Properties of Extremely Heterogeneous Networks

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A network is defined by a set of nodes $\mathcal{N}$ and a set of links between them $\mathcal{L} \subseteq \mathcal{N} \times \mathcal{N}$. The *degree* of a node is the number of links that it is part of. For many real networks, the degrees of nodes are observed to be extremely heterogeneous [7], and it is often claimed that the proportion of nodes with degree k,

$$p_k \sim k^{-\alpha} \text{ for large } k \; . \tag{1}$$

Such graphs can be generated and analysed using a stochastic process known as preferential attachment [1], and are often called 'scale free' because the power-law behaviour of (1) is invariant under a rescaling $k \to \beta k$.

Scale free networks have many unusual properties, for example they are very resilient to random attack but susceptible to targeted attack [5], they are all sparse [3], and asymptotically they do not have an epidemic threshold [8, 5] (although this need not hold for finite networks [6] or if high degree nodes are less transmissible on average [2]).

Recent work has shown that Monte Carlo methods used in sampling heterogeneous networks can lead to unreliable results [3], and that for degree-homogeneous networks the moment closure assumptions used to obtain analytical results can actually lead to qualitatively incorrect conclusions [4]. This project aims to develop the reliable Monte Carlo and analytical approaches proposed as alternatives, and will apply these to the study of highly heterogeneous networks. Of particular interest is their how their connectivity affects their ability to support spreading processes such as epidemics, and how the process dynamics changes as the value of α is varied.

References

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