PCC 2012

22nd Postgraduate Combinatorial Conference

15th - 17th August 2012

University of Warwick

Welcome

Welcome to the Postgraduate Combinatorial Conference 2012 at the University of Warwick! The PCC has always been the best place for postgraduates in combinatorics to get to know one another. That's why it's great to see so many of you here representing universities from around the world. We'd also like to welcome our four excellent invited speakers (in order of appearance): Professor Andrew Thomason, Dr Stefanie Gerke and Dr Konrad Swanepoel.

General Information

Registration

Registration will be in the Mathematics Institute (Zeeman) building from about 10.00 on the first day of the conference. Check-in for those with accommodation is at Rootes Reception between 15.00 and 23.00 on your day of arrival. Rooms must be vacated by 09.30 on the day of departure. Luggage can be stored at Rootes Reception.

Internet Access

Internet access is available in your room. Remember to bring a network cable if you want to use this. Alternatively, you can buy one from Rootes Reception. If you have an eduroam account, you can use that while in the maths department.

Meals

1. Breakfast (for those with accommodation) is served in Rootes Restaurant (apart from those staying in Radcliffe, who will get breakfast in Radcliffe Restaurant)
2. Lunches will be in the Undergraduate Common Room in the Mathematics Institute (Zeeman Building)
3. Dinner on Wednesday is in Radcliffe House from 18.00.
4. The barbecue on Thursday is in the Staff Common Room (Room 103 on the first floor) in the Department of Computer Science (next door to the maths department). Food will be served from 18.00, with drinks from 17.30.

Local Organisers

Konrad Dąbrowski & Chris Purcell
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Acknowledgements

We wish thank the London Mathematical Society, the British Combinatorics Committee and the University of Warwick for helping to reduce the costs of the conference. Also, we'd like to thank the admin staff at Warwick for their help. Particular thanks go to Yvonne Carty, without whose tireless help the conference would not have happened.

Wednesday 15th August 2012

10.00: Registration begins in the Mathematics Institute (Zeeman) building. Tea and coffee will be available.

Invited Talk. Room MS.01

11.00: Andrew Thomason (Cambridge)
Hypergraph Containers

12.00: Break.

First Session. Room MS.01 (Chair: Robert Schumacher)

12.10: Aistis Atminas (Warwick)
Well-quasi-ordering of Subquadratic Classes

12.35: Pierre Aboulker (LIAFA)
2-Clock-free Graphs

13.00: **Lunch in Undergraduate Common Room (photo at 13:50)**

Second Session. Room MS.01 (Chair: Adam Bohn)

14.00: Liam Harris (Glamorgan)
Theocracy on the Chessboard; Bishop Independence and Domination

14.25: Darryl Green (RHUL)
Merging Secret Sharing Schemes

14.50: Andrei-Alin Popescu (UEA)
Lassoing Rooted Phylogenetic Trees

15.15: Tea and Coffee

Third Session. Room MS.01 (Chair: Andrew Collins)

15.45: Robert Powell (Durham)
Skew Bisubmodularity and Valued CSPs

16.10: Robert Crowston (RHUL)
Parameterized Study of the Test Cover Problem

16.35: Anne Hillebrand (Oxford)
Reverse Radar and the K_1, K_2-model

17.00: Talks end

18.00: **Dinner in Radcliffe House**

Thursday 16th August 2012

Invited Talk. Room MS.01

09.30: Stefanie Gerke (RHUL)
Random Graph Processes

10.30: Tea and Coffee

First Session. Room MS.01 (Chair: Aistis Atminas)

11.00: Andrew McDowell (RHUL)
Non-vertex Balanced Factors in Random Graphs

11.25: Fiona Skerman (Oxford)
Row and Column Sums of Random 0-1 Matrices

11.50: Break.

12.00: Michał Przykucki (Cambridge)
Maximum Percolation Time in Bootstrap Percolation

12.25: Annika Heckel (Oxford)
Anomalous Behaviour of Biased Random Walks in Random Environments

12.50: **Lunch in Undergraduate Common Room**

Second Session. Room MS.01 (Chair: Neville Ball)

13.50: Kerstin Weller (Oxford)
Coloured Forests - A Partially Bridge-addable Class

14.15: Basile Morcrette (INRIA and LIP6)
Exactly Solvable Pólya Urn Models via Analytic Combinatorics

14.40: Matthew Wells (Essex)
On Sets with more Restricted Sums than Differences

15.05: Tea and Coffee

Third Session. Room MS.01 (Chair: Anne Hillebrand)

15.35: Fiachra Knox (Birmingham)
Embedding Spanning Bipartite Graphs of Small Bandwidth

16.00: Katherine Staden (Birmingham)
Approximate Hamilton Decompositions of Robustly Expanding Digraphs

16.25: John Lapinskas (Birmingham)
Optimal Packings of Hamilton Cycles in Graphs of High Minimum Degree

16.50: Talks end.

18.00: **Barbecue in Department of Computer Science**

Friday 17th August 2012

Invited Talk. Room MS.01

09.30: Konrad Swanepoel (LSE)
Unit Vectors with Small Subset Sums in Normed Spaces

10.30: Tea and Coffee

First Session. Room MS.01 (Chair: Fiona Skerman)

11.00: David Bevan (The Open University)
Skinny Permutation Grid Classes

11.25: Stuart Hannah (Strathclyde)
Quicksort and Permutation Patterns

11.50: Christopher Harden (Essex)
Fixed-point Polynomials

12.15: Break.

12.25: Adam Bohn (QMUL)
Imaginary Evaluations of the Chromatic Polynomial

12.50: Robert Schumacher (City University)
Acyclic Orientations of Graphs

13.15: **Lunch in Undergraduate Common Room**

Wednesday 15th August 2012
Invited Talk:

Hypergraph Containers

Andrew Thomason

University of Cambridge

<email redacted for online version>

(joint work with David Saxton)

We develop a notion of containment for independent sets in hypergraphs. For every r-uniform hypergraph G, we find a relatively small collection C of vertex subsets, such that every independent set of G is contained within a member of C, and no member of C is large; the collection, which is in various respects optimal, reveals an underlying structure to the independent sets. The containers offer a straightforward and unified approach to many combinatorial questions concerned (usually implicitly) with independence.

We hope to outline the construction of the containers and some of the applications. For example, with regard to colouring, many (including all simple) r-uniform hypergraphs of average degree d have list chromatic number at least $(1/(r-1)^2 + o(1)) \log_r d$. For $r = 2$ this improves Alon's bound and is tight.

In the context of extremal graph theory, for each ℓ-uniform hypergraph H of order k, there is a collection C of ℓ-uniform hypergraphs of order n each with $o(n^k)$ copies of H, such that every H-free ℓ-uniform hypergraph of order n is a subgraph of a hypergraph in C, and $\log |C| \le cn^{\ell-1/m(H)} \log n$ where $m(H)$ is a standard parameter (there is a similar statement for induced subgraphs). This yields simple proofs of many hitherto difficult results: these include the number of H-free hypergraphs, sparsity theorems of Conlon-Gowers and Schacht, and the full KŁR conjecture.

Likewise, for systems of linear equations the containers supply, for example, bounds on the number of solution-free sets (including Sidon sets) and the existence of solutions in sparse random subsets.

Keywords: hypergraph, independent set, list colouring.

Contributed Talks:

Well-quasi-ordering of Subquadratic Classes

Aistis Atminas

DIMAP and Mathematics Insitute, University of Warwick

<email redacted for online version>

(joint work with V. Lozin, I. Razgon)

A hereditary graph property (or a class of graphs) X is subquadratic if there is a subquadratic function $f(n) = o(n^2)$ bounding the number of edges in all n-vertex graphs in X. We show that the presence of a large path in these classes implies the presence of a large induced path. We use this result to prove that any subquadratic property which is well-quasi-ordered by the induced subgraph relation is of bounded tree-width. This verifies (and even strengthens) for subquadratic properties a recent conjecture by Thomassé et al. stating that every hereditary class which is well-quasi-ordered by the induced subgraph relation is of bounded clique-width.

Keywords: well-quasi-ordering, tree-width, clique-width.

2-Clock-free Graphs

Pierre Aboulker

LIAFA

<email redacted for online version>

(joint work with Zhentao Li, Stéphane Thomassé)

A 2-clock is a graph formed by a chordless cycle C and a vertex not in C that is adjacent to exactly two vertices of C. We say that a graph is *2-clock-free* if it does not contain a 2-clock as an induced subgraph. We prove that the class of 2-clock-free graphs is χ-bounded. We also prove that, if a graph G is 2-clock-free and has girth greater than 9, then it contains a vertex of degree 2.

Keywords: χ-boundedness, induced subgraph, degeneracy.

Theocracy on the Chessboard; Bishop Independence and Domination

Liam Harris

University of Glamorgan

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(joint work with S. Perkins, S. K. Jones, P. A. Roach)

In chess, Bishop Independence concerns determining the maximum number of Bishops that can be placed on a board such that no Bishop can attack any other Bishop in a single move. Bishop Domination concerns the minimum number of Bishops that can be placed on a board such that every square on the board can be reached by at least one Bishop in a single move. This presentation explores the solutions to the Bishop Independence problem for all sizes of boards on the following topologies: the rectangle, the cylinder, the Möbius strip, the torus and the Klein bottle. Then the presentation will address the related problem of Bishop Domination and the use of both problems in combinatorics.

Keywords: chess, bishop, domination, independence.

Merging Secret Sharing Schemes

Darryl Green

Royal Holloway, University of London

<email redacted for online version>

In this talk I will present the problem of merging two individual secret sharing schemes into a single new scheme. I will give a brief introduction to the general problem of secret sharing, followed by an overview of the merging problem. Finally, I will present a protocol designed to merge two threshold secret sharing schemes into a new scheme which distinguishes between members of the original groups.

Keywords: secret sharing.

Lassoing Rooted Phylogenetic Trees

Andrei-Alin Popescu

School of Computing, University of East Anglia

<email redacted for online version>

(joint work with Dr. Katharina T. Huber)

The construction of a dendrogram is a crucial step in many genome wide association studies which themselves are an important part of Computational Biology. Such structures are usually constructed from distance information on individuals using e.g. molecular sequence information. However even with modern sequencing techniques not all pairwise distances between individuals may be reliable. Thus it becomes important to reconstruct and understand such structures from partial distance information.

From a formal point of view, a dendrogram is a rooted edge-weighted tree with no degree two interior vertices, except perhaps the root, and leaf set a pre-given set X such that the induced distance between the root and every leaf is the same. Such tree are sometimes called equidistant phylogenetic trees on X. Viewing partial distance information on X as a set of cords, that is, subsets of X of size two, the question becomes when a set of cords lassos, that is, uniquely determines such a tree.

In this talk we first give a brief introduction into phylogenetic tree reconstruction and then present recent results concerning the above question.

Keywords: phylogenetics, incomplete distances, lasso.

Skew Bisubmodularity and Valued CSPs

Robert Powell

Durham University

<email redacted for online version>

(joint work with Anna Huber and Andrei Krokhin)

An instance of the finite Valued Constraint Satisfaction Problem (VCSP) is given by a finite set of variables, a finite domain of values, and a set of finite valued functions, where each function depends on a subset of the variables. The goal is to find an assignment of values to the variables that minimises the total sum of the functions.

We study (assuming that **PTIME** $\neq$ **NP**) how the complexity of this very general problem depends on the functions allowed in the instances. The case when the variables can take only two values was classified by Cohen et al., with submodular functions giving rise to the only tractable case. Any non-submodular function can be used to express, in a certain specific sense, the NP-hard Max Cut problem.

We investigate the case when the variables can take three values. We identify a new infinite family of conditions, that includes bisubmodularity as a special case, which can collectively be called skew bisubmodularity. By a recent result of Thapper and Živný, this condition implies that the corresponding VCSP can be solved by linear programming. We prove that submodularity with respect to a total order and skew bisubmodularity give rise to the only tractable cases, and, in all other cases, Max Cut can be expressed. We also show that our characterisation of tractable cases is tight, that is, none of the conditions can be omitted. Thus, our results provide a new dichotomy theorem in constraint satisfaction research, and lead to a whole series of intriguing open problems in submodularity research.

Keywords: valued constraint satisfaction, submodularity, bisubmodularity, tractability, fractional polymorphisms.

Parameterized Study of the Test Cover Problem

Robert Crowston

Royal Holloway, University of London

<email redacted for online version>

(joint work with Gregory Gutin, Mark Jones, Gabriele Muciaccia, Saket Saurabh and Anders Yeo)

In the Test Cover problem, we are given a set V of vertices, and a collection $\mathcal{E} = \{E_1, ..., E_m\}$ of distinct subsets of the vertices, called tests. We assume that $\mathcal{E}$ is a test cover, i.e., for each pair of vertices there is a test in $\mathcal{E}$ containing exactly one of these vertices.

The objective is to find a minimum size subcollection of $\mathcal{E}$, which is still a test cover.

In parameterized versions of Test Cover, we are given $(V, \mathcal{E})$, and a positive integer k. The objective is to decide whether there is a Test Cover of size at most $p(k, |V|, |\mathcal{E}|)$, where $p(k, |V|, |\mathcal{E}|)$ is the parameter. We aim to decide whether the problem is fixed-parameter tractable, that is, can be solved by algorithms of runtime $f(k) \cdot poly(|V|, |\mathcal{E}|)$ where $f(k)$ is a function of k only.

In this talk, I will look at the complexity status of this problem for different natural choices of $p(k, |V|, |\mathcal{E}|)$: k, $\log |V| + k$, $|V| - k$ and $|\mathcal{E}| - k$.

Keywords: test cover, parameterized complexity.

Reverse Radar and the K_1, K_2-model

Anne Hillebrand

University of Oxford

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(joint work with Colin McDiarmid and Alex Scott)

In the reverse radar problem one tries to determine what platforms (ships) are out there, by looking at the radar pulses they emit. Thales has developed software to map pulses to radar types. Given a library of ships, each associated with a combination of radar types, and a multiset of radar types, each with a geographical location, is it possible to find out which ships are where?

The K_1, K_2-model is a toy model, developed to explore some of the combinatorial properties of the reverse radar problem. Given a cycle C_n, let every node be a bearing along a viewing circle. Let each $v \in V(C_n)$ have a multiset of colours $\mathrm{col}(v)$ associated with it, such that every colour represents a radar type. Let I be a set of small coloured graphs, where each graph represents a ship. Can we find a family of coloured subgraphs $H_1, H_2, \ldots \in I$ such that for each $v \in V(C_n)$

$$\biguplus_{i:v \in V(H_i)} \mathrm{col}_{H_i}(v) = \mathrm{col}_G(v)?$$

It will be shown that these embeddings of coloured subgraphs can be reduced to a matching in the separated graph C_s. Moreover, it will be shown that C_s has bounded treewidth and some algorithms to find matchings in graphs with bounded treewidth will be presented.

Keywords: reverse radar, treewidth, matchings, factors, K_1, K_2-model.

Thursday 16th August 2012
Invited Talk:

Random Graph Processes

Stefanie Gerke

Royal Holloway, University of London

<email redacted for online version>

The classical random graph process starts with the empty graph on n vertices and in each step adds an edge chosen uniformly at random from all non-edges. We consider variants of this process were we only add edges if certain additional requirements are satisfied. For example, we insist that the resulting graph is always triangle-free or planar. We will compare different processes and introduce methods how to investigate such processes. For example we show that with high probability at the end of the random planar process every fixed planar graph is a subgraph whereas in the triangle-process dense triangle-free subgraphs will not appear.

Contributed Talks:

Non-vertex Balanced Factors in Random Graphs

Andrew McDowell

Royal Holloway, University of London

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(joint work with Stefanie Gerke)

For a fixed graph H, an H-factor of a graph G is a set of vertex disjoint copies of H, the union of which cover all the vertices of G. In 2008, Johansson, Kahn and Vu proved exact thresholds for $G(n, p)$ to satisfy this property for strictly balanced H. We outline the key steps of a generalisation of their result that leads to an extension to non-vertex balanced H, and also to partitioned and directed forms of their result.

Keywords: factors, random graphs, partitions, digraphs.

Row and Column Sums of Random 0-1 Matrices

Fiona Skerman

Oxford University

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(joint work with Brendan McKay)

Construct a random $m \times n$ matrix by independently setting each entry to 1 with probability p and to 0 otherwise. We study the joint distribution of the row sums $\mathbf{s} = (s_1, \ldots, s_m)$ and column sums $\mathbf{t} = (t_1, \ldots, t_n)$. Clearly $\mathbf{s}$ and $\mathbf{t}$ have the same sum, but otherwise their dependencies are complicated. We prove that under certain conditions the distribution of $(\mathbf{s}, \mathbf{t})$ is accurately modelled by $(S_1, \ldots, S_m, T_1, \ldots, T_n)$, where each S_j has the binomial distribution $\mathrm{Binom}(n, p')$, each T_k has the binomial distribution $\mathrm{Binom}(m, p')$, p' is drawn from a truncated normal distribution, and $S_1, \ldots, S_m, T_1, \ldots, T_n$ are independent apart from satisfying $\sum_{j=1}^{m} S j = \sum_{k=1}^{n} T_k$. We also consider the case of random 0-1 matrices where only the number of 1s is specified, and also the distribution of $\mathbf{s}$ when $\mathbf{t}$ is specified. In the seminar I will include details of some bounding arguments used in the proof.

These results can also be expressed in the language of random bipartite graphs, group allocation by complexes and coupon collection.

Keywords: random graphs, random binary matrices, degree sequences, group allocation by complexes, coupon collection.

Maximum Percolation Time in Bootstrap Percolation

Michał Przykucki

University of Cambridge

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(joint work with Fabricio Benevides)

Bootstrap percolation is one of the simplest cellular automata. In r-neighbour bootstrap percolation on a graph G an infection spreads according to the following deterministic rule: infected vertices of G remain infected forever and in consecutive rounds healthy vertices with at least r already infected neighbours become infected. Percolation occurs if eventually every vertex is infected.

In this talk we focus on extremal problems in 2-neighbour bootstrap percolation and present our recent results about the maximum time this process can take to percolate in the $n \times n$ square grid and in the n-dimensional hypercube graph.

Keywords: cellular automata, bootstrap percolation, maximum time.

Anomalous Behaviour of Biased Random Walks in Random Environments

Annika Heckel

University of Oxford

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Imagine a small particle, driven for example by Brownian motion, moving randomly through an inhomogeneous medium with randomly open and blocked passageways. A common model for the inhomogeneous medium is a percolation cluster and the movements of the particle are modelled with a discrete time random walk. The particle also has a preferred direction or bias, for example through gravity or an external electrical field, that it is more likely to move in if it has the chance.

In this kind of scenario, the asymptotic speed of the random walk often shows an anomalous behaviour: If there is a small, but existing bias, there is a positive speed. However, if the bias is very large, the asymptotic speed is 0. This is due to traps in the inhomogeneous medium that the random walk spends a long time in if the bias is large.

In this talk, this behaviour will be investigated for finite percolation cluster traps attached to an infinite line. The asymptotic speed and critical values for the bias parameter will be discussed, and a generalisation to different types of traps to an infinite line will be given.

Keywords: Biased random walks, random walks in random environments, percolation.

Coloured Forests - a Partially Bridge-addable Class

Kerstin Weller

University of Oxford

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(joint work with M. Bousquet-Mélou and C. McDiarmid)

A class $\mathcal{A}$ of graphs is called bridge-addable if for all graphs $G \in \mathcal{A}$ and for each pair u and v of vertices in different components of G the graph $G \cup uv$ is also in $\mathcal{A}$. In 2005 McDiarmid et al. proved that for every non-empty brigde-addable class of graphs $\mathcal{A}$ the number of components of a 'typical' graph on n vertices, that is a graph taken uniformely at random from all graphs $\in \mathcal{A}$ on n vertices, is asymptotically stochastically dominated by a Poisson law. However, it is not known what happens if we relax the assumption and look at classes of graphs where many but not all of the possible bridges are allowed to be inserted. In this talk, I discuss one of the examples that we studied recently using tools from Analytic Combinatorics as well as from combinatorics.

Exactly Solvable Pólya Urn Models via Analytic Combinatorics

Basile Morcrette

INRIA Paris-Rocquencourt
LIP6, University of Paris 6

<email redacted for online version>

(joint work with Hosam M. Mahmoud)

In this talk, we will show how the Analytic Combinatorics developed by Flajolet applies to a simple probabilistic system: Pólya urns. A Pólya urn is a very simple dynamical system, where a box with balls of two colors evolves step by step with regards to some deterministic rules. Since 2006, the study of those models has been possible through generating functions and complex analysis. A powerful link exists between the rules of evolution of the urn and some differential system satisfied by the generating function. This can be extended to more general urn models where the evolution is dictated by random variables. When the differential system is solvable, it provides directly, in an automatic way,an exact formula for the distribution of the balls.

Keywords: Pólya urn, analytic combinatorics, random structure, combinatorial probability, differential equations.

On Sets with more Restricted Sums than Differences

Matthew Wells

The University of Essex

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(joint work with Dr. David Penman)

Given a set A of integers, we define its restricted sumset $A\hat{+}A$ to be the set of sums of two *distinct* elements of A and its difference set $A - A$ to be the set of differences of two elements of A. We say A is an MRSTD set if $|A\hat{+}A| > |A - A|$. Though intuition suggests that such sets should be rare, we present various constructions of such sets, prove that a positive proportion of subsets of $\{0, 1, \ldots n - 1\}$ are MRSTD sets, and some related results and commentary.

Keywords: restricted sumset, sumset, difference set, more restricted sums than differences set.

Embedding Spanning Bipartite Graphs of Small Bandwidth

Fiachra Knox

University of Birmingham

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(joint work with Andrew Treglown)

A graph H on n vertices is said to have *bandwidth at most b*, if there exists a labelling of the vertices of H by the numbers $1, \dots, n$ such that for every edge $ij \in E(H)$ we have $|i-j| \leq b$. A theorem of Böttcher, Schacht and Taraz [1] gives a condition on the minimum degree of a graph G on n vertices that ensures G contains *every* r-chromatic graph on n vertices of bounded degree and of bandwidth $o(n)$, thereby proving a conjecture of Bollobás and Komlós [2].

In this talk I will discuss a strengthening of this theorem in the case when H is a bipartite graph. A key definition is the notion of 'robust expansion'. Roughly speaking, a graph G on n vertices is a robust expander if, for every 'reasonably sized' set $S \subseteq V(G)$, G contains at least $|S| + \Omega(n)$ vertices that are adjacent to 'many' vertices in S.

The main result I will present in this talk is the following: Given $\Delta \in \mathbb{N}$ and a positive constant $\eta > 0$, there exist constants $\beta > 0$ and $n_0 \in \mathbb{N}$ such that the following holds. Suppose that H is a bipartite graph on $n \geq n_0$ vertices with $\Delta(H) \leq \Delta$ and bandwidth at most βn. Let G be a graph on n vertices with $\delta(G) \geq \eta n$ which is a robust expander. Then G contains a copy of H.

In fact, there are examples of graphs G that satisfy the hypothesis of this theorem and whose *maximum degree* is small. As corollaries we obtain a similar result for graphs G which satisfy a degree sequence condition and an Ore-type result. All of these results are best possible for many graphs H.

References

[1] J. Böttcher, M. Schacht and A. Taraz, Proof of the bandwidth conjecture of Bollobás and Komlós, *Math. Ann.* **343** (2009), 175–205.

[2] J. Komlós, The Blow-up Lemma, *Combin. Probab. Comput.* **8** (1999), 161–176.

Keywords: embedding, bandwidth, robust expansion, degree sequence, Ore.

Approximate Hamilton Decompositions of Robustly Expanding Digraphs

Katherine Staden

University of Birmingham

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(joint work with Deryk Osthus)

We show that every sufficiently large r-regular digraph G which has linear degree and is a robust outexpander has an approximate decomposition into edge-disjoint Hamilton cycles, i.e. G contains a set of $r - o(r)$ edge-disjoint Hamilton cycles. Here G is a robust outexpander if for every set S which is not too small and not too large, the 'robust' outneighbourhood of S is a little larger than S. This generalises a result of Kühn, Osthus and Treglown on approximate Hamilton decompositions of dense regular oriented graphs. It also generalises a result of Frieze and Krivelevich on approximate Hamilton decompositions of quasirandom (di)graphs. In turn, our result is used as a tool by Kühn and Osthus to prove that any sufficiently large r-regular digraph G which has linear degree and is a robust outexpander even has a Hamilton decomposition.

Keywords: Hamilton cycles, digraphs, regularity lemma, robust expanders.

Optimal Packings of Hamilton Cycles in Graphs of High Minimum Degree

John Lapinskas

University of Birmingham

<email redacted for online version>

(joint work with Daniela Kühn and Deryk Osthus)

We study the number of edge-disjoint Hamilton cycles one can guarantee in a sufficiently large graph G on n vertices with minimum degree $\delta = (1/2 + \alpha)n$. For any constant $\alpha > 0$, we give an optimal answer in the following sense: let $\mathrm{reg}_{\mathrm{even}}(n, \delta)$ denote the degree of the largest even-regular spanning subgraph one can guarantee in a graph G on n vertices with minimum degree δ. Then the number of edge-disjoint Hamilton cycles we find equals $\mathrm{reg}_{\mathrm{even}}(n, \delta)/2$. The value of $\mathrm{reg}_{\mathrm{even}}(n, \delta)$ is known for infinitely many values of n and δ. We also extend our results to graphs G of minimum degree $\delta \geq n/2$, unless G is close to the extremal constructions for Dirac's theorem. Our proof relies on a recent and very general result of Kühn and Osthus on Hamilton decomposition of robustly expanding regular graphs.

Keywords: Hamilton cycles, extremal problems, covering and packing.

Friday 17th August 2012

Invited Talk:

Unit Vectors with Small Subset Sums in Normed Spaces

Konrad Swanepoel

London School of Economics and Political Science

<email redacted for online version>

We consider a circle of problems of which the following questions are representative: *How many unit vectors can be found in a d-dimensional normed space such that the sum of any k of the vectors have norm at most* 1*? How many if we assume in addition that the unit vectors sum to* $\mathbf{o}$*?* We present upper bounds valid for all normed spaces of dimension d. Some of these bounds are sharp, but many open questions remain. We hope to demonstrate some of the tools from graph colourings, linear algebra and convexity that we apply. The origin of this problem lies in the study of singularities (also called Steiner points) in geometric shortest networks in normed spaces.

Contributed Talks:

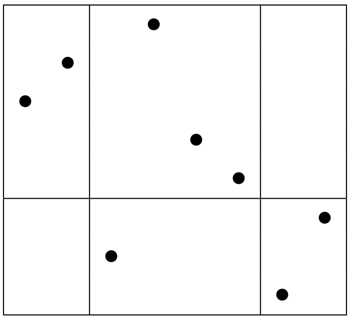
Skinny Permutation Grid Classes

David Bevan

The Open University

`<email redacted for online version>`

A (monotone) *grid class* of permutations Grid(M) is defined by a $0/\pm 1$ matrix M which specifies the shape that permutations in the class must have when plotted. Entries in the matrix correspond to rectangular cells in *griddings* of the permutations in the class. If an entry in the matrix is 1 or -1, any points in the corresponding cell must form an increasing or decreasing sequence respectively; if an entry is 0, the corresponding cell must be empty.



A gridding of permutation 67285413 in $\mathrm{Grid}\begin{pmatrix}1 & -1 & 0\\ 0 & -1 & 1\end{pmatrix}$

As an initial step in attempting to enumerate permutation grid classes and understand something of their structural properties, we consider *skinny* ($k \times 1$) grid classes. Permutations in a particular skinny grid class consist of some specific sequence of (possibly empty) ascending and descending sequences of values. We present a method for determining the ordinary generating function for any specified skinny grid class, and discuss the issues that need to be addressed in extending the work to arbitrary grid classes.

Keywords: permutation, permutation class, generating function.

Quicksort and Permutation Patterns

Stuart Hannah

University Of Strathclyde

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An occurrence of a permutation pattern is a sub-permutation contained within a permutation with restrictions made on the relative order or position of its elements. The subject of permutation patterns was popularized by Donald Knuth who considered the classification of permutations that could be sorted in a single pass through a stack. In this talk we will present an analysis of quicksort with respect to underlying permutation patterns on the permutation sorted.

Keywords: quicksort, permutations, permutation patterns.

Fixed-point Polynomials

Christopher Harden

The University of Essex

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(joint work with Dr David B. Penman)

The fixed point polynomial of a finite permutation group G acting on a set Ω of order n is defined by $P_G(x) = \sum_{i=0}^{n} f_i x^i$ where f_i is the number of elements of G which fix exactly i elements of Ω. In this talk, we will calculate the fixed point polynomials for a variety of examples, note some general theorems about polynomials which often give partial information about where the roots of these polynomials are, and discuss some observations we have made about the whereabouts of roots of the polynomials for various families.

Keywords: group theory, permutation groups, fix, fixed points.

Imaginary Evaluations of the Chromatic Polynomial

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(joint work with Bill Jackson)

The *chromatic polynomial* $P_G(q)$ of a graph G counts the number of proper q-colourings of the vertices of G, and a zero of $P_G(q)$ is known as a *chromatic root* of G. Much is known about the distribution of chromatic roots in the real line and complex plane, however the question of which specific algebraic integers are chromatic roots of graphs is still very much open. In particular, it is not even known if there are any purely imaginary chromatic roots.

Bill Jackson has recently shown that no series-parallel graph has a purely imaginary chromatic root, and we believe that this result can be extended to all graphs. Moreover, given some graph G, there is some evidence that there may exist a pleasing combinatorial interpretation for the squared absolute value of $P_G(i)$. If this is true, then the correct interpretation is likely to be closely linked to Stanley's famous discovery that the number of acyclic orientations of G is given by the absolute value of $P_G(-1)$, and it might be possible to generalise it analogously to arbitrary integer multiples of i.

This is still very much work in progress, however I will briefly discuss our conjectures and the evidence we have obtained in support of them, including a simple proof of an interpretation for complete (and empty) graphs using results on Jacobi-Stirling numbers recently obtained by Gelineau and Zeng.

Keywords: chromatic polynomial, chromatic roots, acyclic orientations, roots of unity.

Acyclic Orientations of Graphs

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An acyclic orientation of a graph G is obtained by taking an undirected graph and assigning a direction to each edge of the graph without creating any directed cycles. This is always possible for any graph. I am interested in how many ways this is possible for any given graph. Let $a(G)$ be the number of acyclic orientations of G. I want to be able to efficiently approximate $a(G)$. It is known that $a(G)$ is the evaluation of a polynomial related to the chromatic polynomial of a graph at -1, but computing the chromatic polynomial is hard work (# P hard) in itself.

A new factor method of counting acyclic orientations of a graph is presented. In this method one looks at the contribution of each edge to the total number of acyclic orientations. In some cases local information is enough to determine the exact contribution. Calculation can thus be simplified, and it may also be possible to approximate the factors efficiently, which would give and efficient approximation of $a(G)$.

Keywords: directed graphs, acyclic orientations, approximation.

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Wednesday	Thursday	Friday
	Gerke	Swanepoel
Tea & Coffee	Tea & Coffee	Tea & Coffee
Thomason First Session (chair: Schumacher) Atminas Aboulker	First Session (chair: Atminas) McDowell Skerman	First Session (chair: Skerman) Bevan Hannah Harden
	Przykucki Heckel	Bohn Schumacher
Lunch	Lunch	Lunch
Second Session (chair: Bohn) Harris Green Popescu	Second Session (chair: Ball) Weller Morcrette Wells	
Tea & Coffee	Tea & Coffee	
Third Session (chair: Collins) Powell Crowston Hillebrand	Third Session (chair: Hillebrand) Knox Staden Lapinskas	
Dinner	Barbecue	