

Properties of geodesics and fixation in Last Passage Percolation.

Last Passage Percolation (LPP) constitutes a very active research area on the interface of probability theory, mathematical physics and material sciences, see e.g. [4]. LPP originated from physical studies on growth models and passage times of liquids inside corroded materials. Soon enough it was realized that LPP could be used to model a much larger number of physical phenomena, but only recently its far reaching and absolutely surprising connections to diverse areas of mathematics (random matrices, concentration, particle systems etc.) have been realized. Still, both the mathematics and the physics communities lie far from a complete understanding of its rich structure.

Details.

Assume that on each site of the lattice $\mathbb{N} \times \mathbb{Z}^d$ there is a nonnegative random variable $\omega(n, x)$ with finite mean and assume that all these variables are i.i.d. Consider, also, all possible simple random walk paths of length N , i.e. $\mathbf{X}_N = (X_n : n = 0, \dots, N)$. The energy of a path \mathbf{X}_N is defined as $H_{\omega, N}(\mathbf{X}_N) := \sum_{n=1}^N \omega(n, X_n)$. Last Passage Percolation is concerned with the study of the statistical properties of the path $\mathbf{X}_{\omega, N}^*$ with maximal energy $H_{\omega, N}^* := H(\mathbf{X}_{\omega, N}^*) := \max_{\mathbf{X}_N} H(\mathbf{X}_N)$, as well as the properties of the associated maximal energy $H_{\omega, N}^*$. Unlike the fluctuations of a typical random walk path, which are of order \sqrt{N} , the maximal path will have fluctuations of order N^χ . The exponent χ has been conjectured from physicists to be equal to $2/3$ in dimension one and its validity has been proved mathematically in a few cases using techniques from particle systems and random matrices. Contrary to the one dimensional case, the value of χ has been elusive in higher dimensions.

In this project we will study some of the properties of these geodesics. In particular, we will focus on the following question: Consider the sequence of maximal paths $(\mathbf{X}_{\omega, N}^*)_{N \geq 1}$. Does the first step of these maximal paths become constant for all $N > N_0(\omega)$ large enough (*fixation*), or will it keep changing among the possible $2d$ directions (*lack of fixation*) ?

As a first step we will study the analogous question on trees, i.e. in this case the lattice $\mathbb{N} \times \mathbb{Z}^d$ will be replaced by a tree structure. The question will be again whether the first step of the maximal paths of increasing length fixate or not. The question in this setting should be easier due to presence of a greater amount of independence.

Aims of the project.

The first part of the project will provide an introduction to LPP which will be mainly achieved by reading selected topics from the recent and classical literature, eg [2],[3]. The part of the study of fixation on trees will be reduced to a question of branching random walks and therefore the candidate will have the chance to familiarize him/her self with some selected topics from that theory, e.g. [1]. The practical part will consist of doing a computer simulation to test whether fixation occurs or not. Computer simulations in this setting should be feasible (simulations of related nature in the more complicated setting of random polymers were achieved by a Complexity student - Oliver Humbrey - two years ago in the frame of the mini project).

Scope for PhD.

This mini project will provide an excellent step towards a PhD thesis on LPP and the more general field of random polymers. In fact, both the numerical testing and the rigorous mathematical treatment of the present project will be of great interest on their own. The MSc candidate will be introduced in this way to an area, which lies in the center of the modern probability theory, and at the same time is of great significance for the corresponding statistical mechanics community.

References.

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- [2] E. Cator, L. Pimentel, *A shape theorem and semi-infinite geodesics for the Hammersley model with random weights*, arXiv:1001.4706
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- [4] J. Krug, *Origins of scale invariance in growth processes*. Adv. Phys. **46**(2), 139-282 (1996)

Contact details:

Nikolaos Zygouras, Department of Statistics (C0.11), phone: 75870

N.Zygouras@warwick.ac.uk