# Complexity Science Doctoral Training Centre

## CO903 Complexity and Chaos in Dynamical Systems

## Assignment I

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1. Consider the following equation

$$\dot{x} = rx + \frac{x^3}{1+x^2}$$

Find the fixed points of this equation and determine their stability. Sketch a bifurcation diagram, find the value (or values) of $r$ at which bifurcations occur and classify the bifurcation. [40%]

2. Consider the following system

$$\dot{x} = h + 2rx - x^2.$$

(a) Plot the bifurcation diagram for each of three cases $h < 0$, $h = 0$, $h > 0$. [10%]

(b) Sketch the regions in the $(r, h)$ plane that correspond to qualitatively different behaviour, and identify the bifurcations that occur on the boundaries of these regions. [30%]

3. Consider the insect outbreak model in dimensionless form

$$\dot{x} = rx\left(1 - \frac{x}{k}\right) - \frac{x^2}{1+x^2},$$

where $x$ is a variable, $r$ and $k > 0$ are parameters. The model is described in the section 3.7 of Strogatz's book. This section is available as the appendix to the assignment.

(a) Show that the fixed point $x^* = 0$ is always (for any $r$ and any positive $k$) unstable. [10%]

(b) Using computer and appropriate software reproduce the figure3.7.5. Show your work. [5%]

(c) Using computer and appropriate software reproduce the figure3.7.6. Show your work. [5%]