Semi-Robust Communication Complexity of Maximum Matching

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Joint work with: Adithya Diddapur and Pavel Dvorak

Random Order Streaming Model

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Sequence of edges of input graph $G = (V, E)$ in *uniform random order*

$$S = e_2 e_1 e_4 e_3$$
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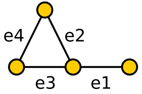
$$\dots$$



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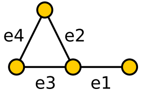


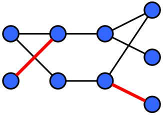
Goal: One-pass streaming algorithms ($n = |V|$)

- Space $o(n^2)$ (semi-streaming: $O(n \cdot \mathrm{polylog}\, n)$)
- **Quality guarantee:** In expectation (or w.h.p.) over the stream order

Motivation: Practically relevant, often easier than adversarial order

The Maximum Matching Problem

Matching: Vertex-disjoint subset of edges



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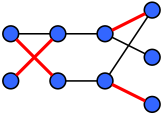

Maximum Matching
(largest size)

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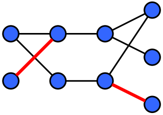

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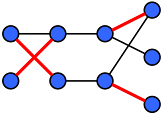

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Adversarial Order Streams: GREEDY

1. $M \leftarrow \varnothing$
2. **for each** edge e in stream: Insert e into M if $M \cup \{e\}$ is a matching
3. **return** M

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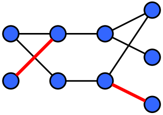

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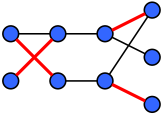

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$\frac{1}{2}$-approximation using $O(n \log n)$ space (semi-streaming)

Semi-Streaming Random Order Maximum Matching

	Bipartite	**General**	**Note**
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Gamlath et al. **[GKMS19]**	0.512	0.506	
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2. Short augmenting paths
3. Edge-degree-constraint-subgraphs (EDCS)

Streaming Limitation Results: One-Way Communication

Lower Bound Technique: One-way two-party Communication

$$\underset{E_A}{\textbf{Alice}} \xrightarrow{\quad M \quad} \underset{E_B}{\textbf{Bob}} \longrightarrow \text{large matching in } E_A \cup E_B$$

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Random Edge Partitioning = Robust Communication Setting

(Each edge assigned to either Alice or Bob with probability $\frac{1}{2}$ each)

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Communication Protocols

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Q: How about simple protocols?

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Analysis holds in the semi-robust setting, and is tight in this setting!

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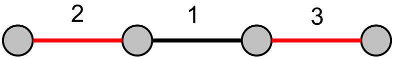

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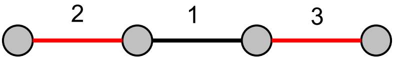

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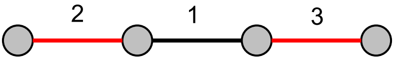

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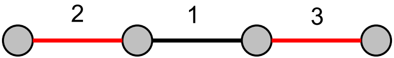

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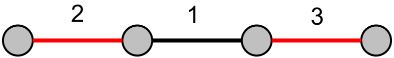

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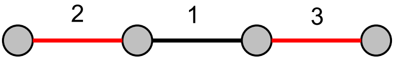

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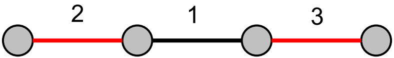

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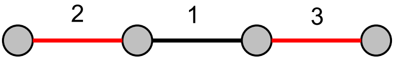

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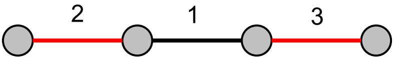

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$\mathbb{E}\ |\text{output}| = (\frac{1}{2} + \frac{1}{4}) \cdot 2 + \frac{1}{4} \cdot 1 = \frac{7}{4} \rightarrow 7/8 \approx 0.875$-approximation

Robust Setting: Hard Instance

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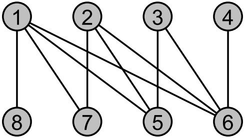

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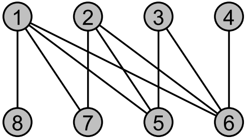

$\mathbb{E}$ approximation factor $= 0.832 < 5/6 = 0.8\overline{3}$

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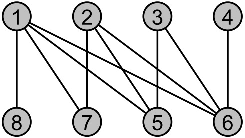

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Open Question:

What is the approximation factor of Π in the Robust Setting?

Analysis:
Π in the Semi-Robust Setting

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Setup: $G = (V, E)$, OPT maximum matching

- M: Lexicographic-first Maximum Matching of E_A
- $H = (V, M \cup OPT)$ (paths and cycles)

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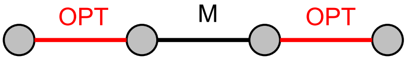

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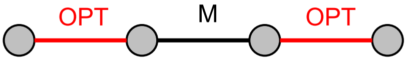

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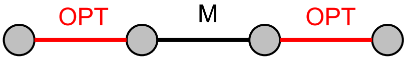

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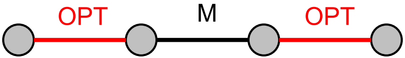
  Solution size on P: $1/4 \cdot 2 + 3/4 \cdot 1 = 5/4 \rightarrow 5/8$-approximation

Distribution of OPT Edges



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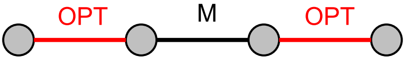

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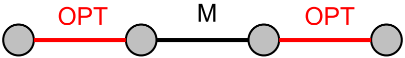

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(as M would not be maximum)

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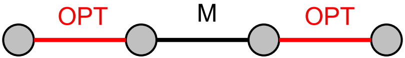

Observation

Alice cannot hold both *OPT* edges conditioned on P existing!

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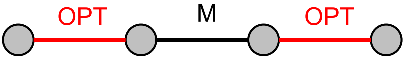
If distribution of *OPT* edges was uniform conditioned on P existing (excluding the case that Alice holds both *OPT* edges) then Bob holds both *OPT* edges with probability $1/3$ and can report solution of size 2

Distribution of OPT Edges



Observation

Alice cannot hold both OPT edges conditioned on P existing!

(as M would not be maximum)

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Solution size on P: $1/3 \cdot 2 + 2/3 \cdot 1 = 4/3 \rightarrow 2/3$-approximation

Distribution of *OPT* Edges (2)

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Theorem

The probability that Bob holds all OPT edges on an augmenting path of length $2k - 1$ is at least $\frac{1}{2^k - 1}$.

(Uniform distribution is the worst case)

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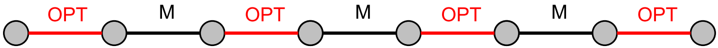

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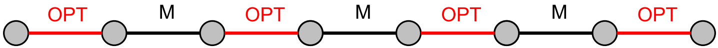

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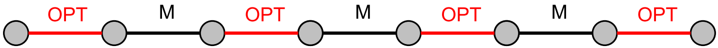

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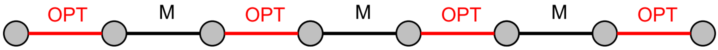

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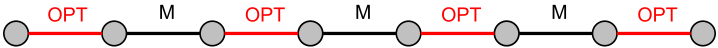

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$\rightarrow$ **This theorem already yields $2/3$-approximation!**

Idea 2: Limited Number of Augmenting Paths

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The expected number of *OPT* edges that Bob holds on an augmenting path of length $2k-1$ is at least $k\frac{2^{k-1}}{2^k-1}$.

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- Worst-case structures: Augmenting paths of lengths 3 and 5! □

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- Sending a lexicographic-first *Maximal Matching* yields a $5/8$-approximation in expectation
- Path of length 3 with middle edge given to Alice establishes tightness

Open Problems

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Thanks!