Online Capacity Maximization in Wireless Networks

Sascha Geulen

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Joint work with
Alexander Fanghänel, Martin Hoefer, and Berthold Vöcking

Problem

Situation

- Wireless network
- Request (s_i, r_i) from sender s_i to receiver r_i:

$$s_i \longrightarrow r_i$$

Objective: Accept as many requests as possible

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Competitive Analysis: Acceptance based on previous requests only

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(s_i, r_i)

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- Receiver can interpret the signal if SINR (signal-to-interference-plus-noise-ratio) constraint holds:

$$\frac{S}{N + I} \geq \beta$$

α	-	path-loss exponent
β	-	gain
N	-	ambient noise
I	-	interferences caused by other senders

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- Metric space: d-dimensional Euclidean space

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Power Assignments

(s_i, r_i)

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Power Assignments:

- Distance-based: $P = \phi(\text{distance}_{ii})$
- Polynomial: $P = \text{distance}_{ii}^{r\alpha}$ for a fix $r \in \mathbb{R}$
  - $r = 0$ (uniform)
  - $r = 1$ (linear)
  - $r = \frac{1}{2}$ (square root)

Greedy Algorithm

Algorithm 1 (GREEDY)

Accept if SINR constraint still holds at each accepted receiver

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$\longrightarrow$ Safe distance σ
$\longrightarrow$ Conflict between correctness and competitive ratio
$\longrightarrow$ Worst-case scenario

Safe Distance Algorithm

$P = \text{distance}_{ii}^{r\alpha}$

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Algorithm 2 (SAFE-DISTANCE)

Accept if $\min\{\text{distance}_{ij}, \text{distance}_{ji}\} \geq \sigma$ for each scheduled request

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SAFE-DISTANCE has competitive ratio

$$\begin{cases} O\left(\Delta^{d}\right), & \text{if } r \in [0,1] \\ O\left(\Delta^{d \cdot \max\{r, 1-r\}}\right), & \text{else} \end{cases}$$

Square-root power assignment

$P = \text{distance}_{ii}^{r\alpha}$

$\text{distance}_{ii} \in [1, \Delta]$

$r \in (0, 1)$
$\longrightarrow$ Short requests can be scheduled in longer ones
$\longrightarrow$ Use m length classes

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Theorem

MULTI-CLASS SAFE-DISTANCE has competitive ratio

$$O\left(\Delta^{\frac{d}{2}+\epsilon}\right)$$

for any constant $\epsilon > 0$

Lower bound

$P = \text{distance}_{ii}^{r\alpha}$

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Theorem

Every deterministic online algorithm has at least competitive ratio

$$\Omega\left(\Delta^{d \cdot \max\{r, 1-r\}}\right)$$

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- It blocks some area around sender and receiver
- Offline Algorithm can place requests into these blocked areas

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Corollary

MULTI-CLASS SAFE-DISTANCE with $\log\Delta$-channels has competitive ratio

$$O(\log\Delta)$$

Randomization

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Simulate k-channels by one channel using randomization

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RANDOM SAFE-DISTANCE has competitive ratio

$$O(\log \Delta)$$

Conclusion

- GREEDY is not competitive
- Usage of safe distance leads to local effect
- MULTI-CLASS SAFE-DISTANCE approaches lower bound
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Thank you for your attention

Safe distance computation

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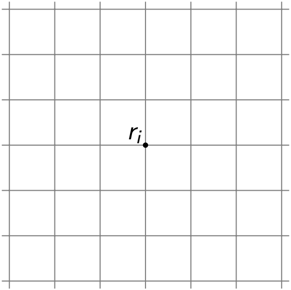

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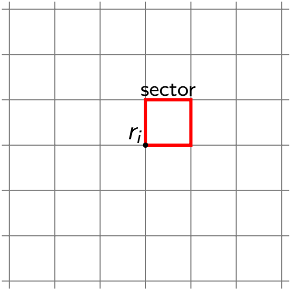

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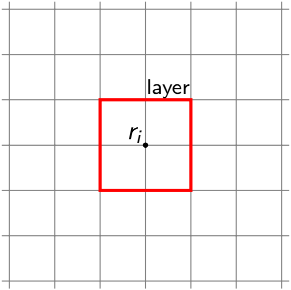

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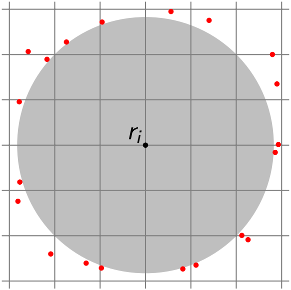

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