Complexity Science Doctoral Training Centre

CO903 Complexity and Chaos in Dynamical Systems

Assignment I

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1. Consider the following equation

$$\dot{x} = rx + \frac{x^3}{1+x^2}$$

   Find the fixed points of this equation and determine their stability. Sketch a bifurcation diagram, find the value (or values) of r at which bifurcations occur and classify the bifurcation.

2. Consider the following system

$$\dot{x} = h + 2rx - x^2.$$

   (a) Plot the bifurcation diagram for each of three cases $h < 0$, $h = 0$, $h > 0$.

   (b) Sketch the regions in the (r, h) plane that correspond to qualitatively different behaviour, and identify the bifurcations that occur on the boundaries of these regions.

3. Consider the following Lotka-Volterra competition model for the population of two species, u_1 and u_2:

$$\begin{aligned} \dot{u}_1 &= u_1(1 - u_1 - a_{12}u_2), \\ \dot{u}_2 &= \rho u_2(1 - u_2 - a_{21}u_1), \end{aligned}$$

   where ρ, a_{12} and a_{21} are positive constants. Find the fixed points, investigate their stability and sketch plausible global solutions.

   [Hint: consider 4 cases (i) $a_{12} < 1$, $a_{21} < 1$, (ii) $a_{12} > 1$, $a_{21} > 1$, (iii) $a_{12} > 1$, $a_{21} < 1$, (iv) $a_{12} < 1$, $a_{21} > 1$].

Y Timofeeva 2009