Nonequilibrium phase transitions in perturbed particle systems

Phase transitions are a common collective phenomenon observed in complex interacting systems, and there is a well developed mathematical theory for systems in thermal equilibrium. However, many phenomena of interest in applications are not in equilibrium. Think about traffic, such as cars on a highway or ribosomes moving along messenger RNA in protein synthesis, or spreading of infectious diseases and various other examples. Typical phase transitions in these applications can be rather annoying, such as traffic jams or epidemics. To avoid or influence their occurrence, a systematic understanding of the underlying mechanisms is vital and a current topic of major research in statistical mechanics. In this context one often studies mathematical models which capture the qualitative features of the phenomenon.

One of these models that has recently attracted attention is the zero-range process, which exhibits a condensation transition that is mathematically well understood [1]. These results are widely applied to a range of applications (see [1] and references therein), such as clustering in shaken granular gases, traffic jams or growing and rewiring networks [3]. An intriguing question is the robustness of the results under small random perturbations. This is very relevant for applications, since real systems are typically not perfectly homogeneous. The aim of the project is to complement recent results [4] on that question by Monte-Carlo simulations, and to study dynamical properties of the phase transition which are not accessible by exact calculations.

Details.

The zero-range process is a continuous-time ergodic Markov chain, where identical particles jump on a lattice with a rate that depends on the local particle configuration. When the particle density is higher than a critical value ρ_c, the system phase separates in the stationary state into a homogeneous background with density ρ_c and a condensate site, containing all the excess particles. The phase diagram of the process in the stationary state is well understood mathematically (see e.g. [1] and [2]), and recent results indicate that it is very sensitive to small random perturbations of the parameters [4]. Monte Carlo simulations of the model should complement these results and get new insight on the time evolution. Starting the simulation from a homogeneous state with particle density higher than ρ_c, the process is expected to show an interesting coarsening phenomenon with increasing roughness of density profiles (see e.g. [3] chapter 6). This behaviour should follow a scaling law, which should be derived heuristically and confirmed by simulations.

Suggested workplan.

An introduction to the zero-range process and its applications is given in [1] which could be read together with the first two chapters of [2], where connections to other interacting particle systems and typical questions of interest are explained. [4] and chapter 6 of [2] contain detailed information directly relevant for the project, and should be consulted to identify interesting quantities to be measured. Then write a computer program for a Monte-Carlo simulation of the zero-range process. Measure the critical density as a function of the system parameters to map out the stationary phase diagram, and dynamical quantities to verify the coarsening scaling law.

Collaboration, PhD project.

The project is part of an ongoing research collaboration with Gunter Schütz from the Research Centre at Jülich, Germany. The study of nonequilibrium phase transitions in interacting particle systems is an active field with various applications. The project is meant to be an accessible introduction in this area of research, which offers many opportunities for PhD projects in applied as well as more theoretical directions.

References.

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[4] S. Grosskinsky, G.M. Schütz, *Condensation in a zero-range process with random interaction*. In preparation, available online at: `http://www.warwick.ac.uk/~masgav/` under publications

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