

Amalgamating Personal Information Through Sequential Voting

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A team of n agents (a jury, perhaps), has to decide a verdict as to which of two states of Nature (A or B=not A) holds. Maybe 'innocent or guilty' (in a trial), or 'in or out' (for a tennis line call by linesman and then umpire). Each agent i has a subjective probability of A, which he gets through a 'signal'. Different agents have different signal strengths, higher signal strengths make them more informed (more likely to have subjective probabilities near 0 or near 1). A strategy for each player is determined by threshold numbers $x_i(S)$, where agent i will vote A after a voting sequence S if his subjective probability p_i satisfies $p_i > x_i(S)$. A team strategy is determined by a complete set of thresholds which tells each agent how to vote after every possible prior voting sequence S and his probability p_i .

Question 1: Given the set of signal strengths (a_1, a_2, \dots, a_n) , what are the optimal thresholds for voting, and what is the maximum probability of achieving the correct verdict?

Question 2: What is the best way to order the agents (the voting order), given their signal strengths?

With a colleague Prof Bo Chen, I have answered these questions for a particular family of signals. We find, surprisingly, that for a jury of 3, the best informed and least informed agents (those with lowest and highest signal strengths) should go middle and last in voting order.

A particular version a student could study is the following: There are three balls in an urn. Equiprobably there are two white and one black (state A), or one white and two black (state B). An agent with signal strength k is allowed to 'sample a ball, observe its color, and replace it' k times. Clearly higher k give a more informed agent.

Question 1: Suppose a jury of 3 has signal strengths say $(3,5,1)$, in order of voting. What are the optimal voting thresholds and what is the maximum probability they can achieve a correct verdict by majority vote?

Question 2: What is the best voting order for these jurors, assuming they can choose the order.

At first, this could be done numerically using Mathematica or some other program. If patterns emerge (as they have done in the past), the student could try to establish them algebraically or otherwise. Since there is sampling involved, a simulation approach could also be tried.