Complexity Science Doctoral Training Centre

CO903 Complexity and Chaos in Dynamical Systems

Exam questions for preparation part (preparation time for two questions: 20 min)

1. First-order systems $\dot{x} = f(x),\ x \in \mathbb{R}$: fixed points, stability, graphical analysis of vector field.
2. Linear stability analysis for $\dot{x} = f(x),\ x \in \mathbb{R}$.
3. Existence and uniqueness theorem in $\mathbb{R}$. Potential for $\dot{x} = f(x),\ x \in \mathbb{R}$.
4. Euler and Improved Euler methods in numerical simulations.
5. Bifurcations in 1D (saddle-node, transcritical, pitchfork).
6. Multistability and hysteresis.
7. Flow on the circle (uniform oscillator, non-uniform oscillator).
8. Harmonic oscillator.
9. Solving linear system in $\mathbb{R}^2$.
10. Classification of fixed points for linear systems in $\mathbb{R}^2$.
11. Solving linear systems in $\mathbb{R}^n$ (normal forms).
12. Existence and uniqueness theorem in $\mathbb{R}^n$. Definitions of stability (Lyapunov, asymptotic).
13. Linear stability analysis for $\dot{x} = f(x, y),\ \dot{y} = g(x, y)$. Linear stability theorem.
14. Lyapunov theorem.
15. Hopf bifurcation (supercritical, subcritical).
16. Poincare-Bendixson theorem.
17. Relaxation oscillators.
18. Regular perturbation method.
19. Perturbation method of multiple scales.
20. Coupled oscillators.
21. Poincare maps and Floquet multipliers.
22. Defining chaos and attractor (strange attractor).
23. Main properties of the Lorenz equations: $\dot{x} = \sigma(y - x),\ \dot{y} = rx - y - xz,\ \dot{z} = xy - bz$.
24. Lyapunov exponent and a time when prediction breaks down.
25. Lorenz map.
26. One-dimensional maps, cobwebs.

27. Fixed points and linear stability of 1D maps.

28. A flip bifurcation in the logistic map.

29. Lyapunov exponent for 1D map.

30. Routes to chaos.

31. Fractal dimensions.

32. Global bifurcation (saddle-node bifurcation of cycles, homoclinic bifurcation, heteroclinic bifurcation).

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