Complexity Science Doctoral Training Centre

CO903 Complexity and Chaos in Dynamical Systems

Assignment II

Issue date: 19 November
Submission date: 26 November (2pm)

1. For **any two** of the following systems,
(i) find the fixed points; [10%]
(ii) classify them; [10%]
(iii) sketch the neighboring trajectories, [10%]
and try to fill in the rest of the phase portrait (without using computer).
Then compare your sketch with a computer-generated phase portrait;
(iv) provide a brief description of behaviour of trajectories. [10%]
(a) $\dot{x} = \sin y$, $\dot{y} = \cos x$, (b) $\dot{x} = \sin y$, $\dot{y} = x - x^3$,
(c) $\dot{x} = x - y$, $\dot{y} = x^2 - 4$, (d) $\dot{x} = 1 + y - e^{-x}$, $\dot{y} = x^3 - y$,
(e) $\dot{x} = y + x - x^3$, $\dot{y} = -y$, (f) $\dot{x} = xy - 1$, $\dot{y} = x - y^3$.

2. Consider the following system $\dot{x} = bx - x^3/3 - y$, $\dot{y} = x - a$. Find the curve in (a, b) space at which Hopf bifurcation occurs. Using a computer, check the validity of the curve and determine whether the bifurcation is subcritical or supercritical. Plot typical phase portraits above and below the Hopf bifurcation. [30%]

3. For each of the following systems, a Hopf bifurcation occurs at the origin when $a = 0$. Using a computer, plot the phase portrait and determine whether the bifurcation is subcritical or supercritical.
(a) $\dot{x} = ax + y - x^2$, $\dot{y} = -x + ay + 2x^2$, [10%]
(b) $\dot{x} = ax + y - x^3$, $\dot{y} = -x + ay + 2y^3$, [10%]
(c) $\dot{x} = y + ax$, $\dot{y} = -x + ay - x^2y$. [10%]