Synchronization in complex networks

Synchronized neural activity plays a fundamental role in the brain, mostly a functional one when processing information and external stimuli. But also 'malfunctions' such as epileptic seizures or Parkinson's tremor are know to be connected to synchronized neural activity on large scales. Trying to understand such phenomena, the collective behaviour of interacting dynamical systems on complex networks has recently attracted considerable research interest [1]. These systems can also be power station grids or stock markets, and synchronization is an important aspect in various fields of applications [2].

In general there are two important questions for synchronization. The level, i.e. which fraction of the system is in a synchronized state, and the synchronization time, i.e. the time it takes to reach such a state. Both features depend on the structure of the network. Regarding the level, results indicate that small world networks perform better than deterministic or purely random graphs [3]. With respect to synchronization times there are analytic results for random graphs [4], but the dependence on the network structure has not been studied systematically so far. The aim of the project is to conduct such a study by computer simulations for pulse-coupled oscillator networks, which are directly inspired by neural networks in the brain.

Details.
Consider a random neural network, where each neuron i is represented by an oscillator, whose state is specified by a single phase variable $\phi_i(t)$. In the absence of interactions the phase increases linearly with time, e.g. $\frac{d}{dt}\phi_i(t) = 1$. When it reaches a certain threshold ϕ_i is reset to 0 and the neuron 'fires' a signal to all connected neurons. They receive the signal after a delay time τ, and the incoming signal induces a phase jump according to a specified rule, which depends on the synaptic connection between the neurons. Under mild conditions on the model parameters, such systems are known to converge to a fully synchronized state. The time to reach this state depends on the structure as well as the size of the network. In the given context, the most reasonable measure of the size seems to be the typical graph distance on the network rather than just counting the numbers of neurons or synapses. To study the effect of the structure we would like to compare average synchronization times for different network topologies of the same size.

Suggested workplan.
First get an overview over random networks and synchronization by reading e.g. parts of [2] and [5]. Have a look at literature on synchronization times such as [3] and [4] to pin down interesting network types to look at. Write a computer program to simulate a pulse-coupled oscillator network as described above. Measure the average synchronization time for different network topologies with a fixed typical distance and analyze the data.

Collaboration, PhD project.
This research project is a collaboration initiated by the group of Marc Timme at the Max-Planck Institute for Dynamics and Self-Organization in Göttingen, Germany. A following PhD project would be carried out in joint supervision with Marc Timme and close collaboration with his research group. The structure and in particular the dynamics of complex networks is an active field of research with many interesting applications. So there is ample room for a direct extension of this project to a PhD in applied as well as more theoretical direction.

References.   (There are many more, these are just suggestions.)

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