Research project proposal February 2011

Competition interfaces in expanding populations

In addition to selective pressure, genetic drift is now widely accepted to be a major evolutionary force. The term summarizes the effective stochasticity from chance effects in reproduction events, and can have significant impact in small populations or at the growth front of growing populations. This was studied under controlled experimental conditions in [1] for flourescently labeled microbial organisms, including E. coli and yeast. Under rich growth conditions the only limiting factor is available space, and a coarsening process driven by genetic drift gives rise to segregation patterns for the two labels, starting with well mixed initial conditions. In [2] a mathematical description of this phenomenon was established using principles of local scale invariance, leading to a mapping between circular and linear growth geometries.

To fully understand the underlying principles of this phase segregation, we study a simplified, lattice-based mathematical growth model with a single boundary between two species. This model can be mapped on a well-studied particle system in non-equilibrium statistical mechanics, the totally asymmetric simple exclusion process (TASEP). The location of a special particle in that process marks the position of the species boundary, also called competition interface. It can be studied by rigorous analytic methods (see [3]), and the dynamics of the interface is also related to so-called 'directed polymer models' and 'last passage percolation' (see [4] for a survey, more references will be provided).

Details.

The TASEP is a continuous-time ergodic Markov chain, where identical particles jump to the right with rate one subject to an exclusion interaction (at most one particle per site). A so-called second class particle follows the same rules, but can be overtaken by the other particles. It has been shown to mark the location of a particular type of competition interface [3], which is directly related to last-passage percolation [4]. The latter roughly consists of finding a path of maximal weight in a given random environment. To be relevant in the biological context of species segregation, the dynamics of the interface have to be modified, leading to a new type of particle in the TASEP which has not been studied so far.

After some introductory reading (which mainly consists of references [2] to [4]), the aim of the project is to perform a Monte Carlo simulation of the TASEP with periodic boundary conditions and study the dynamics of the particle marking the competition interface. The most interesting question to start with is the scaling of the mean-squared displacement of the particle depending on the initial conditions. The theoretical part of the project will be to connect the dynamics of this particle to a last-passage percolation/directed polymer model, in analogy to the results in [3].

Collaboration, prospect for PhD project.

The project is an extension of a successful miniproject of Adnan Ali, who is currently doing his PhD on questions related to the influence of reproduction time statistics on the macroscopic behaviour. The project is meant to provide an accessible introduction and offers opportunities for PhD projects in applied as well as more mathematical directions. A PhD project continuing in a theoretical direction related to exclusion processes and last passage percolation would be part of a collaboration with Patrik Ferrari at the University of Bonn.

References.

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[3] P.A. Ferrari, J.B. Martin, L.P.R. Pimentel, *Roughening and inclination of competition interfaces*. Phys. Rev. E **73**, 031602 (2006), arXiv:math/0412198

[4] J.B. Martin, *Last-passage percolation with general weight distribution*. Markov Processes and Related Fields **12**, 273-299 (2006),
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