

Rough paths and sound recognition

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Rough paths theory provides a tool, the signature of iterated integrals,

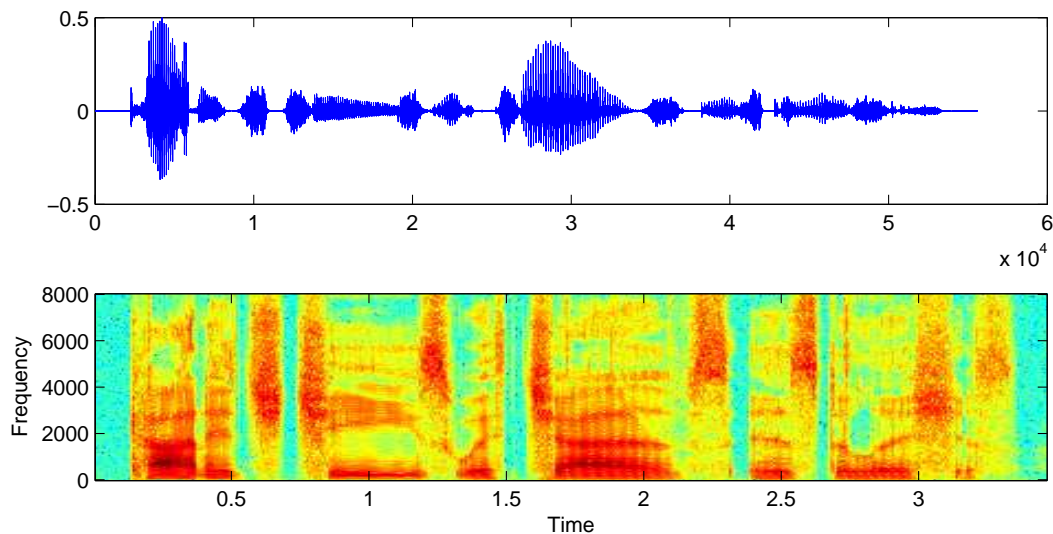
$$X_{0,1}^n = \int_{0 < u_1 < \dots < u_n < 1} 1 dX(u_1) \otimes \dots \otimes dX(u_n) \in \mathbb{R}^{d^n}$$

for describing the shape of a path. If you think of the path as a driving signal for a differential equation

$$dY(t)f(Y(t)) = dX(t)$$

then the signature seems to provide a very concise way of describing the impact the path will have.

For example, think of a sound wave making your ear drum vibrate—the signature of the path is related to what you will actually hear. The mathematical difficulty with signatures is that the inverse problem, calculating the path from the signature, is very difficult. The goal of the project would be to explore the many possible applications of machine learning to studying the link between paths and their signatures.



References:

- T. J. Lyons, N. Sidorova: Sound compression - a rough path approach, Proceedings of the 4th International Symposium on Information and Communication Technologies, 223-229, Cape Town, January 2005
- Terry Lyons and Zhongmin Qian, System Control and Rough Paths, Clarendon Press, Oxford Mathematical Monographs