Compositional Temporal Synthesis

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What Good is Model Checking?

Model Checking:

- *Given*: Program P, Specification φ.
- *Task*: Check that $P \models \varphi$

Success:

- *Algorithmic methods*: temporal specifications and finite-state programs.
- *Also*: Certain classes of infinite-state programs
- *Tools*: SMV, SPIN, SLAM, etc.
- *Impact* on industrial design practices is increasing.

Problems:

- Designing P is hard and expensive.
- Redesigning P when $P \not\models \varphi$ is hard and expensive.

Automated Design

Basic Idea:

- Start from spec φ, design P such that $P \models \varphi$.

  Advantage:

  - No verification
  - No re-design

- Derive P from φ algorithmically.

  Advantage:

  - No design

In essence: Declarative programming taken to the limit.

Program Synthesis

The Basic Idea: Mechanical translation of human-understandable task specifications to a program that is known to meet the specifications.

Deductive Approach (Green, 1969, Waldinger and Lee, 1969, Manna and Waldinger, 1980)

- Prove *realizability* of function,
  e.g., $(\forall x)(\exists y)(Pre(x) \rightarrow Post(x, y))$
- Extract *program* from realizability proof.

Classical vs. Temporal Synthesis:

- *Classical*: Synthesize transformational programs
- *Temporal*: Synthesize programs for ongoing computations (protocols, operating systems, controllers, etc.)

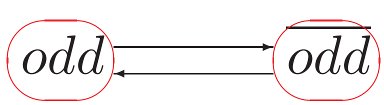
Synthesis of Ongoing Programs

Specs: Temporal logic formulas

Early 1980s: Satisfiability approach
(Wolper, Clarke+Emerson, 1981)

- *Given*: φ
- *Satisfiability*: Construct $M \models \varphi$
- *Synthesis*: Extract P from M.

Example: $always\ \ (odd \rightarrow next\ \neg odd) \wedge$
$always\ \ (\neg odd \rightarrow next\ odd)$



Reactive Systems

Reactivity: Ongoing interaction with environment (Harel+Pnueli, 1985), e.g., hardware, operating systems, communication protocols, etc.

Example: Printer specification –
J_i - job i submitted, P_i - job i printed.

- *Safety*: two jobs are not printed together
  $always \ \neg(P_1 \wedge P_2)$

- *Liveness*: every job is eventually printed
  $always \ \bigwedge_{j=1}^{2}(J_i \rightarrow eventually \ P_i)$

Satisfiability and Synthesis

Specification Satisfiable? Yes!

Model M: A single state where J_1, J_2, P_1, and P_2 are all false.

Extract program from M? No!

Why? Because M handles only one input sequence.

- J_1, J_2: input variables, controlled by environment
- P_1, P_2: output variables, controlled by system

Desired: a system that is receptive to *all* input sequences.

Conclusion: Satisfiability is inadequate for synthesis.

Realizability

I: input variables, O: output variables

Game:

- *System*: choose from 2^O
- *Env*: choose from 2^I

Infinite Play:

$i_0, i_1, i_2, \ldots$

$0_0, 0_1, 0_2, \ldots$

Infinite Behavior: $i_0 \cup o_0, i_1 \cup o_1, i_2 \cup o_2, \ldots$

Win: behavior $\models$ spec

Specifications: LTL formula on $I \cup O$

Strategy: Function $f : (2^I)^* \rightarrow 2^O$

Realizability: Abadi+Lamport+Wolper, 1989
Dill, 1989, Pnueli+Rosner, 1989
Existence of winning strategy for system.

Synthesis: Pnueli+Rosner, 1989
Extraction of winning strategy for system.

Church's Problem

Church, 1957: Realizability problem wrt specification expressed in MSO (monadic second-order theory of one successor function)

Büchi+Landweber, 1969:

- Realizability is decidable.
- If a winning strategy exists, then a *finite-state* winning strategy exists.
- Realizability algorithm *produces* finite-state strategy.

Rabin, 1972: Simpler solution via Rabin tree automata.

Question: LTL is subsumed by MSO, so what did Pnueli and Rosner do?
Answer: better algorithms!

Post-1972 Developments

- Pnueli, 1977: Use LTL rather than MSO as spec language.
- V.+Wolper, 1983: Elementary (exponential) translation from LTL to automata.
- Safra, 1988: Doubly exponential construction of tree automata for strategy trees wrt LTL spec (using V.+Wolper).
- Pnueli+Rosner, 1989: 2EXPTIME realizability algorithm wrt LTL spec (using Safra).
- Rosner, 1990: Realizability is 2EXPTIME-complete.

Standard Critique

Impractical! 2EXPTIME is a horrible complexity.

Response:

- 2EXPTIME is just worst-case complexity.
- 2EXPTIME lower bound implies a doubly exponential bound on the size of the smallest strategy; thus, hand design cannot do better in the worst case.

Real Critique

- Algorithmics not ready for practical implementation.
- Complete specification – unrealistic.
- Construction from scratch – unrealistic.

Response: More research needed!

- Better algorithms
- Incremental synthesis – write spec incrementally.
- *Compositional synthesis* – synthesis from components.

Synthesis from Components

Basic Intuition: [Lustig+V., 2009]

- In practice, systems are typically not built from scratch; rather, they are constructed from existing components.
  - *Hardware*: IP cores, design libraries
  - *Software*: standard libraries, web APIs
  - *Example*: mapping application on smartphone – location services, Google maps API, graphics library
- Can we automate "construction from components"?

Setup:

- *Library* $L = \{C_1, \ldots, C_k\}$ of *component types*.
- *Linear temporal specification*: φ

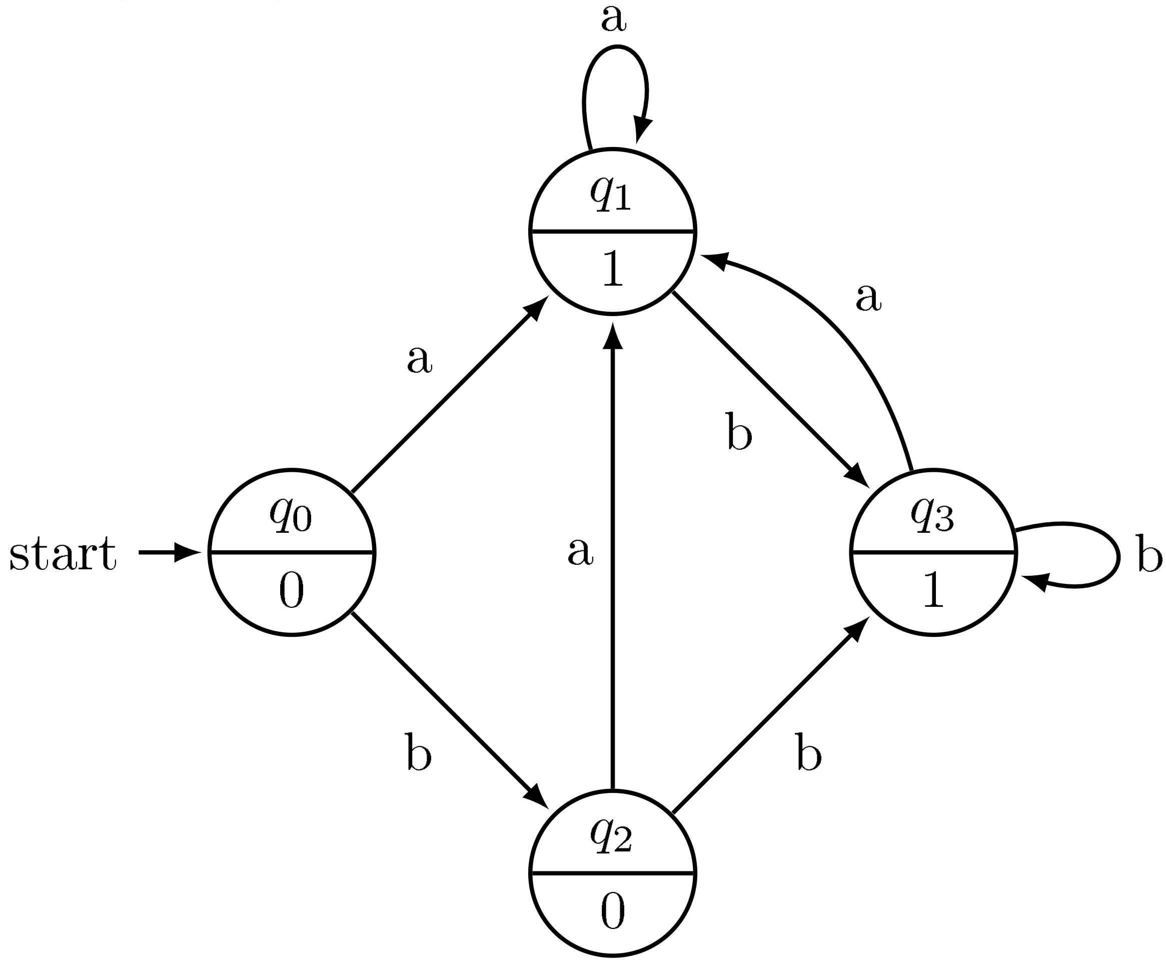
Problem: Construct a finite system S that satisfies φ by composing components that are *instances* of the component types in L.

Question: What are components? How do you compose them?

Components I: Transducers

Transducer: A simple model of a reactive system – a finite-state machine with inputs and outputs (*Moore machine*).

- Transducers are receptive.
- Output depends on state alone.



Dataflow Synthesis from Components

Setup:

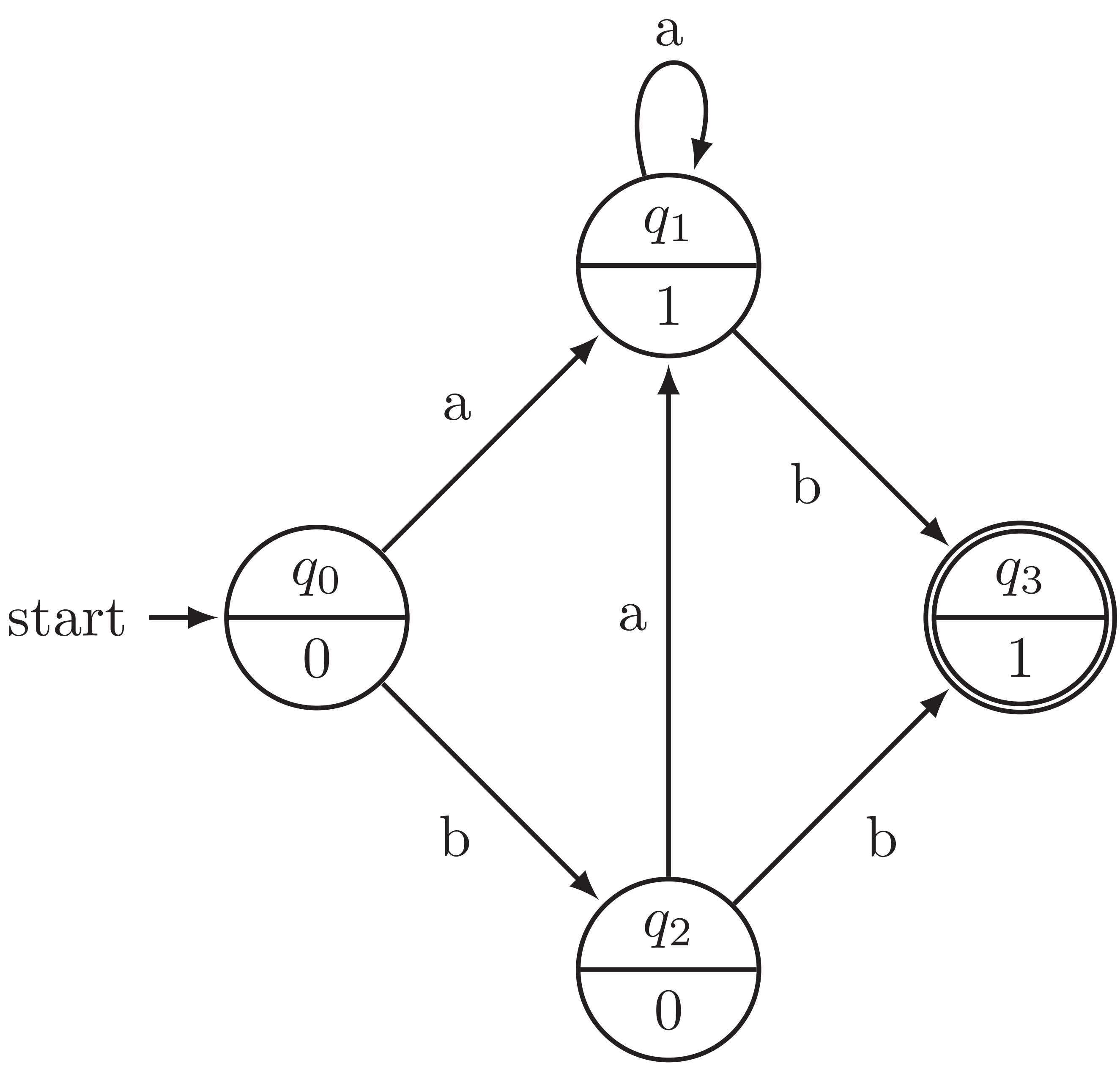
- *Components*: multi-input multi-output transducers e.g., hardware IP blocks
- *Dataflow composition*: connect input and output ports so outputs become inputs, e.g., connect sequential circuits

Theorem: [Lustig+V.,2009]
Dataflow synthesis from components is undecidable.

Crux:

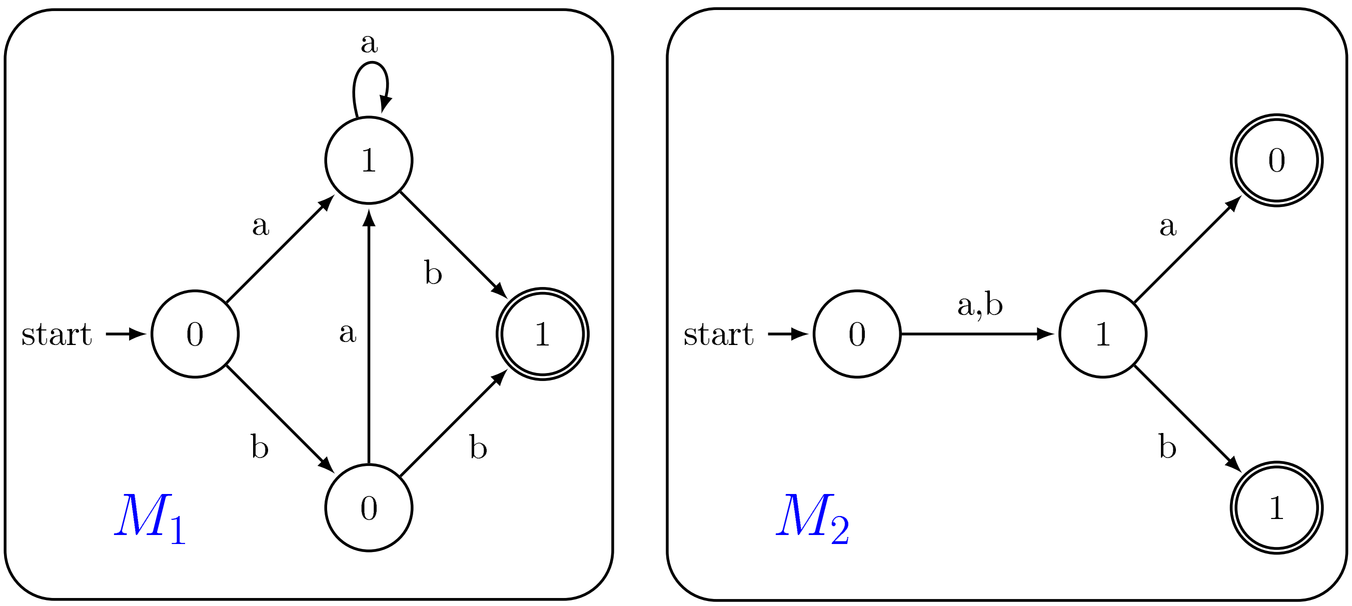
- Number of component instances not bounded, a priori.
- Cell of Turing-machine tape can be viewed as a component, connected to cells to its left and right.

Components II: Transducers with Exits



Control-flow Composition I

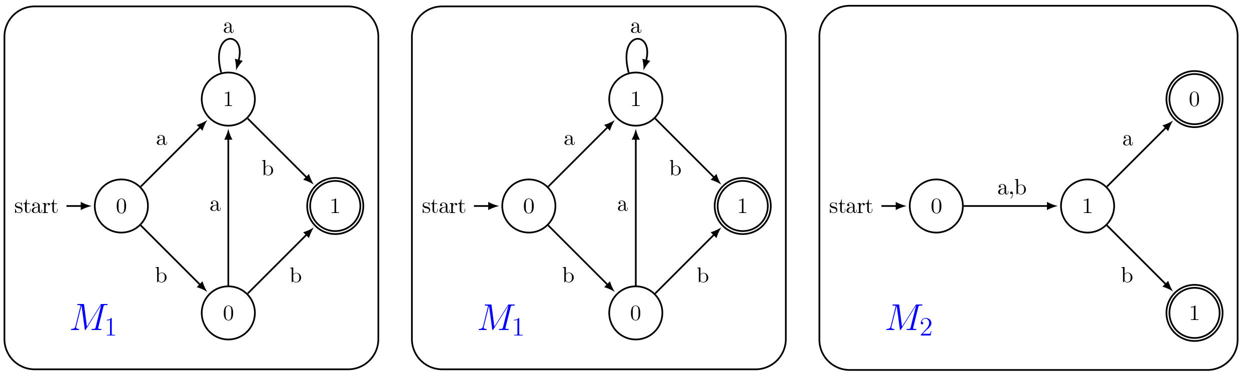
Motivation: Software-module composition – exactly one component interacts with environment at one time; on reaching an exit state, *goto* start state of another component.



A library of two components: $L = \{M_1, M_2\}$

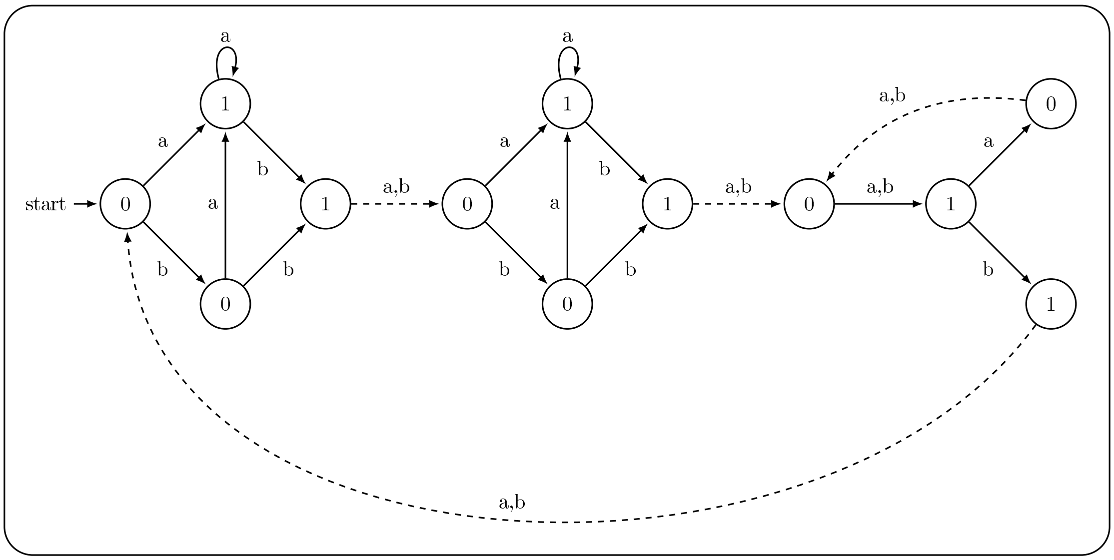
Control-flow Composition II

Pick three component instances from L:



Control-flow Composition III

Connect each exit to some start state – resulting composition is a transducer and is receptive.



Controlflow Synthesis

Setup:

- *Components*: single-input single-output transducers with *exit states*, e.g., software module
- *Controlflow composition*: upon arrival at an exit state, *goto* start state of another component – composer chooses target of branch.
- *No* a priori bound on number of component instances!

Theorem: [Lustig+V.,2009]
Controlflow synthesis from components is 2EXPTIME-complete.

Crux:

- Consider general (possible infinite) composition trees, that is, unfoldings of compositions
- Use alternating automata to check that all possible computations wrt composition satisfy φ
- Show that if general composition exists then finite composition exists.

Controlflow Synthesis from Recursive Components

Key Idea: Use *call* and *return*, instead of *goto*.

- An online store may call the PayPal web service, which receives control of the interaction with the user until it returns the control to the online store with approval/disapproval of payment.

Modeling:

- *calls*: component has a set of *call states*; when a *call state* is reached, another component is called.
- *returns*: component has a set of *return states*; when a *return state* is reached, control returns to the calling component.
- *re-entry*: component has a set of *re-entry states*; when control returns to a component, the component enters a *re-entry state*.
- *return value*: modeled by means of *re-entry states*.
- *call value*: not modeled explicitly here.

Recursive Components

Setup:

- *Components*: single-input single-output transducers with *call states*, *return states*, and *re-entry states*
- *Controlflow composition*: calls and returns

Related: *recursive state machines* of Alur at el.

- The result of composing recursive components is a recursive state machine.
- Equivalent to an infinite-state transducer.

Specifying Call-and-Return Computations

Need: In a call-and-return computation, specification may need to refer to call-and-return structure [Alur-Etessami-Madhusudan, 2004]

- E.g., “if the pre-condition p holds when a procedure A is *called*, then if A terminates, then the post-condition p is satisfied upon *return*.

Solution: Alur et al.

- *Nested Word*: Description of call-and-return computations – sequence of letters, plus *calls*, and matching *returns*, when exist
  - Traces of pushdown machines with *pushes* and *pops* made visible
- *Nested-Word Temporal Logic* (NWTL): logic refers to call-and-return structure
  - *next*: refers to next state
  - *next*$_\mu$: refers to return that matches a call

Now: Controlflow synthesis from recursive components wrt NWTL properties.

Automata-Theoretic Approach

Key Idea of Temporal Synthesis:

- Use tree automata to accept "good" strategy trees
- Use word automata to accept "good" tree branches

V.+Wolper, 1983: Exponential translation from LTL to Büchi automata

Needed Aere: automata-theoretic counterpart to NWTL

Answer: NWBA – Nested-Word Büchi Automata [Alur et al., 2008]

- *Standard* transition relation
- *Call* transition relation
- *Return* transition relation

Theorem: Exponential translation from NWTL to NWBA

Automata-Theoretic Approach to Controlflow Synthesis

Key Idea Lustig+V, 2009

- Composition tree is bad if it enables a computation that violates φ, i.e., accepted by NBW $A_{\neg\varphi}$.
- Construct NBT that searches for a bad computation by guessing a computation and simulating $A_{\neg\varphi}$.
- Complement NBT and test for nonemptiness.

Extending to Recursive Components:

- Computations go up and down the composition tree – use *2-way automata* to track them.
- Need to have an NBT simulate NWTL – NBT needs to track *cycles*, from call to return and back to call.

Bottom Line: Doable, but construction is rather messy. Complexity: 2EXPTIME-complete.

Question: Can construction be simplified?

- *Note*: using alternation and 2-wayness simplified earlier messy automata-theoretic constructions.

Controlflow Synthesis from Probabilistic Components

Goal: Build reliable systems from unreliable components.

- **Example**: How do you turn a fair coin into a comletely biased coin?

- What are probabilistic components?

- How are they connected together?

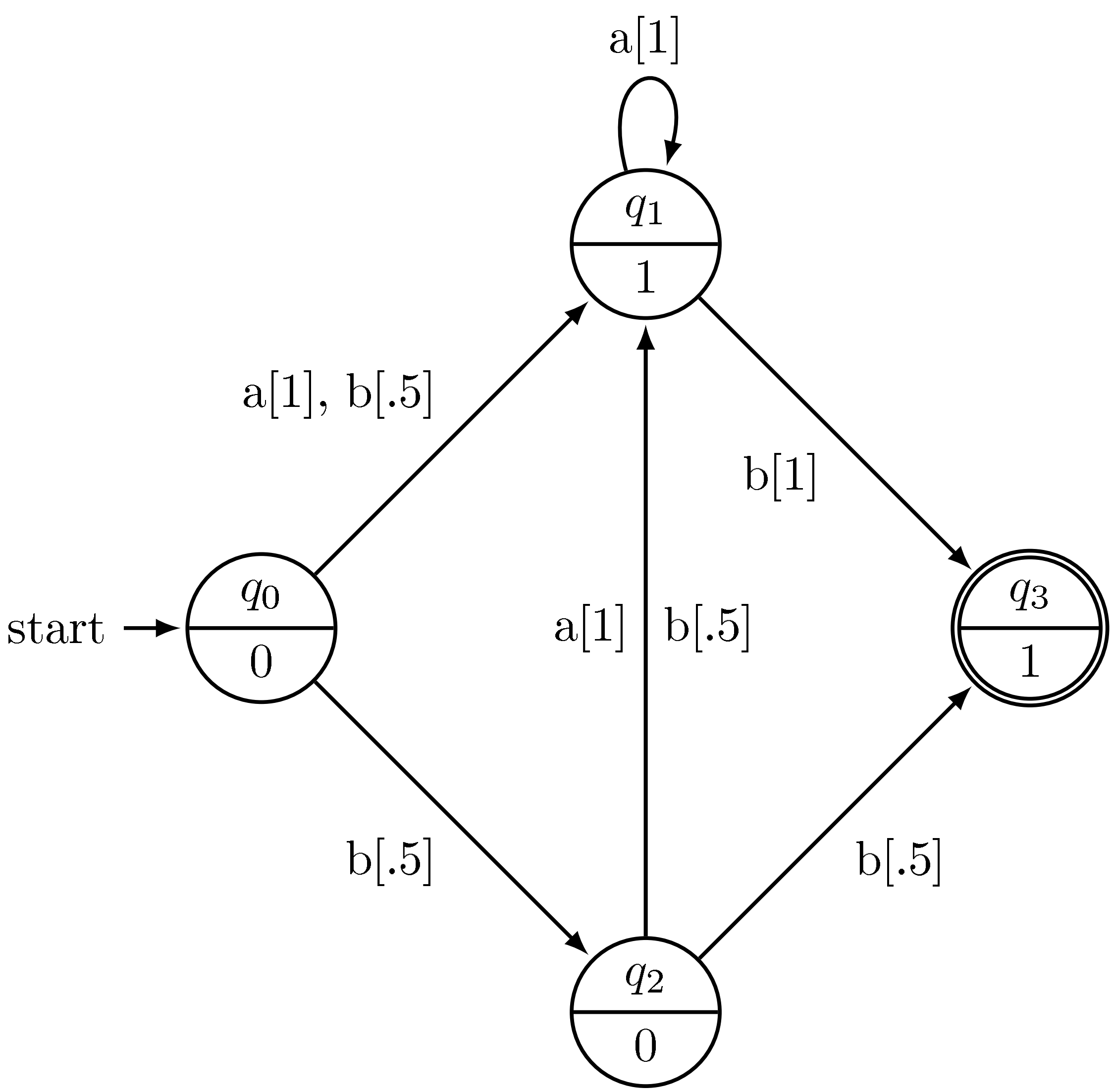
- What is the specification formalism?

- What is the appropriate notion of realizability?

Probabilistic Components

Examples: noisy sensors, probabilistic CMOS

Probabilistic Components: transducers with *exit states* and *probabilistic transition function*.



Probabilistic Components

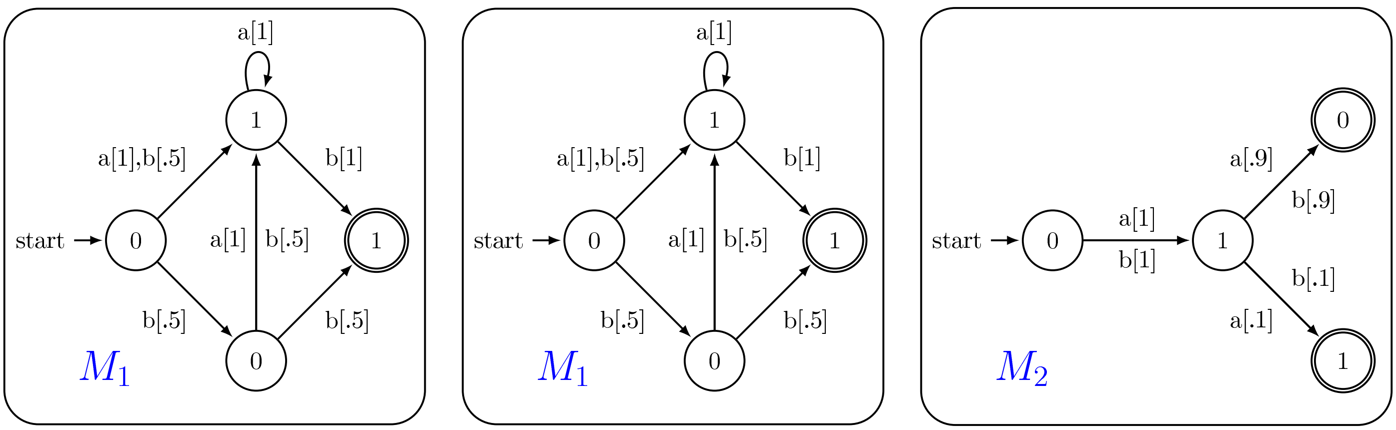
A probabilistic component is a *probabilistic transducer w. exits* – $(\Sigma_I, \Sigma_O, Q, q_0, \delta, F, L)$:

- Q — finite set of states
- q_0 — start state
- $F \subseteq Q$ — set of exit states
- Σ_I and Σ_O —- input and output alphabets
- $\delta : Q \times \Sigma_I \rightarrow Dist(Q)$ — Transition function that assigns a *prob. distribution* to state/input pairs
- $L : Q \rightarrow L$ — output function

Input: $w \in \{a, b\}^\omega$
Output: probability distribution on $\{0, 1\}^\omega$

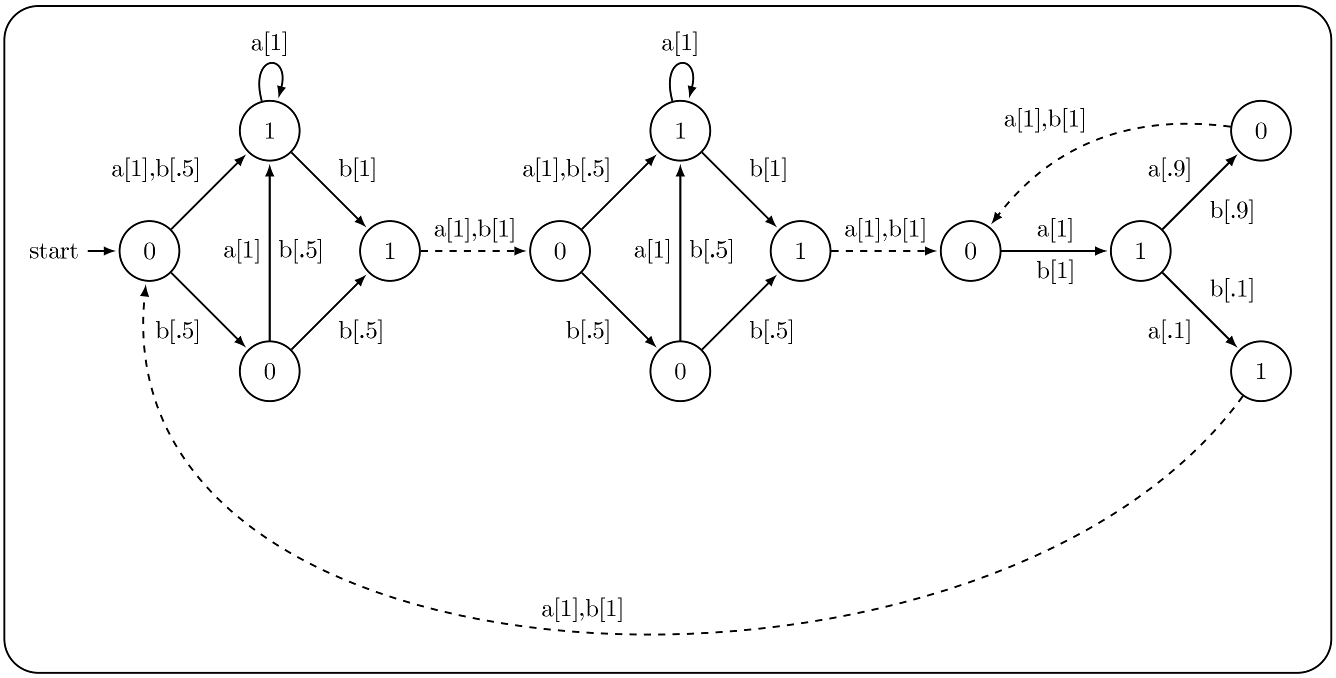
Control-flow Composition I

Pick three component instances from library $L = \{M_1, M_2\}$:



Controlflow Composition

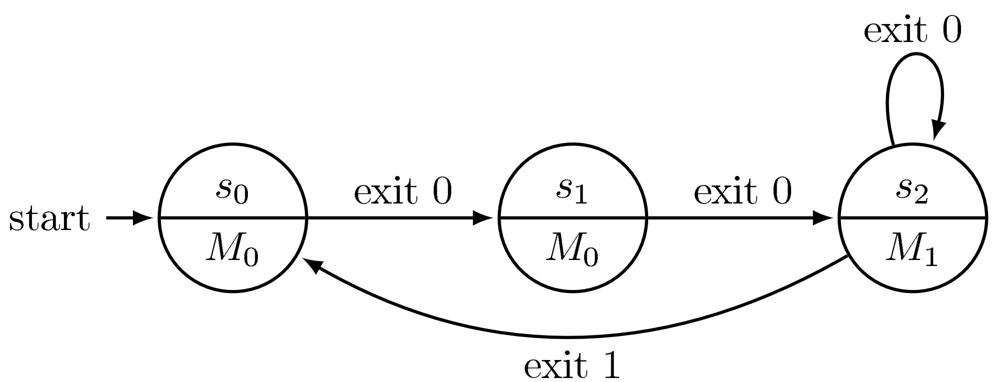
Connect each exit to some start state – resulting composition is a probabilistic transducer.



Modeling Controlflow Composition

Crux: (Current component, Exit state) $\longrightarrow$ Next state

> **Composer**: A *deterministic* transducer that captures controlflow in a composition.



- Composer — describes how to connect components
- Composition — resulting probabilistic transducer

DPW Specification

DPW — Deterministic Parity Word Automaton A

- Each state of A has a *priority* (a natural number).
- A *accepts* an infinite word if the corresponding run of A satisfies the *parity condition*.
- *Parity condition*: the lowest priority that occurs infinitely often is even.

DPW can express all ω-regular specifications.

LTL can be translated to DPW.

Probabilistic Correctness

Key Idea: System must satisfy DPW specification in face of *every* possible input.

- With prob 1, the run of the system is accepted by DPW.
- Probability defined by input: so which input?
- We assume adversarial environment: for every possible input, with prob 1, DPW must accept.

Probabilistic Realizability

Environment Strategy: The environment probabilistically chooses the next input depending upon history of the system.

- Environment: a function $f : Q^* \rightarrow Dist(\Sigma_I)$
- Each strategy f induces a probability distribution μ_f on the set of runs of M.
- Environment *wins* if run of M is rejected by A with probability > 0.

Realizability: System M *realizes* spec A iff the environment has no winning strategy against M.

Controlflow Synthesis from Probabilistic Components

- What are probabilistic components? *Probabilistic Transducers w. Exits*
- How are they connected together? *Deterministic Controlflow*
- What is the specification formalism? *DPW*
- What is the appropriate notion of realizability? *Probabilistic*
- What is the object being synthesized? Composer

Note: Components are now probabilistic, but controlflow is still deterministic.

DPW Synthesis problem: Given library L and DPW A, find composer C over L such that the composition defined by C realizes A.

Theorem: [Lustig-Nain-V., 2011] DPW synthesis from probabilistic components is *decidable*

Embedded-Parity Synthesis:
Simplifying DPW Synthesis

Key Idea: Instead of using a DPW as spec, assign priorities directly to each state of each component in the library and use the parity condition.

DPW Synthesis	Embedded-Parity Synthesis
Specification given as DPW	Specification embedded as priorities of component states
Environment wins if output rejected by DPW with prob. > 0	Environment wins if output satisfies parity condition with prob < 1
Natural problem	Artificial problem

Embedded-Parity Synthesis

Theorem: [Lustig-Nain-V., 2011] embedded-parity synthesis from probabilistic components is decidable in EXPTIME.

Proof Idea:

- Composer is finite, so composition is finite.
- Suffices to focus on pure, memoryless environment strategies.
- Finite probabilistic transduer + pure, memoryless environment strategiy = Markov chain.
- Apply ergodic analysis: with prob 1, limit behavior in ergodic set.
- Unfold chain into tree, translating ergodicity onto tree.
- Construct Büchi tree automaton for bad composition trees.
- Complement automaton and check nonemptiness.

DPW Synthesis

Theorem: [Lustig-Nain-V., 2011] DPW synthesis from probabilistic components is decidable in 2EXPTIME.

- **Proof Idea**: Take product of components in library L with DPW A and reduce to embedded-parity synthesis.
- **Difficulty**: Transitions of composers must depend only on components, cannot depend on states of A.
- **Solution**: Use techniques from synthesis with incomplete information, pay another exponential in complexity.
- **Note**: Upper bound in 4EXPTIME for LTL spec.

Controlflow Composition

Questions:

- If components are probabilistic why not allow probabilistic controlflow?
- Is probabilistic controlflow more powerful than deterministic controlflow?

Theorem: [Nain&V., 2012] Probabilistic and deterministic composers have the same expressive power for embedded-parity specifications.

Theorem: [Nain&V., 2012] Probabilistic composers are more expressive than deterministic composers for DPW specifications.

Similar to memory vs randomness tradeoff in games [[Chatterjee-De Alfaro-Henzinger, 2004].

Synthesizing Probabilistic Composers

Main difficulties:

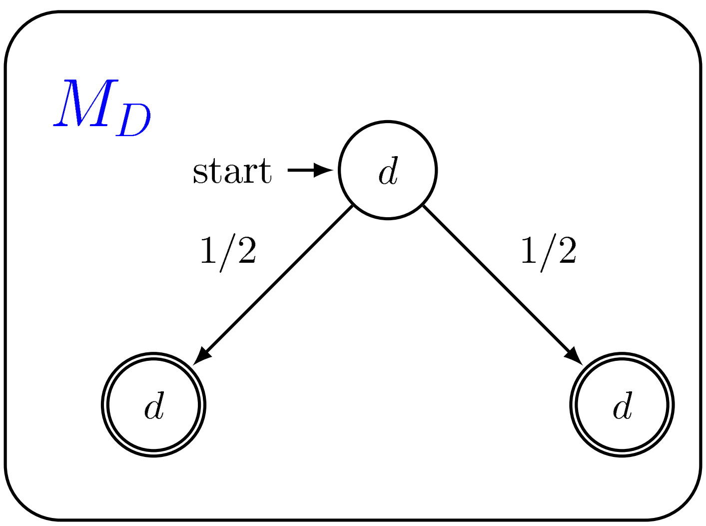
- **Expressiveness Barrier**:
  - For deterministic composers, DPW synthesis is solved via embedded parity.
  - Expressiveness result rules this out for probabilistic composers.
- **Unbounded Branching of Tree Representation**:
  - For deterministic composers:
    - branching of transition function is bounded.
    - depends on number of exits, fixed for given library.
    - So automata-theoretic techniques can be used.
  - For probabilistic composers:
    - branching of transition function is potentially unbounded.
    - depends on size of composition.

Synthesizing Probabilitis Composers

Theorem: [Nain&V., 2012] Controlflow synthesis of probabilistic composers from probabilistic components is decidable.

Proof Idea: Simulate probabilistic controlflow via deterministic controlflow.

- Add to library a component M_D whose sole purpose is to express probabilitic branching.
- Modify spec to ignore M_D.



In Conclusion

Framework: Compositional Synthesis = Synthesis from Component Libraries:

- What types of components?
- How are components composed?
- How are requirements specified?

Future Work

- Connection to games with incomplete information
- Tighter bounds
- Better algorithms