

## Random media, random polymers and random surfaces

Motion in random media constitutes a very active research area on the interface of probability theory, mathematical physics and material sciences, see e.g. [3]. In fact the presence of randomness in the medium is the source of a wide variety of effects. For example, one may consider a long polymer chain, the shape of which might be drastically altered by the presence of impurities (randomness) in the medium, e.g. localization/delocalization transition. The growth of random surfaces can also be described as a (zero temperature) motion in a random medium [1],[4].

The present state of our understanding is rather limited, based on non rigorous physical theories and some recent, rather special, mathematically rigorous results [4].

### Details.

A directed random polymer is the path  $X = (X_n : n = 0, \dots, N)$  of a random walk in  $\mathbb{Z}^d$  starting in the origin, with probability distribution

$$\mu_N(X) = \frac{1}{Z_N} \exp\left(\beta \sum_{n=0}^N V(n, X_n)\right) P(X). \quad (1)$$

where the normalization  $Z_N$  is called the partition function and  $P(X)$  is the probability of a simple random walk with trajectory  $X$ . The interpretation is that the space-time points  $(n, x)$  carry independent identically distributed random weights  $V(n, x)$ , altering the distribution of the walk. Depending on the inverse temperature  $\beta$  and the distribution of the  $V(n, x)$ , the end of the polymer can exhibit usual Gaussian fluctuations, i.e.  $X_N \sim \sqrt{N}$  or (larger) fluctuations of order  $N^\gamma$ , for some exponent  $\gamma$  different than  $1/2$ . Intuitively, the influence of the random environment increases with increasing  $\beta$ . Indeed, it has been shown that for space dimensions  $d \geq 3$  there exists a critical value  $0 < \beta_c(d) < \infty$  such that fluctuations are Gaussian for  $\beta < \beta_c(d)$  and non-Gaussian for  $\beta > \beta_c(d)$ . The second case falls into the so called KPZ universality class [1] (instead of the central limit theorem regime). For  $\beta = \infty$  (zero temperature), the polymer distribution will be dominated by the one with the highest weight; this is known as last passage percolation [2].

Providing good estimates of the actual value of  $\beta_c(d)$  and its dependence on the random environment is an active area of research, with scope for mathematical as well as computational investigations.

### Aims of the project.

The first part of the project provides an introduction to the area of critical phenomena in kinetic roughening and random media, which is mainly achieved by reading relevant literature. The practical part consists of doing a computer simulation on one of the following open questions:

- Estimate the critical temperature  $\beta_c(d)$  of the roughening transition for  $d = 3$ , depending on the distribution of the random environment.
- Investigate a conjectured cross-over for last passage percolation ( $\beta = \infty$ ) in dimension  $d = 1$ . For small parameter  $p$  of a Bernoulli random environment, i.e.  $V(n, x) = 1$  with prob.  $p$  and  $V(n, x) = 0$  with prob.  $1 - p$ , fluctuations should be Gaussian, for large  $p$  an  $N^{1/3}$ -scaling is expected.

The first problem might be computationally challenging due to high space dimension, whereas the second one should clearly be feasible.

### Scope for PhD.

Motion in random media and critical phenomena within this framework are a very active area of research. There is scope for mathematically challenging PhD projects as well as practical ones in connection to real-world phenomena, where precise numerical estimates of critical values are of great importance.

### References.

- [1] J. Krug, *Origins of scale invariance in growth processes*. Adv. Phys. **46**(2), 139-282 (1996)
- [2] J.B. Martin, *Last-passage percolation with general weight distribution*, Markov Processes and Related Fields **12**, 273-299 (2006). (<http://www.stats.ox.ac.uk/~martin/papers.html>)
- [3] A.S. Sznitman, *Brownian motion, obstacles and random media*. Springer Monographs in Mathematics. Springer-Verlag, Berlin, 1998. xvi+353 pp.
- [4] K. Johansson, *Discrete polynuclear growth and determinantal processes*. Comm. Math. Phys. **242** (2003), no. 1-2, 277-329

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