Robust Shattering Arguments (part II)

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joint work with

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Magnús M. Halldórsson
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With inputs from

Sebastian Brandt
Christoph Grunau
Václav Rozhoň

Topic of the talk

Two kinds of talk

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↑ today ↑

Our topic

Well known result by Fischer and Ghaffari

Theorem ([FG17])

There is an algorithm of complexity $O(d^2 + T_{\mathrm{LLL}}^{\mathsf{det}}(\operatorname{poly}\log n))$ *for the constructive Lovász Local Lemma under criterion* $8pd^9 < 1$ *in the randomized LOCAL model.*

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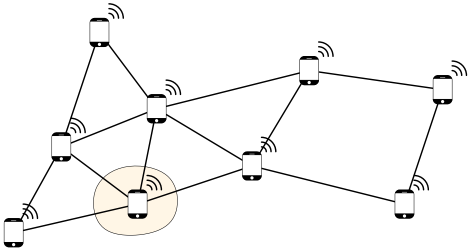
Problem definition

LOCAL model



- synchronous,
- messages of unbounded size,
- no limit on computational power,
- nodes given unique identifiers, between 1 and poly(n)
- polynomial upper bound on n known by the nodes (n: size of the graph)

Goal: minimizing the number of communication rounds (time unit).

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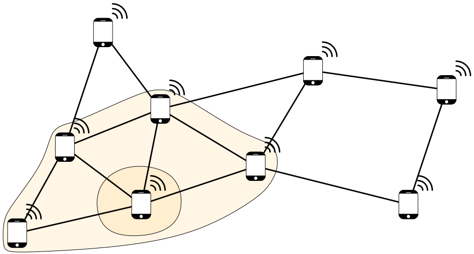

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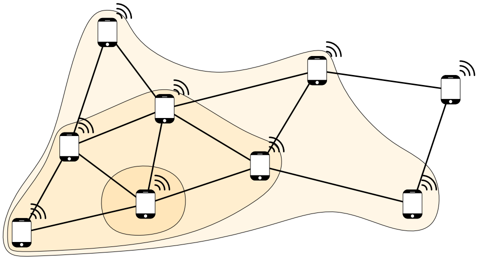

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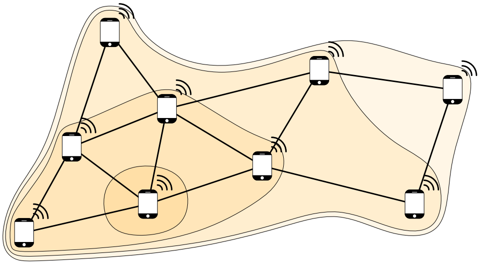

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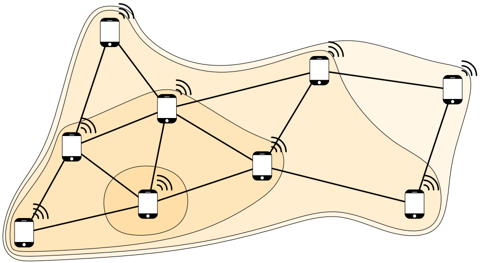

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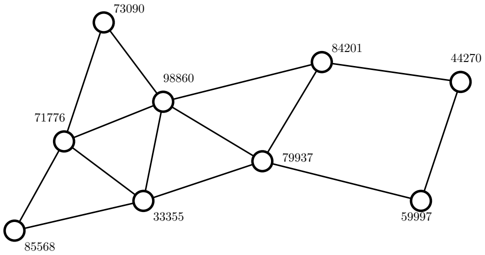
Randomized variant: allowed to fail with probability $1/n$

What is LLL?

Answer: A kind of probabilistic method



Graph G where nodes have huge identifier (up to 2^n).

Want to update $h(x) = x \mod p$ everywhere without having a collision.

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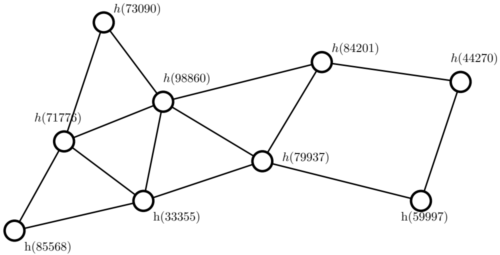

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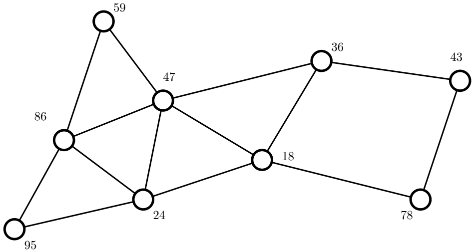

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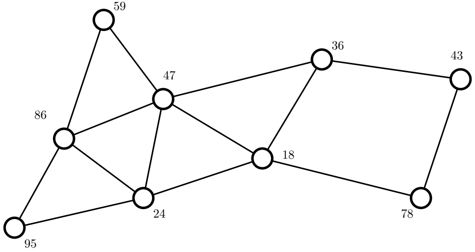

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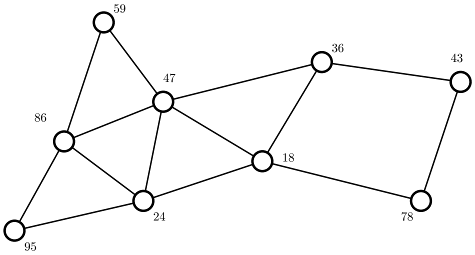

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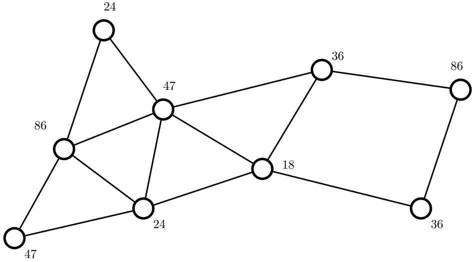
There are $\binom{n}{2}$ pairs of nodes: $\binom{n}{2} \cdot n^{-2} < 1$, so there exists p s.t. all $h(x)$ differ.

What is LLL? cont.

What if: bad events are more likely, but only local?



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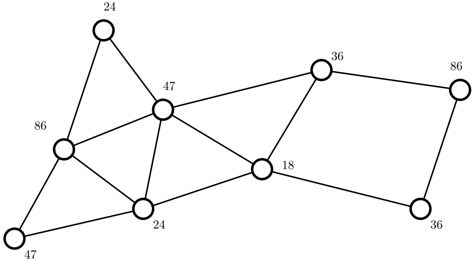

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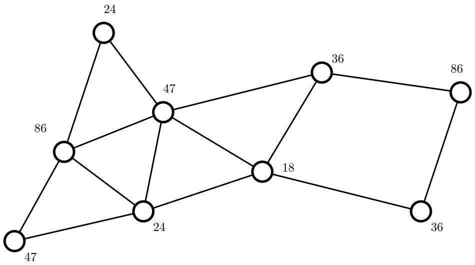

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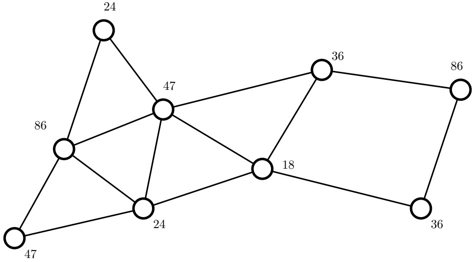

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But: every event is only non-independent with $\leq \Delta^2$ other events.

Lovász Local Lemma (symmetric version)

Assume:

- Set of "bad events" $\mathcal{E}$,
- $p \in [0, 1]$ s.t.: Each event has probability $\leq p$,
- Dependency graph: an edge between two events if non-independent,
- d: max degree of dependency graph.

Theorem (LLL)

If $ep(d+1) < 1$*, with probability* > 0*, no bad event occurs.*

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Constructive LLL: finding an assignment of random variables s.t. no bad event occurs.

Importance of LLL

LLL complete for LCLs of complexity $o(\log n)$ in randomized LOCAL [CP19]

Several problems are easily cast / reduced as solving an LLL: sinkless orientation, splitting problems, edge coloring problems...

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Several problems are easily cast / reduced as solving an LLL: sinkless orientation, splitting problems, edge coloring problems...

For many problems, LLL is the only reason that we know that a solution even exists: we have no other approach!

Shattering LLL

Fischer-Ghaffari result

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What could shattering look like for LLL?

Recall: valid shattering lemma

Lemma

If for every x-independent set S, the probability that all nodes in S join a "post-shattering" set B is at most $\Delta^{-(x+4)|S|}$, then w.h.p., B contains no connected component of size $\geq \Delta^x \log n$

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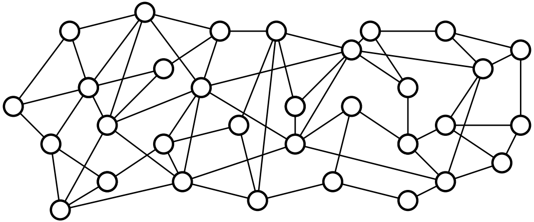
2. a set of distant nodes should become solved roughly independently (to apply above lemma)
3. what remains unsolved should be solvable, and fast (in our case: it's still an LLL)

What happens if we just sample everything?

Consider a set S of bad events, at pairwise distance ≥ 2 in the dependency graph.
Prob. that they all hold: $< p^{|S|}$.



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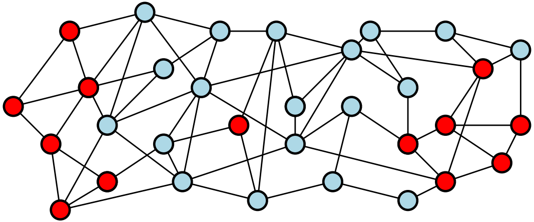

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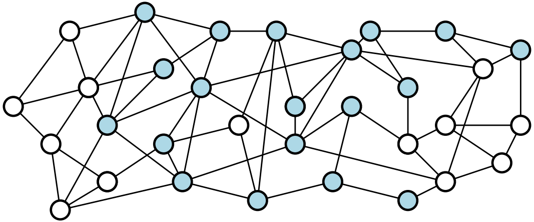

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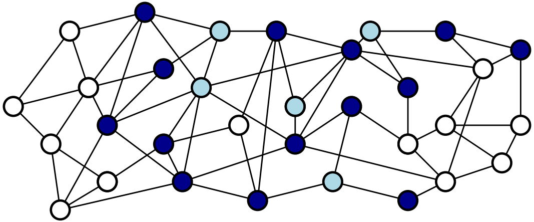

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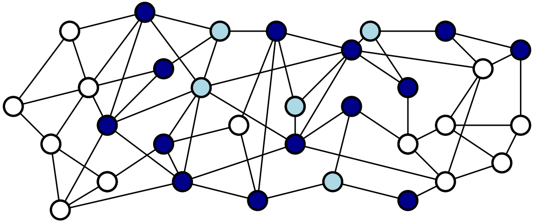

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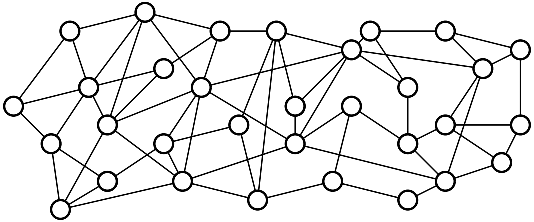
And if resampling: some events might need $\Omega(\log_p n)$ resamplings

Hint from sinkless orientation

Sample edges, but not ALL edges [GS17].



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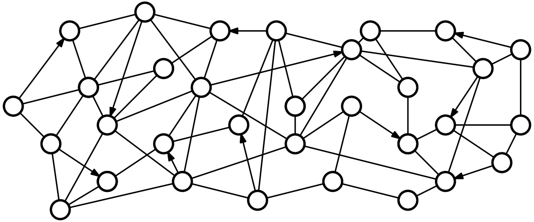

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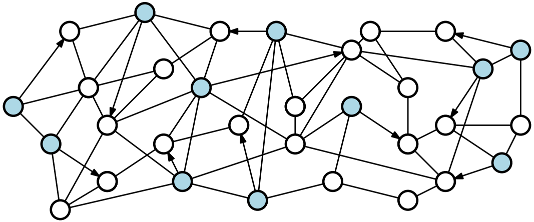

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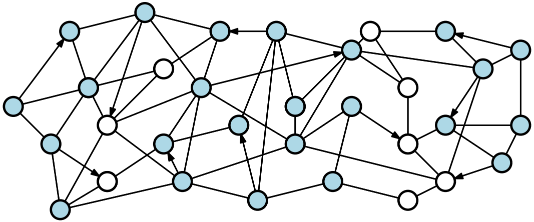

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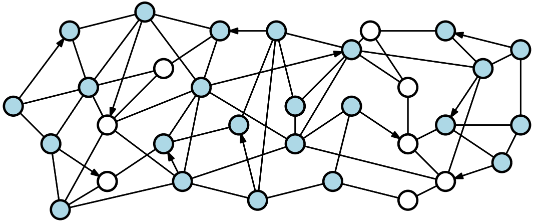
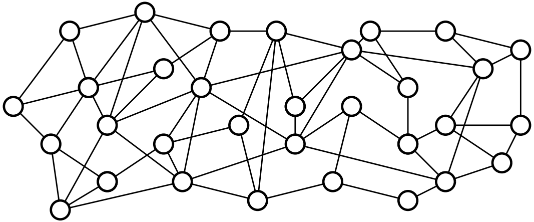


For LLL: sample many events and variables, but not ALL events and variables.

Freezing variables

Idea: progressively sample, stop when marginal probability becomes too high [MR98].



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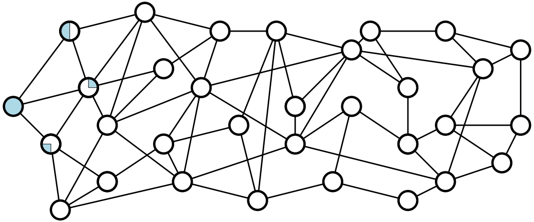

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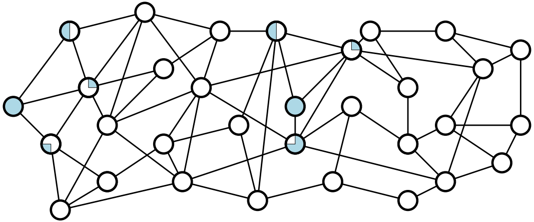

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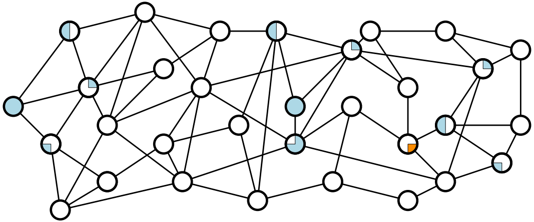

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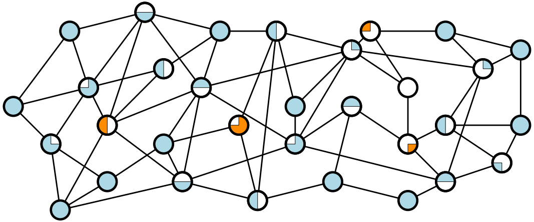

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Fischer Ghaffari : distributed version

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- Use it as a schedule for sampling

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Reason for the d^2 in the runtime.

Why distance-2 ? an event never has two neighbors sampling some its variables being sampled at the same time.

This allows to do progressive sampling in $O(1)$ rounds even with many variables in an event.

In what sense should it shatter?

Intuitively, the probability that a node still has some unsampled variables with probability $\leq \frac{2p(d+1)}{q}$ at the end of the process, with freezing threshold $q/2$.

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All we need is independence now...

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What is NOT independent: when progressively sampling variables according to some rules in a process that takes $\Theta(d^2)$ rounds, whether two $O(d^2)$-distant events have unsampled variables at the end.

Independence does not hold

What is independent: if we sample the variables of two non-adjacent events, whether the events holds is independent.

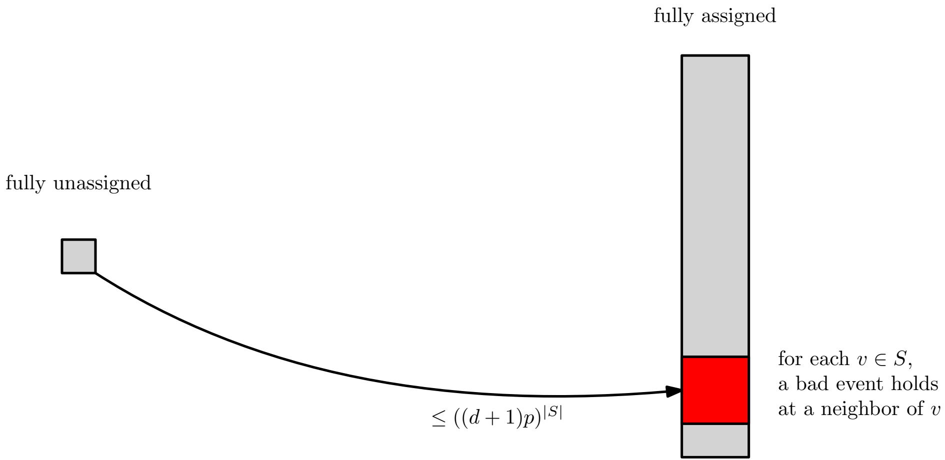
What is NOT independent: when progressively sampling variables according to some rules in a process that takes $\Theta(d^2)$ rounds, whether two $O(d^2)$-distant events have unsampled variables at the end.

We have a (tedious) concrete example.

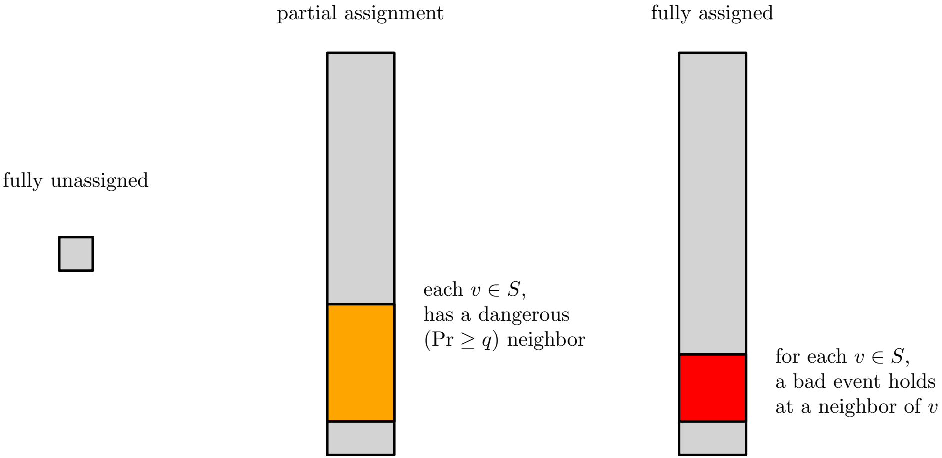
Visually


fully assigned
fully unassigned
for each $v \in S$,
a bad event holds
at a neighbor of v

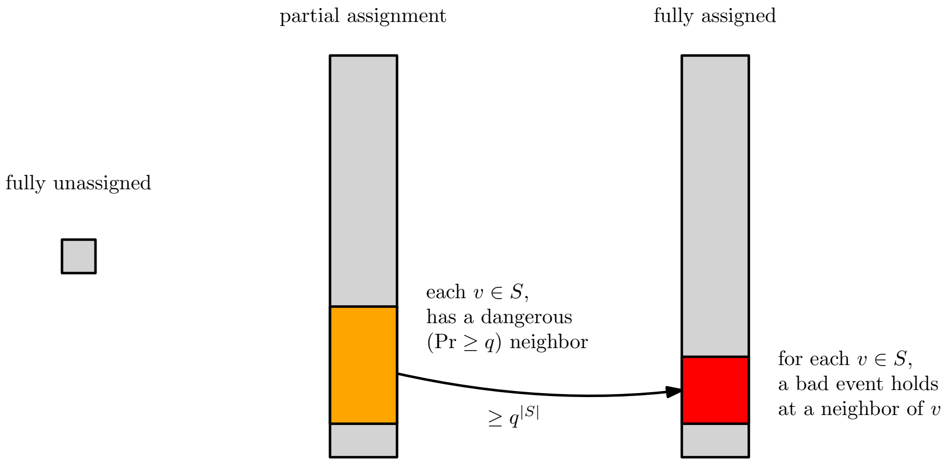
Visually


fully assigned
fully unassigned
for each $v \in S$,
a bad event holds
at a neighbor of v
$\leq ((d+1)p)^{|S|}$

Visually


partial assignment
fully assigned
fully unassigned
each $v \in S$,
has a dangerous
($\Pr \geq q$) neighbor
for each $v \in S$,
a bad event holds
at a neighbor of v

Visually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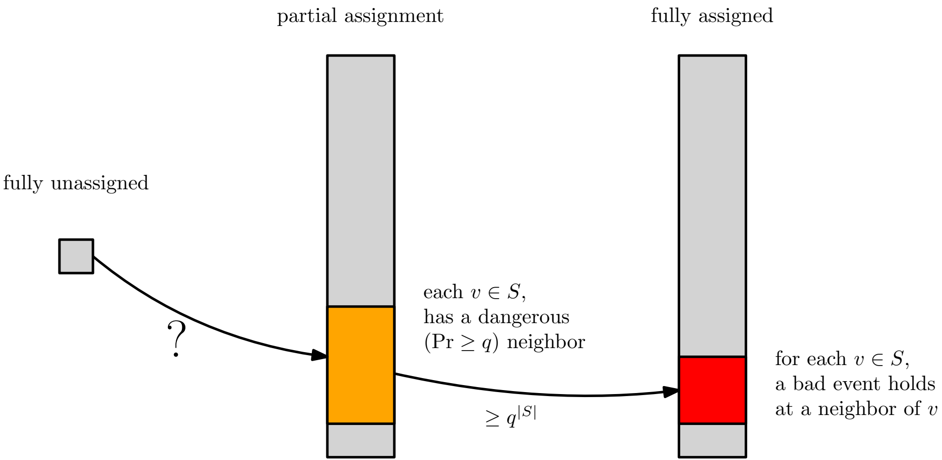
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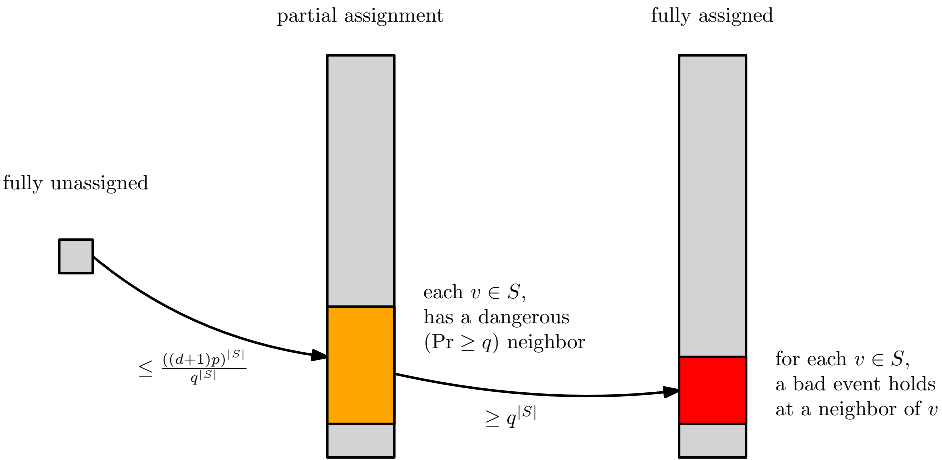
partial assignment
fully assigned
fully unassigned
each $v \in S$,
has a dangerous
$(\Pr \geq q)$ neighbor
$\leq \frac{((d+1)p)^{|S|}}{q^{|S|}}$
$\geq q^{|S|}$
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Whitebox fix

1. Consider a set S of 4-distant nodes. Consider the event that they all have unsampled variables at the end of the partial sampling.

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3. But if sampling everything at once initially, probability to have a bad event next to each node in S? At most $((d+1)p)^{|S|}$.

Whitebox fix

1. Consider a set S of 4-distant nodes. Consider the event that they all have unsampled variables at the end of the partial sampling.
2. Probability that starting from the partial sampling, we have a bad event next to each node in S? At least $q^{|S|}$.
3. But if sampling everything at once initially, probability to have a bad event next to each node in S? At most $((d+1)p)^{|S|}$.
4. **Conclusion:** probability to reach a partial assignment where all nodes in S have unsampled variables is at most

$$\frac{((d+1)p)^{|S|}}{q^{|S|}}\ .$$

pick $q = 1/(d+1)$, $p = d^{-10}/2$... done

Fischer-Ghaffari restored

Theorem ([FG17])

There is an algorithm of complexity $O(d^2 + T^{\mathsf{det}}_{\mathrm{LLL}}(\mathrm{poly}\log n))$ *for the constructive Lovász Local Lemma under criterion* $8pd^9 < 1$ *in the randomized LOCAL model.*

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If $d^2 \geq \operatorname{poly}\log n$: use Moser-Tardos [MT10], faster than $O(d^2)$ in that case. Only shatter for $\Delta \in O(\operatorname{poly}\log n)$.

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Making progress can be: simply getting solved, or, e.g., having its degree decrease by a constant factor.

We give a generic argument that captures many algorithms for greedy problems (MIS, MM, coloring...)

More concretely

If each node in a set has x guaranteed steps where it gets solved w.p. $\geq 1 - p$, conditioned on an arbitrary past.

Probability that no one in the set gets solved: $\leq p^{|S|}$.

Examples: $O(\log \Delta)$ shattering by Ghaffari for MIS, [Gha16], coloring and MIS by Barenboim, Elkin, Pettie and Schneider [BEPS16].

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Is there a more generic argument for LLL?

Thanks!

Small announcement: PODC 2027 in Iceland

June 22 to 24, 2027.
Reykjavik University

Leonid Barenboim, Michael Elkin, Seth Pettie, and Johannes Schneider.
The locality of distributed symmetry breaking.
J. ACM, 63(3):20:1–20:45, 2016.

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Sublogarithmic distributed algorithms for lovász local lemma, and the complexity hierarchy.
In *Proceedings of the International Symposium on Distributed Computing (DISC)*, LIPIcs, pages 18:1–18:16. Schloss Dagstuhl - Leibniz-Zentrum für Informatik, 2017.

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In *Proceedings of the ACM-SIAM Symposium on Discrete Algorithms (SODA)*, pages 270–277. SIAM, 2016.

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On derandomizing local distributed algorithms.
In *Proceedings of the Symposium on Foundations of Computer Science (FOCS)*, pages 662–673. IEEE Computer Society, 2018.

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Robin A. Moser and Gábor Tardos.
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