Condensation in disordered stochastic particle systems

Phase transitions are a common collective phenomenon observed in complex interacting systems, and there is a well developed mathematical theory for systems in thermal equilibrium. However, many phenomena of interest in applications are not in equilibrium. e.g. think about traffic on a highway or spreading of infectious diseases. Typical phase transitions in these applications can be rather annoying, such as traffic jams or epidemics. To avoid or influence their occurrence, a systematic understanding of the underlying mechanisms is vital and a current topic of major research in statistical mechanics. In this context one often studies simplified mathematical models which capture the qualitative features of the phenomenon.

One of these models that has recently attracted attention is the zero-range process, which exhibits a condensation transition that is mathematically well understood [1]. These results are widely applied to a range of applications (see [1] and references therein), such as clustering in shaken granular gases, traffic jams or growing and rewiring networks. The more recently introduced inclusion process shows a similar critical behaviour, which is only partly understood so far [4]. An intriguing question is the robustness of the transition under random perturbations of the dynamics. This is very relevant for applications, since real systems are typically not perfectly homogeneous. The aim of the project is to extend first results in [3,4], towards answering the following questions:

→ Does the phase transition persist in the thermodynamic limit in the presence of disorder?

→ Is the disorder relevant, i.e. does it change critical properties or could it just be replaced by its average?

These are the most basic (and interesting) questions in statistical mechanics of disordered systems in general, and still need to be established in the context of condensation models.

Details.

Zero-range and inclusion processes are continuous-time ergodic Markov chains, where identical particles jump on a lattice with a rate that depends on the local particle configuration. When the particle density is higher than a critical value ρ_c, the system phase separates in the stationary state into a homogeneous background with density ρ_c and a condensate site, containing all the excess particles. An introduction to the zero-range process and its applications is given in [1] which could be read together with the first two chapters of [2], where connections to other interacting particle systems and typical questions of interest are explained. [3,4] and chapter 6 of [2] contain detailed information directly relevant for the project.

In the homogeneous case the transition is well understood mathematically (see e.g. [1] and [4]), and the relevance of perturbations should be inverstigated in this project using one or both of two possible routes: Existence of condensation (equivalent to $\mathbb{E}\rho_c < \infty$) can be established by mathematically rigorous estimates for the free energy. A study of the actual value of the mean and the full distribution of the critical density (which is a random variable due to disorder) can be obtained using Monte Carlo simulations. First results indicate that it shows an interesting power law behaviour with an exponetial cut-off at large values.

Collaboration, prospect for PhD project.

The project is an extension of a successful miniproject of Paul Chleboun, who is currently doing his PhD on related questions in condensing particle systems. He agreed to co-supervise the project and help in particular with computational questions arising, a previous version of his code could be used as a starting point. The project is meant to provide an accessible introduction to phase transitions in stochastic particle systems, which is an active field and offers many opportunities for PhD projects in applied as well as more theoretical directions.

References.

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Contact details:

Stefan Grosskinsky office: D1.10

Mathematics Institute phone: +44 2476522673

University of Warwick S.W.Grosskinsky@warwick.ac.uk