ($C$ or $D$),

5There are various options for the procedure of types’ change. For example, if conformist agents are judged to be the wealthier in terms of payoffs, a maxi agent could introduce a small proportion of conformism in its strategy ; or simply become conformist. Because we want to capture the essence of the consequences of types endogenization, we will consider the simpler option, which is the latter.
4. each agent plays the game $G$ with its neighbours using the same behaviour ($C$ or $D$). Then for each agent, the scores of all its pairwise games are computed and the sum is the new payoffs of the agent.

This dynamics is illustrated by figure 6. It is important to note that an agent which is the best of its neighbourhood according to its utility function will be satisfied and will not engage in an imitation process at the rule level or at the behavioural level (here the metamimetic chain only has two levels).
B Settings and previous results

In [6] and [7], Chavalarias perform a metamimetic game based in a social dilemma. In this, agents are displayed at the nodes of a two dimensional toric grid and $\Gamma_A$ is the Moore neighbourhood (height adjacent cells). The game is initialized with a random uniform distribution of rules and a random distribution of behaviours with an average level of cooperation named in the figures as $\text{IniCoop}$. The model has thus two parameters, $p$ and $\text{IniCoop}$.

The behavior of the agent only consists in an action and a rule that determine the behavior. In terms of the metamimetic structure this means that the metamimetic chain only has a metamimetic level (actions are considered as the level zero).

This game has been studied in details through computational and analytical methods. The main results are the following:

• Populations reach very quickly an heterogeneous attractor (within dozens of periods) where all rules are represented (cf. figure 7),

• Populations at the attractor are well structured with patterns which reflect the cognitive endowment of the agents (mixed groups of mini and maxi, dense groups of conformists, isolated anti-conformists),

• Cooperation is always present at the attractor (between 10% and 90% according to the settings) and the level of cooperation depends both on $p$ and the initial level of cooperation. The influence of the initial level of cooperation decreases when the strength of the social dilemma increases.

• Variations in types proportions at the attractor are well predicted by the spatial dominance of the rules [7], which is an index approximating for each type, the probability of an agent to be satisfied.

In order to introduce ergodicity in the system we implement the replacement of agents. We decided to represent "one year" by eight games and used data from the 2010 US Census Bureau [22] to initialize agents age and compute at each period the probability for an agent to die in function of its age, thus defining a replacement dynamics. Every died agent is then replaced by a new agent which rule and behaviour are randomly selected among the set of possible choices.

When the model include the time traits, the particular values for the initialisation of $\theta_r$ and $\theta_a$ are not important. The only thing which matters is the order of magnitude of the ratio between these initial values, which determines the baseline time-scale of the cultural dynamics, and the lifespan expectancy of the agents. For that reason, we choose to initialise $\theta_r$ and $\theta_a$ at 1 which have the advantage of the simplicity and don’t impose any particular hierarchy between rule and behaviour updates.

For agents with mobility capabilities the initial distribution of $\mu$ is also irrelevant but, in contrary to the time parameters, is taken as uniform as it does not determine any particular length of the system.

The agents are supposed to be honest about their traits and records. The simulations are done with 10000 agents and the presented data corresponds to the mean of 30 repetitions at the attractor (reached before 1000 games).

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6The number of games per year is a neutral variable from the moment it is sufficiently large for the cultural dynamics to really play its role.
Figure 7: Emergence of heterogeneous attractors.
A - right): Evolution of the spatial distribution of rules (upper part) and behaviours (lower part) from the initial disordered state to a structured attractor. Each small square represents an agent. This configuration is globally stable (only few oscillators remaining at the attractor). Legend: Upper part - white: conformists, black: anti-conformists, light grey: mini, dark grey: maxi. Lower part - light grey: cooperators, dark grey: defectors.
B - left): Influence of the strength of the prisoner’s dilemma on distribution of rules at attractor for an initial rate of cooperation of 50%. We can observe that environmental factors like the strength of the social dilemma do influence types distribution and that in this example, materialistic types are favoured by strong social dilemma. Discontinuities are due to the discrete character of the neighbourhoods.

Figure 8: Sensibility study for the structure of the attractor: dependence on $p$ and the initial rate of cooperation at 100 time steps, averaged over 30 independent runs
Rate of cooperators at the attractor. The rate of cooperation is always above 9.5%.
A -Right: Results obtained by Chavalarias in [7]
B -Left: Reproduced results