

Statistical Mechanics of Complex Systems

PX439 and CO904

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Module web page: `go.warwick.ac.uk/px439`

`go.warwick.ac.uk/co904`

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About this module

PX439:

- lecture only (classwork optional)
- assessment: 1.5 hour written exam

CO904

- lecture + classwork
- assessment: classwork (50%) + oral exam on 10 Dec 2007 (50%).

Lecture:	Tue 10-11	F 1.11	(Engineering)
	Tue 13-14	B 2.13/4	(Science concourse)
	Fri 09-10	PS 1.28	(Physical Sciences)
	Fri 14-15	PS 1.28	

Classwork:	Tue 11-12	PS 0.17A	(Physical Sciences)
	Tue 14-15	PS 0.17A	
	Fri 10:30-12:30	PS 0.17A	

About this module

- broad student background
level of material will vary
- some material closer to research:
survive with incomplete information, closer to real life
- feedback:
questionnaires
email → E.Somfai@warwick.ac.uk
- literature:
Books:
 - ▶ Sethna: Statistical Mechanics: Entropy, Order Parameters and Complexity, OUP 2006 (online: <http://pages.physics.cornell.edu/sethna/StatMech/>)
 - ▶ E. T. Jaynes: Probability Theory: The Logic of ScienceJournal papers: to follow

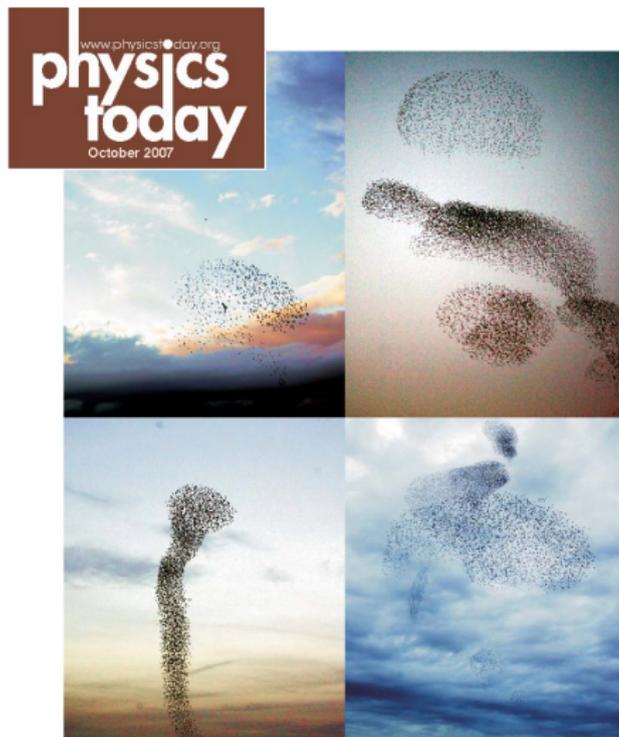
What are complex systems?

- Wikipedia:

Complex system is a system composed of interconnected simple parts, that together exhibit a high degree of complexity from which emerges a higher order behavior.

- collective behaviour is not easily predictable from constituents
- typical examples:
 - ▶ (economy) stock market
 - ▶ (bio) cells, immune system
 - ▶ (social) political parties
 - ▶ (physics) phase transition

What are complex systems?



The physics of flocking

What is statistical mechanics?

- number of particles:
 - ▶ 1: very easy
 - ▶ 2: easy (two body problem)
 - ▶ 3: difficult (three body problem)
 - ▶ many: very difficult
 - ▶ 10^{23} : getting easy again (statistical mechanics)
only aggregate information, not about individual particles
- microscopic foundation of thermodynamics
- powerful tools — not just for ideal gas!

Physicist approach

- based on models
probably the most fundamental concept in science
- fundamentally quantitative
- but not rigorous

Contents of this module

Equilibrium:

- Statistical Mechanics from the Maximum Entropy Principle
- Fluctuations, the large number limit and Thermodynamics
- Equilibrium ordering and phase transitions

Dynamics:

- Modelling non-equilibrium states, granular physics
- Transport: fluxes and forces
- Dynamical ordering: examples in collective biological motion, flocking, social ordering

Probability distributions

- an event can have different outcomes: A_1, A_2, \dots, A_n
trying out N times we obtain N_1 times A_1, \dots

- define probability:

$$p_i = \lim_{N \rightarrow \infty} \frac{N_i}{N}$$

- sum of probabilities:

$$\sum_{i=1}^n p_i = 1$$

- suppose each outcome has an associated quantity (eg. profit): X_i
on a “long run” how much is the profit per event?

Average or expectation:

$$\langle X \rangle = \sum_{i=1}^n X_i p_i$$

- different interpretation of probability: degree of belief

Information entropy

- given a discrete probability distribution, what is the amount of information an observation gives? (Amount of uncertainty of the random variable)
- properties required for $H_n(p_1, \dots, p_n)$:
 - ▶ continuity: H_n is continuous function of p_i 's
 - ▶ 'sense of direction':

$$h(n) = H_n \left(\frac{1}{n}, \dots, \frac{1}{n} \right)$$

is monotonically increasing with n

- ▶ consistency: if it can be calculated two ways, they should agree in particular,

$$H_3(p_1, p_2, p_3) = H_2(p_1, q) + qH_2\left(\frac{p_2}{q}, \frac{p_3}{q}\right) \quad \text{where } q = p_2 + p_3$$

Information entropy

- solution (up to scale factor):

$$H_n(p_1, \dots, p_n) = - \sum_{i=1}^n p_i \log(p_i)$$

information entropy or **Shannon entropy**
of a discrete probability distribution