# Convex hulls, anomalous diffusion & the home range of foraging animals

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**Research objectives of the mini-project:**

• Write programs to numerically confirm the main results of Majumdar et al [*PRL*, **103**, 140602 (2009)] for the convex hull of Brownian motion.
• Generalise the above to allow numerical, and ideally analytical, investigation of the effect of anomalous diffusion on the convex hull.

**Why it is interesting:** The problem of how *n* random walkers "cover the ground" has many applications. In ecology it is directly related to the size of the home range (http://en.wikipedia.org/wiki/Home_range) of animals, and how this quantity scales with the number of animals in a given region. A classic ecological approach to the problem has been through modelling animals as Brownian walkers. One can define an area and perimeter for the walkers, using the definition of a convex hull as the minimal convex polygon enclosing a set of points. Your knowledge about random walks will already tell you that the area and perimeter scale as the duration T of the walk, and the square root of T, respectively, but the prefactors which control the exact magnitude of the area and perimeter are much harder to calculate. Important recent work has obtained them for the Brownian case [Majumdar et al, op cit; Majumdar et al, arXiv: 0912:0631].

Brownian motion may not, however, be a very good model for the motion of animals (or humans for that matter [c.f. Brockmann et al, *Nature* **439**, 462 (2006)]), so it is a natural and topical problem to explore how "anomalous" diffusion alters both the scaling relations and prefactors. Anomalous diffusion can arise both from memory effects, in which the size or direction of a step is no longer independent of those that preceded it, and from jumps taken from a magnitude distribution such as power laws which are "heavier" than Gaussian, with a correspondingly greatly increased likelihood of large steps. Paradigmatic examples which you will have encountered are fractional Brownian motion and Levy flights respectively.

In this project you will explore a toy model which combines the two effects and will allow us to numerically explore some conjectures about the consequent scaling of the convex hull. There will be substantial scope for analytical approaches, and the great current interest in anomalous diffusion [e.g. G. M. Zaslavsky, *Hamiltonian Chaos and Fractional Dynamics*, OUP, 2004; R. Balescu, *Aspects of Anomalous Transport in Plasmas* IoP, 2005; *Anomalous Transport*, R. Klages, G. Radons, and I. M. Sokolov eds., Wiley, 2008] seems likely to make the results potentially publishable. The proposer and his colleagues have a good track record of developing summer project work for papers with student co-authors.

**Background to be assimilated:** Literature on stochastic processes and random walks; a representative example would be_the physics book by Paul and Baschnagel **http://preview.tinyurl.com/ykq5rld**

**Techniques required:** Basic knowledge of Applied Mathematical methods & Brownian Motion Theory from your Msc courses, and some ability at programming in a package like Matlab or R should suffice.

**Relation to end/downstream users- who should benefit from this line of research:** The Natural Complexity Group at the British Antarctic Survey is working on related issues for various observed datasets. Also the wider complexity, anomalous diffusion and ecology research communities will benefit from this research, and are likely to find it interesting, based on discussions which the proposer has had, and the "open problems" section of Majumdar's Warwick talk in 2010 [http://www2.warwick.ac.uk/fac/sci/maths/research/events/2009_2010/symposium/neqwks/programme/sm/conhull_satya_majumdar.pdf].

**Prospects for this mini-project leading into a PhD project:** The British Antarctic Survey are interested in developing the work into a complexity PhD project, depending on results.