Complexity Science Doctoral Training Centre

CO903 Complexity and Chaos in Dynamical Systems

Assignment II

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1. Using the method of multiple scales find the leading order solution to Duffing's equation

$$\ddot{x} + \epsilon x^3 + x = 0, \quad 0 < \epsilon \ll 1$$

with the initial conditions $x(0) = a$, $\dot{x}(0) = 0$.

Hint: you might find it useful to remember the following formulae during your calculation

$$e^{i\Theta} = \cos\Theta + i\sin\Theta \qquad \text{and} \qquad (\cos\Theta + i\sin\Theta)^3 = \cos(3\Theta) + i\sin(3\Theta)$$

2. Numerically analyse the so-called Atri model for intracellular calcium dynamics given by the following equations:

$$\begin{aligned} \frac{dc}{dt} &= \overbrace{\mu\, k_f\, h\left(b + \frac{V_1 c}{k_1 + c}\right)}^{J_{channel}} - \overbrace{\frac{\gamma c}{k_\gamma + c}}^{J_{pump}} + \overbrace{\beta}^{J_{leak}} \\ \tau_h \frac{dh}{dt} &= \frac{k_2^2}{k_2^2 + c^2} - h \end{aligned}$$

where c denotes Ca^{2+} concentration and h is an inactivation variable. μ is the concentration of inositol trisphosphate (IP_3) and is treated as the main bifurcation parameter. Other parameters are

$$\begin{aligned} b &= 0.111,\ V_1 = 0.889,\ \beta = 0.02\ \mu M \cdot s^{-1},\ \gamma = 2\ \mu M \cdot s^{-1},\ \tau_h = 2\ s, \\ k_1 &= 0.7\ \mu M, k_\gamma = 0.1\ \mu M,\ k_2 = 0.7\ \mu M,\ k_f = 8.1\ \mu M \cdot s^{-1}. \end{aligned}$$

[You might want to look at this review article on models of calcium dynamics:
http://www.scholarpedia.org/article/models_of_calcium_dynamics]

Compute the bifurcation diagram (including the branches of periodic solutions from Hopf bifurcation points) for calcium concentration using μ as a parameter. Window the diagram with $0 < \mu < 1.5$ and $0 < c < 3$. Classify Hopf bifurcation points.

Compute the two-parameter curve of Hopf bifurcation points using β as the second parameter. Find approximate β^* such that if $\beta < \beta^*$, then there are two Hopf bifurcation points.

3. Consider the following map:

$$f(n) = -rn\ln(n)$$

Find fixed points and determine the first period-doubling bifurcations of this map. Compute the bifurcation diagram of this map when r varies from 1 to 2.7. Compute Lyapunov exponents over the same range of parameter r.

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