Modelling Resonant Wave Interactions in Harbours

Supervisor: P. Denissenko (Engineering)
Co-supervisor: C. Connaughton (Complexity and Mathematics)

Project Outline and Objectives

Gravity waves at wavelengths of tens of metres represent a serious problem for ships docked in oceanic harbours. The energy is pumped in to the wave modes existing in a harbour by the waves coming in from an ocean storm. The main mechanism for the removal of the wave energy from the large waves is the progressive generation of shorter and shorter waves via nonlinear interactions. These shorter waves eventually break, thus removing energy from the system. If one supposed that the wavelengths of the long waves excited by the storm are known, one can ask the question of which shorter wavelenght modes are most easily excited and whether, by articfially exciting these modes, one could enhance the loss of energy by the large waves, thus reducing their amplitude and corresponding danger to ships. This project is a continuation of a miniproject which ran last year which made some progress towards answering these questions. By analysing timeseries data from experiments in a 6×12 metre wave tank [1] at Hull, it was suggested that the modes which are likely to be excited first for a given pair of excitation mode are those which satisfy a certain resonance condition involving quartets of waves while simulataneously maximising the nonlinear interaction coefficient describing energy exchange among the modes consituting the quartet.

The limitations of the experimental work were that only data in the frequency domain was possible and the underlying equations describing the nonlinear interactions of gravity waves are horrendously complicated making it difficult to derive a direct relationship between the observations and the theory. In this continuation, we propose to validate the above mechanism in the case of a reduced PDE model describing resonant gravity wave interactions:

$$\frac{\partial a_k}{\partial t} + i\omega_k a_k = \int T_{kk_1k_2k_3} a^*_{k_1} a_{k_2} a_{k_3} \delta(k - k_1 - k_2 - k_3) \tag{1}$$

where $\omega_k = \sqrt{k}$ (this is the dispersion relation for surface gravity waves on water) and $T_{kk_1k_2k_3}$ will be chosen as a simple function which will mimic the (very complicated) true nonlinear interaction coefficient.

Required Background and Methodology

The student will be required to solve (numerically) the set of coupled nonlinear differential equations:

$$\begin{aligned}
\partial_t a_p + i\omega_p a_p &= 2T(p,q,r,s) a^*_q a_r a_s \\
\partial_t a_q + i\omega_q a_q &= 2T(p,q,r,s) a^*_p a_r a_s \\
\partial_t a_r + i\omega_r a_r &= 2T(r,s,p,q) a^*_s a_p a_q \\
\partial_t a_s + i\omega_s a_s &= 2T(s,r,p,q) a^*_r a_p a_q
\end{aligned} \tag{2}$$

obtained by galerkin truncation of Eq. (1) to a single quartet (p,q,r,s). The next step will be to solve the PDE, Eq. (1) using a pseudo-spectral algorithm and compare the results with the truncated system, Eq. (2) and qualitatively compare with experimental data.

Research Outcomes and Outlook

We will use this project to supplement the results from the earlier research in order to make them publishable. The project may initiate further wave experiments at Hull in the end of 2009. The end users are engineers at Warwick and Hull. If initiation of a wave mode different from that excited by the ocean waves will prove to reduce the amplitude of waves in a harbour, major implications in harbour design and management may follow.

Contact Information: Petr Denissenko – School of Engineering, University of Warwick. Phone: 28010; Rm. F337; E-mail: P.Denissenko@warwick.ac.uk

References

[1] Petr Denissenko, Sergei Lukaschuk, and Sergey Nazarenko. Gravity Wave Turbulence in a Laboratory Flume. *PRL* **99**, 014501 (2007); doi:10.1103/PhysRevLett.99.014501