Statistical Mechanics of Complex Systems

PX439 and **CO904**

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Module web page: `go.warwick.ac.uk/px439`
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About this module

PX439:

- lecture only (classwork optional)
- assessment: 1.5 hour written exam

CO904

- lecture + classwork
- assessment: classwork (50%) + oral exam on 10 Dec 2007 (50%).

Lecture:	Tue 10-11	F 1.11	(Engineering)
	Tue 13-14	B 2.13/4	(Science concourse)
	Fri 09-10	PS 1.28	(Physical Sciences)
	Fri 14-15	PS 1.28	

Classwork:	Tue 11-12	PS 0.17A	(Physical Sciences)
	Tue 14-15	PS 0.17A	
	Fri 10:30-12:30	PS 0.17A	

About this module

- broad student background
  level of material will vary
- some material closer to research:
  survive with incomplete information, closer to real life
- feedback:
  questionnaires
  email → `E.Somfai@warwick.ac.uk`
- literature:
  Books:
  - Sethna: Statistical Mechanics: Entropy, Order Pararmeters and Complexity, OUP 2006 (online: `http://pages.physics.cornell.edu/sethna/StatMech/`)
  - E. T. Jaynes: Probability Theory: The Logic of Science

  Journal papers: to follow

What are complex systems?

- Wikipedia:

> Complex system is a system composed of interconnected simple parts, that together exhibit a high degree of complexity from which emerges a higher order behavior.

- collective behaviour is not easily predictable from constituents
- typical examples:
  - (economy) stock market
  - (bio) cells, immune system
  - (social) political parties
  - (physics) phase transition

What are complex systems?

What is statistical mechanics?

- number of particles:
  - 1: very easy
  - 2: easy (two body problem)
  - 3: difficult (three body problem)
  - many: very difficult
  - 10^{23}: getting easy again (statistical mechanics)
    only aggregate information, not about individual particles
- microscopic foundation of thermodynamics
- powerful tools — not just for ideal gas!

Physicist approach

- based on models
  probably the most fundamental concept in science
- fundamentally quantitative
- but not rigorous

Contents of this module

Equilibrium:

- Statistical Mechanics from the Maximum Entropy Principle
- Fluctuations, the large number limit and Thermodynamics
- Equilibrium ordering and phase transitions

Dynamics:

- Modelling non-equilibrium states, granular physics
- Transport: fluxes and forces
- Dynamical ordering: examples in collective biological motion, flocking, social ordering

Probability distributions

- an event can have different outcomes: $A_1, A_2, \ldots, A_n$
  trying out N times we obtain N_1 times $A_1, \ldots$
- define probability:
$$p_i = \lim_{N\to\infty} \frac{N_i}{N}$$
- sum of probabilities:
$$\sum_{i=1}^{n} p_i = 1$$
- suppose each outcome has an associated quantity (eg. profit): X_i
  on a "long run" how much is the profit per event?
  Average or expectation:
$$\langle X \rangle = \sum_{i=1}^{n} X_i p_i$$
- different interpretation of probability: degree of belief

Information entropy

- given a discrete probability distribution,
  what is the amount of information an observation gives?
  (Amount of uncertainty of the random variable)

- properties required for $H_n(p_1, \ldots, p_n)$:
  - continuity: H_n is continuous function of p_i's
  - 'sense of direction':

    $$h(n) = H_n\left(\frac{1}{n}, \ldots, \frac{1}{n}\right)$$

    is monotonically increasing with n
  - consistency: if it can be calculated two ways, they should agree
    in particular,

$$H_3(p_1, p_2, p_3) = H_2(p_1, q) + qH_2\left(\frac{p_2}{q}, \frac{p_3}{q}\right) \qquad \text{where } q = p_2 + p_3$$

Information entropy

- solution (up to scale factor):

$$H_n\left(p_1, \ldots, p_n\right) = -\sum_{i=1}^{n} p_i \log(p_i)$$

information entropy or **Shannon entropy**
of a discrete probability distribution