Distributed Algorithms for Local Potential Problems

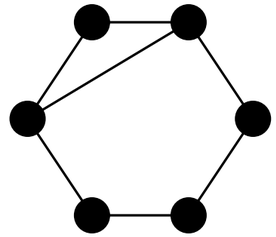
Francesco d'Amore

Joint work with A. Balliu, T. Boudier, F. Kuhn, D. Olivetti, G. Schmid, and J. Suomela
To appear at PODC 2026

Workshop on Foundations of Distributed and Parallel Graph Algorithms (Venice 2026)
18 May 2026

Locally Optimal Cut (LOC)

Input: - graph $G = (V, E)$
- two colors red and green



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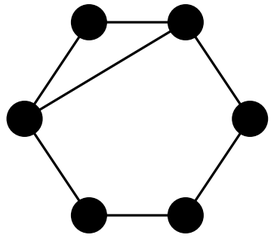

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- for each $v \in V$, at least $\geq \deg(v)/2$ *neighbors of different color* (w.r.t. v) ⟶ *locally checkable*

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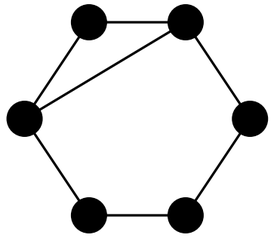

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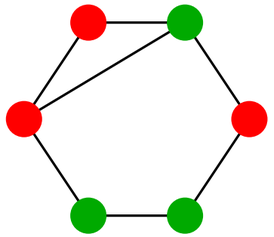

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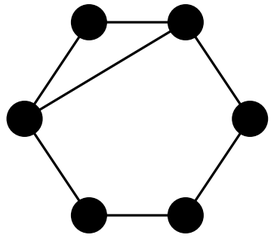

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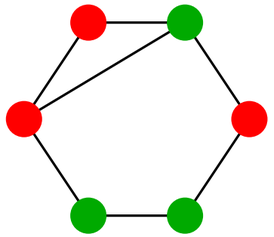

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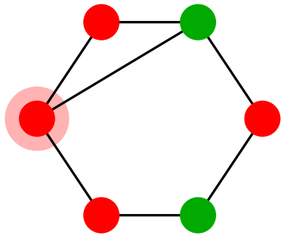

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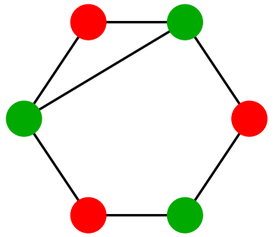
fixing procedure ⟶

flip color of invalid node



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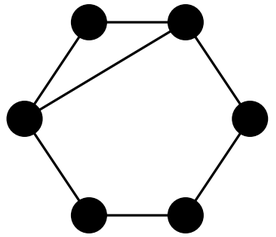

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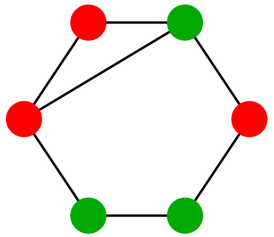

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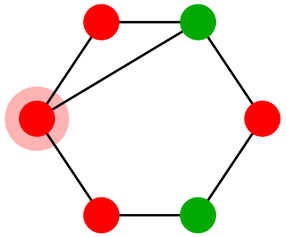

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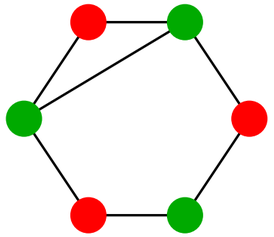

Question: can we *always* solve the problem?

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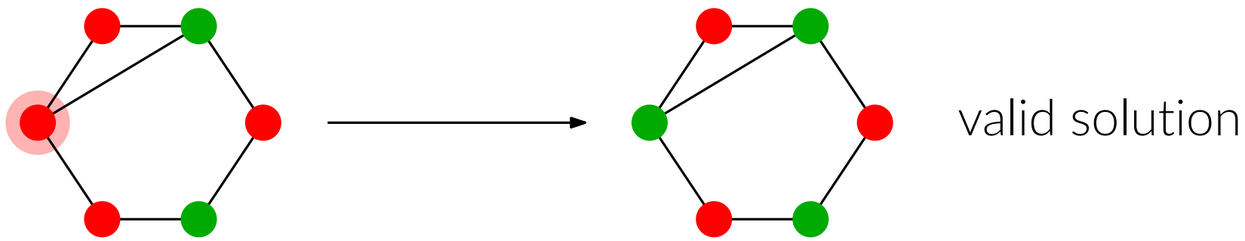

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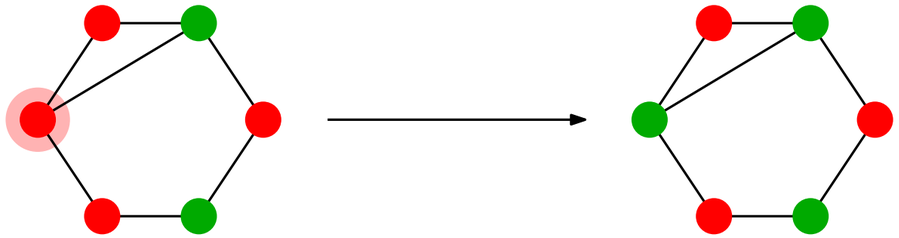

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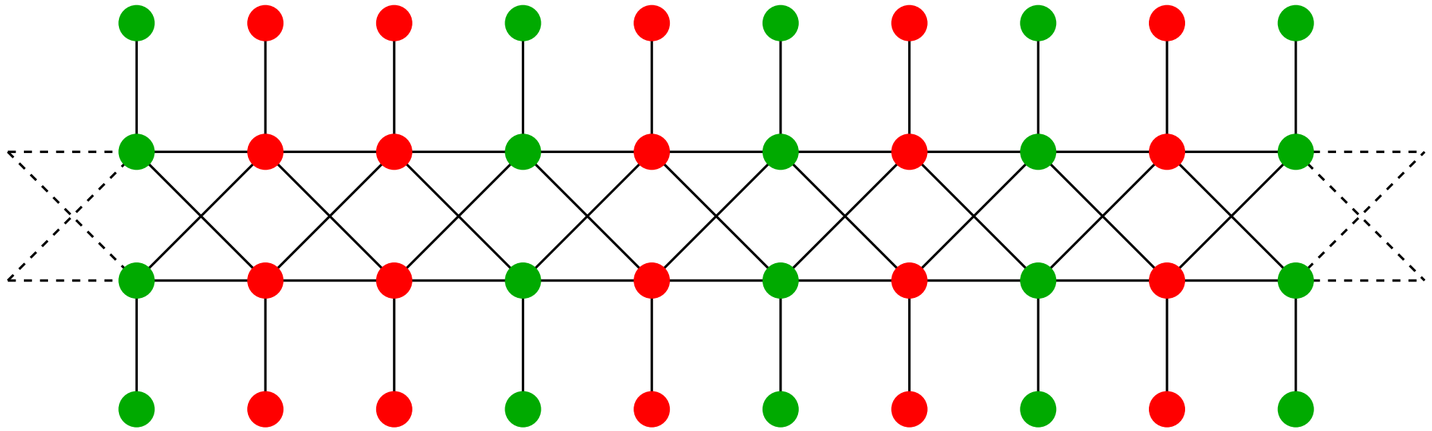

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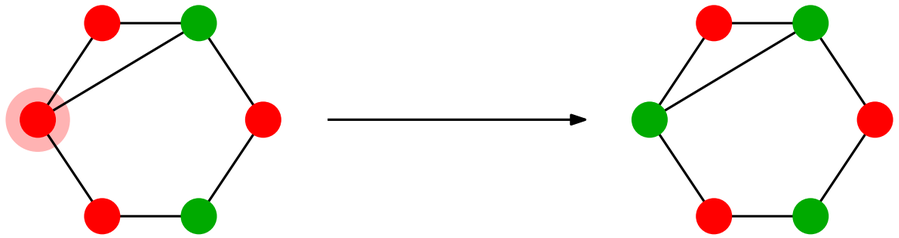

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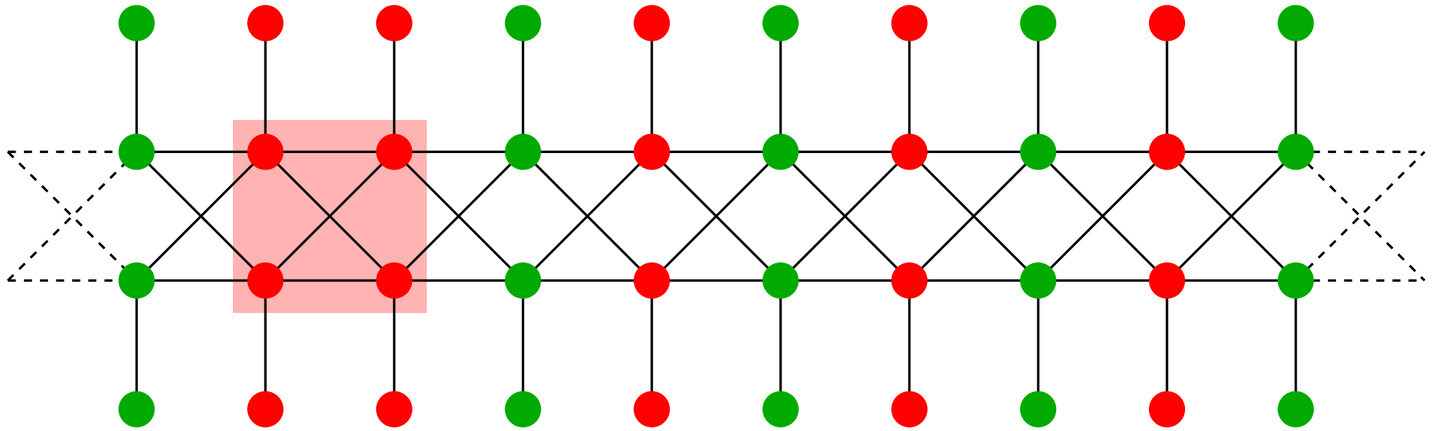

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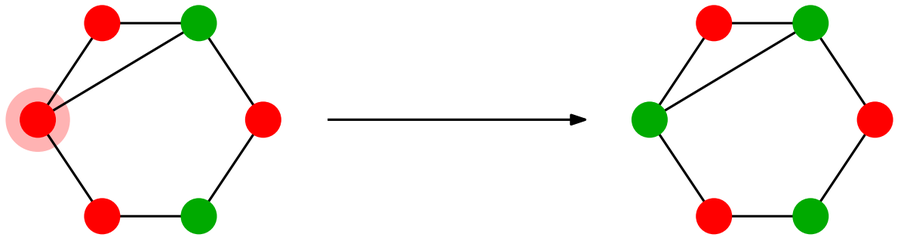

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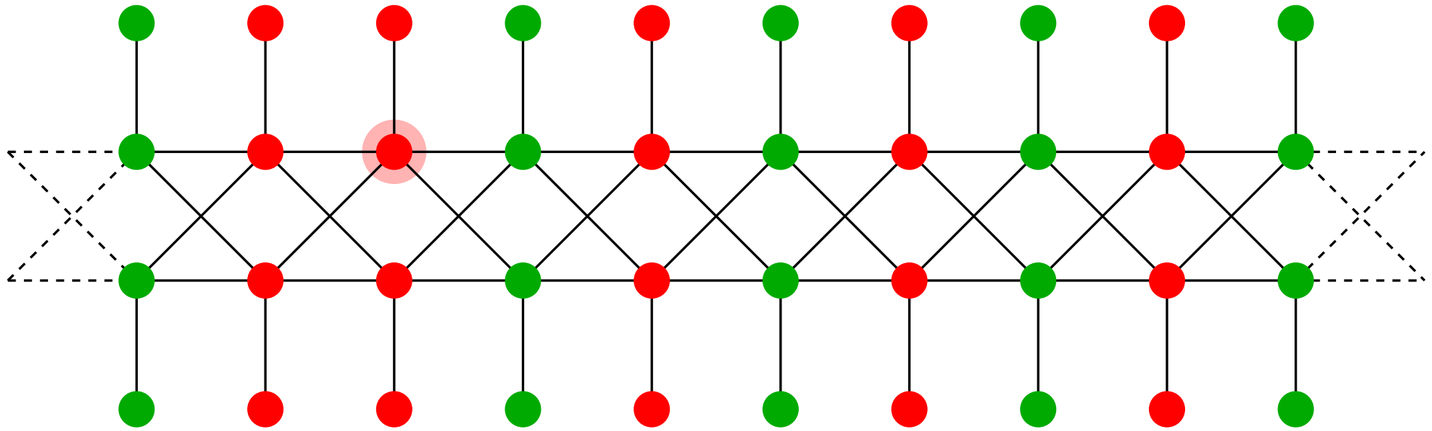

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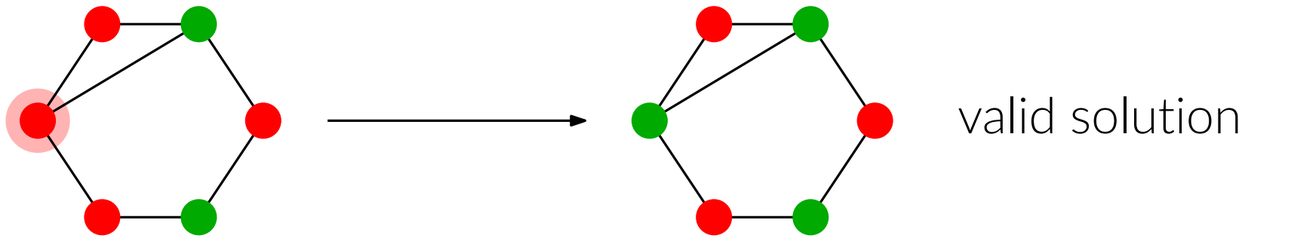

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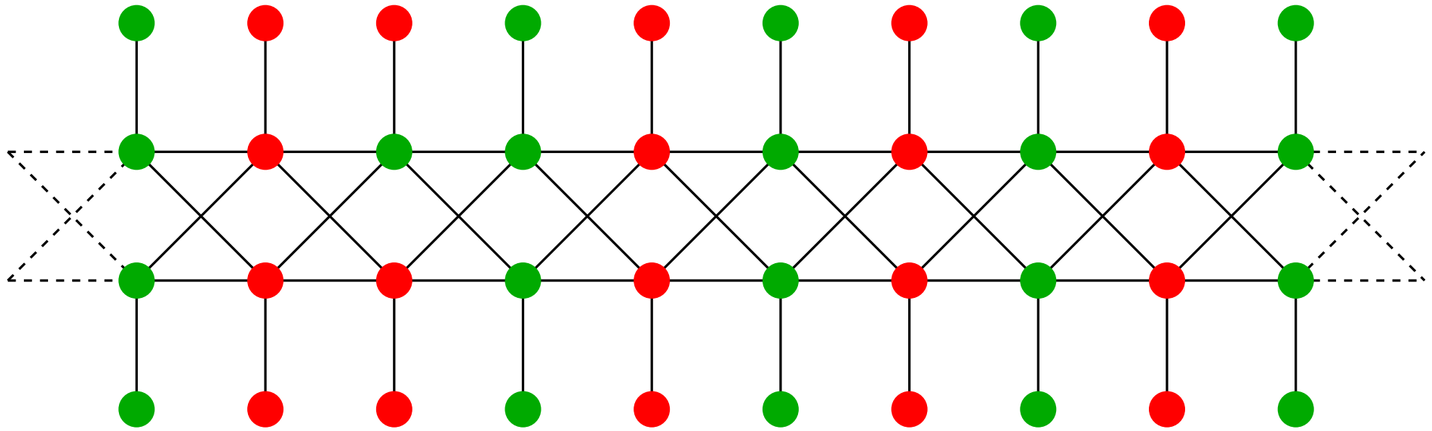

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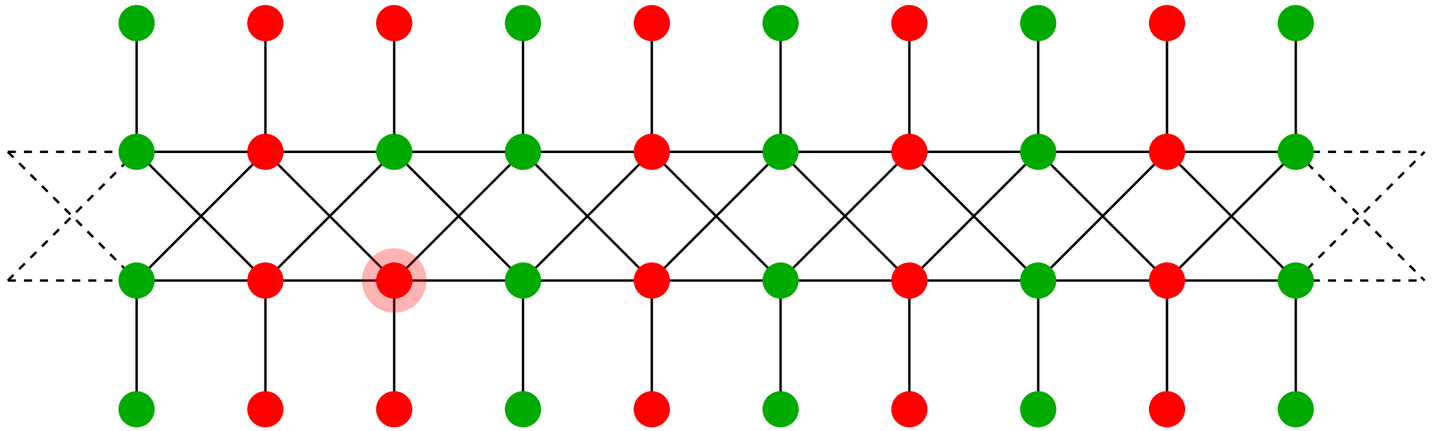

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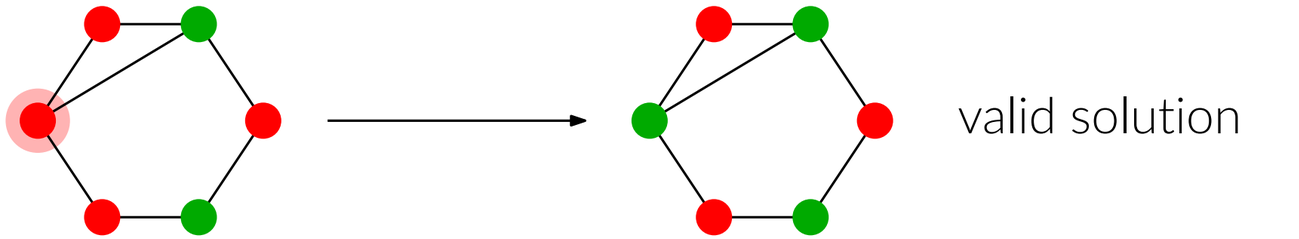

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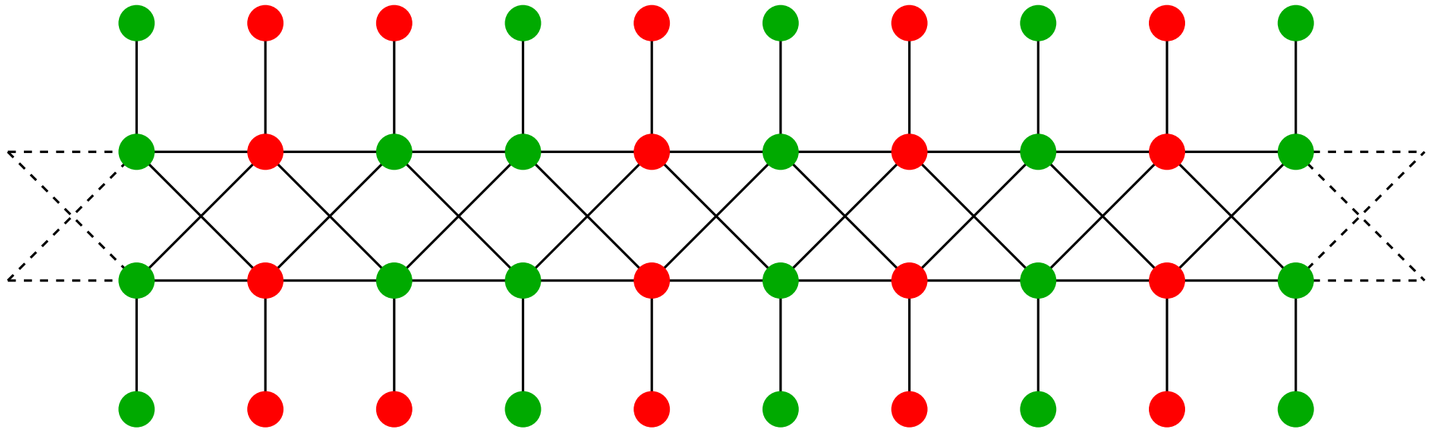

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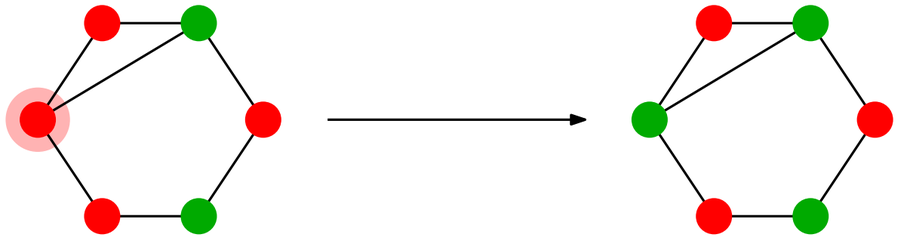

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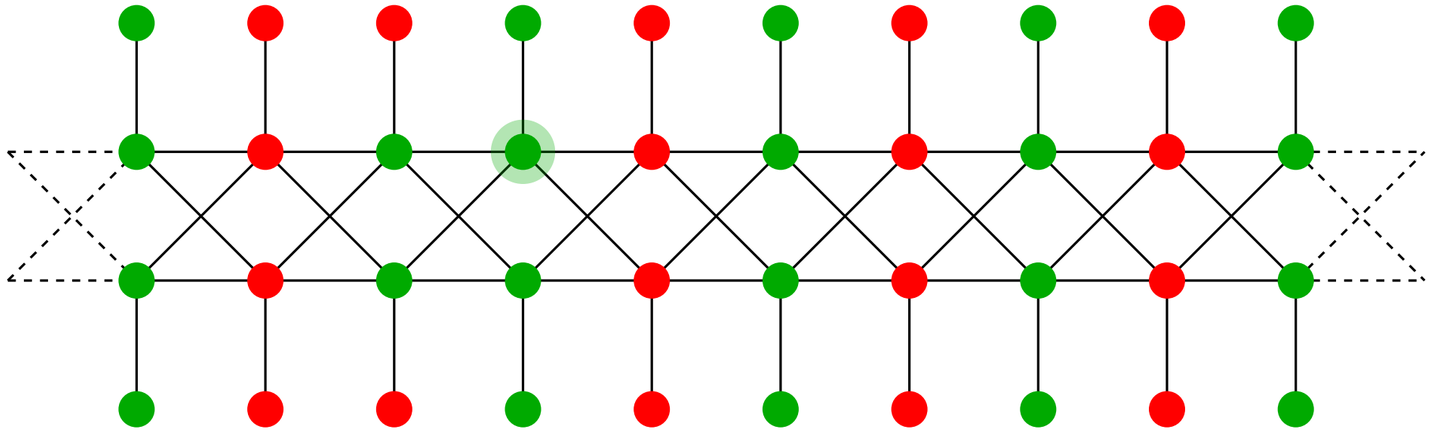

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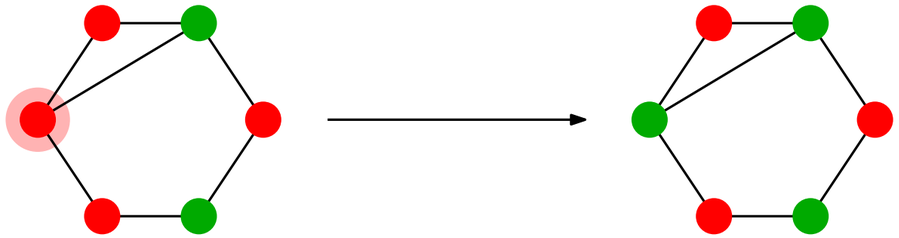

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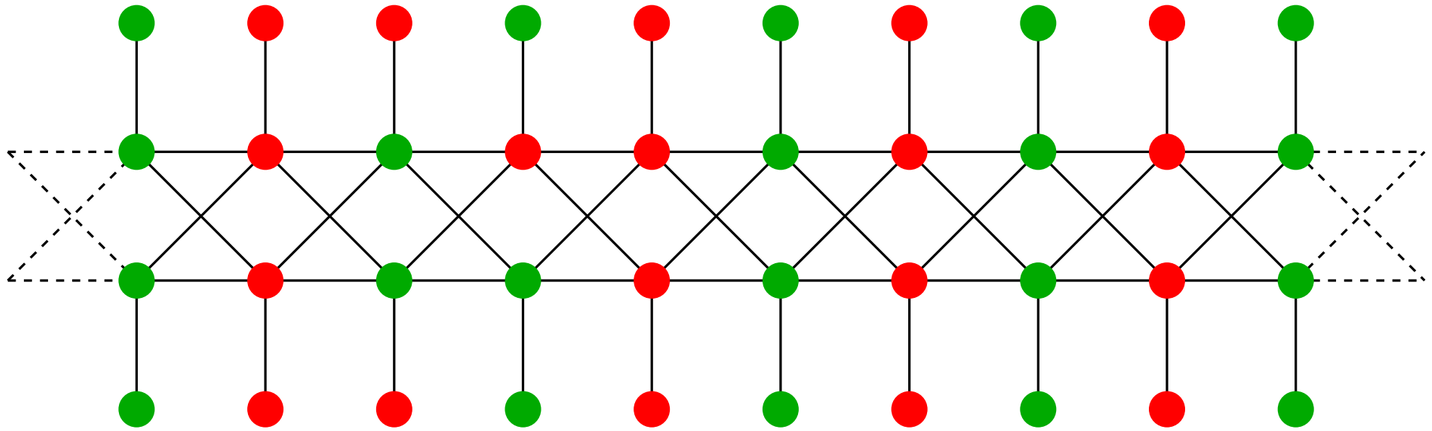

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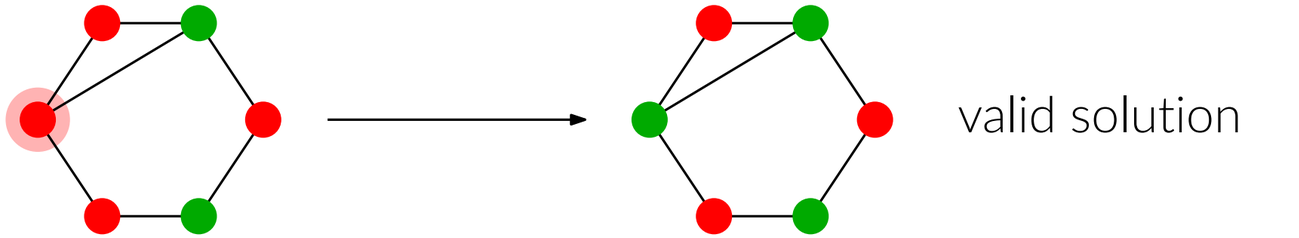

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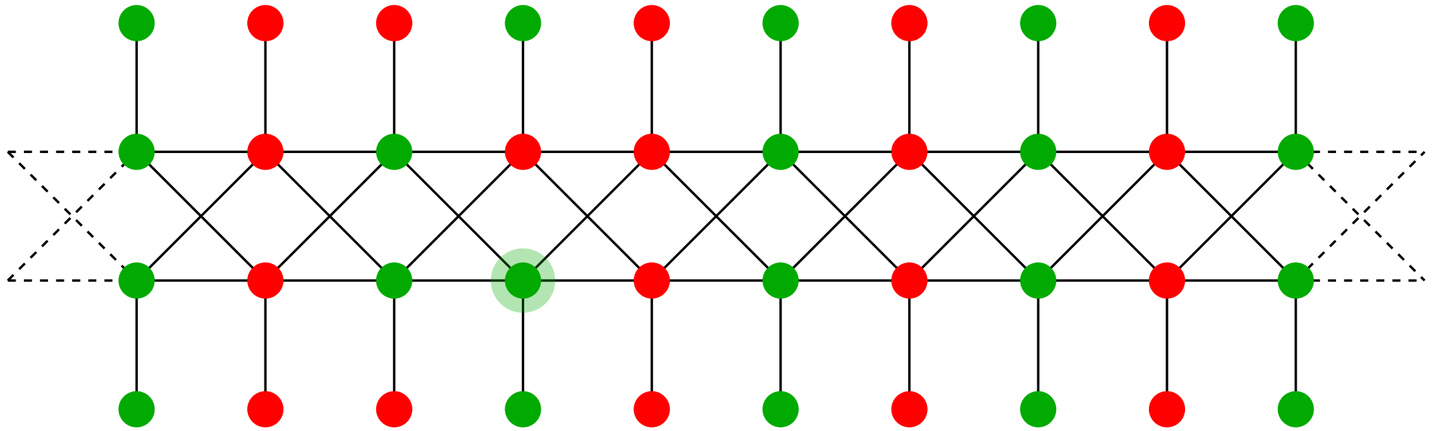

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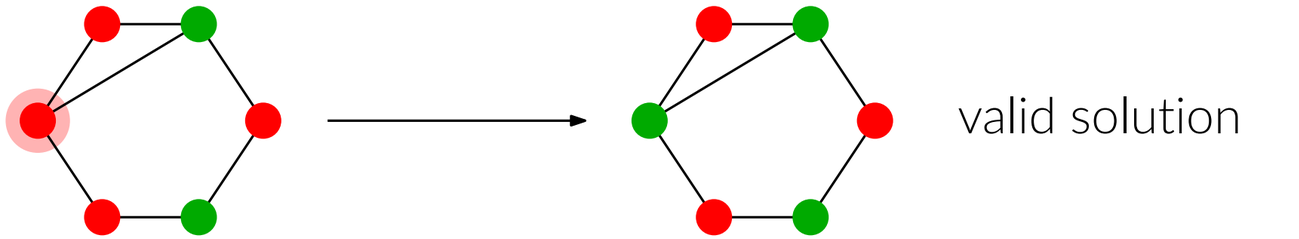

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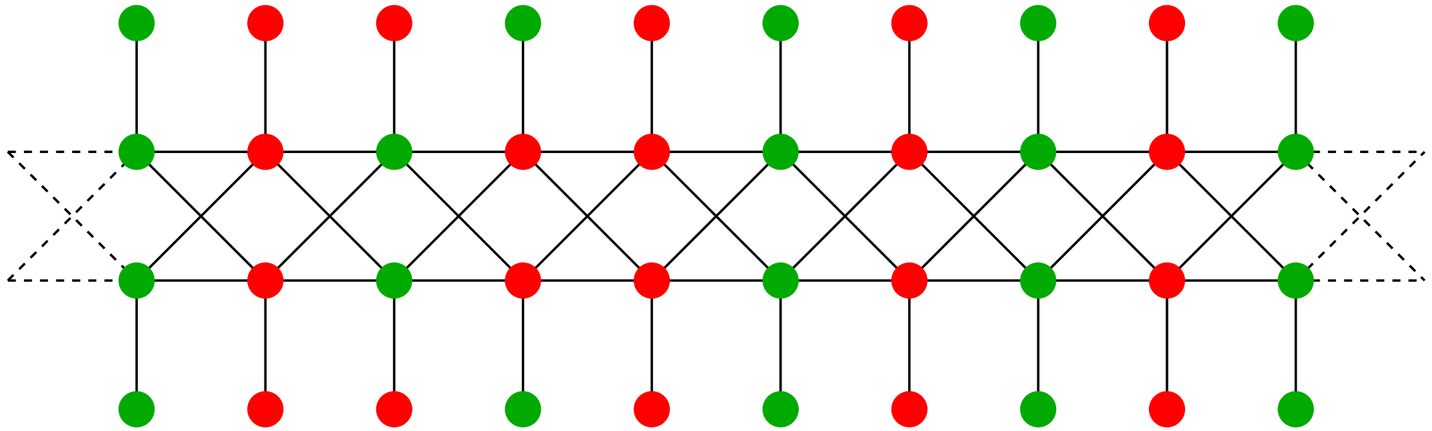

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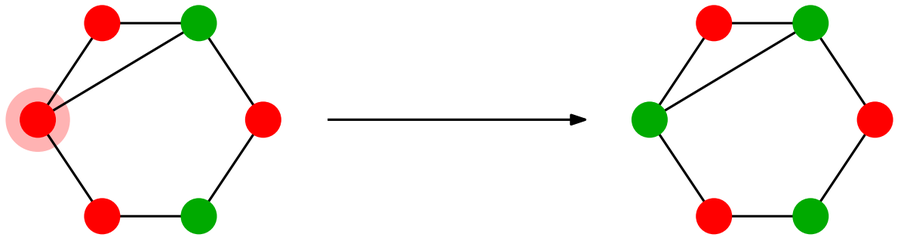

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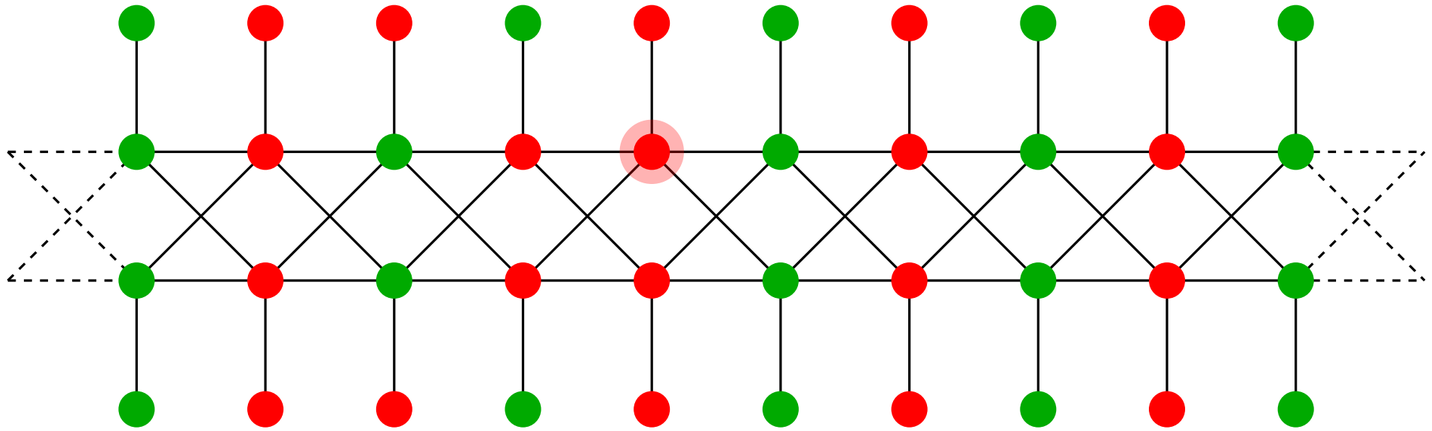

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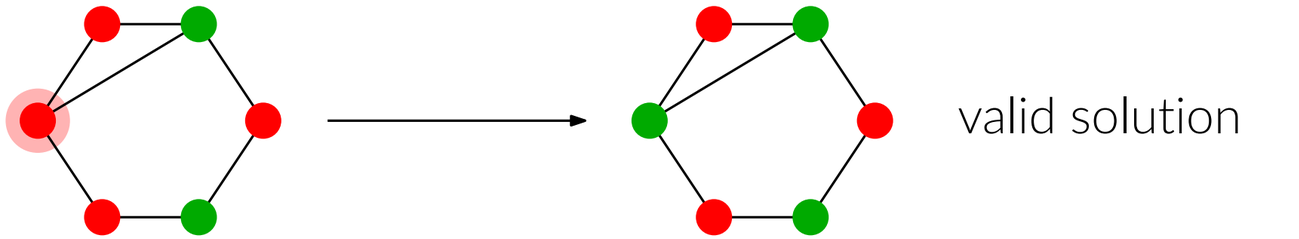

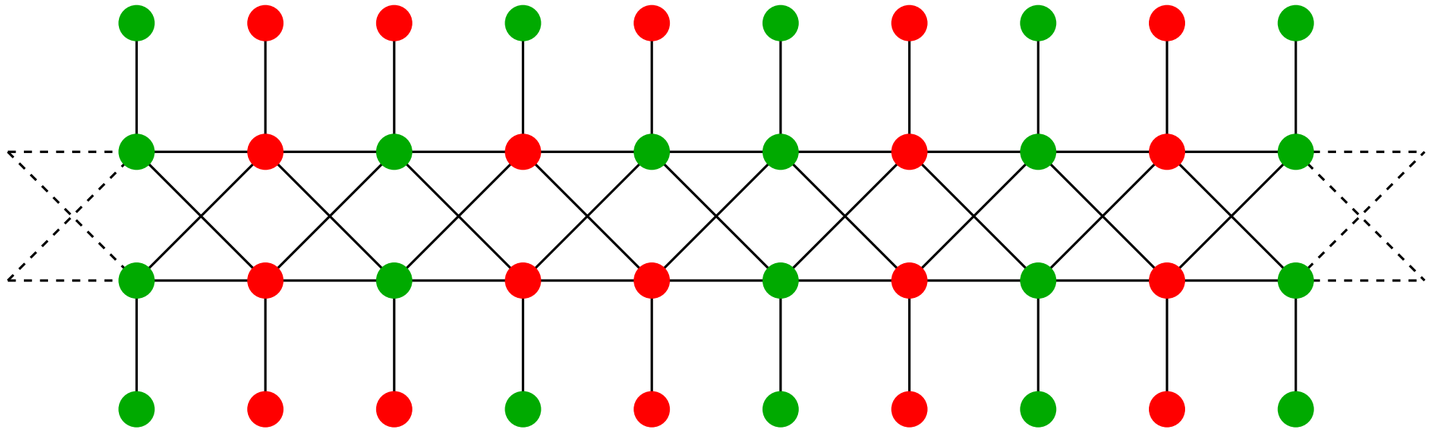

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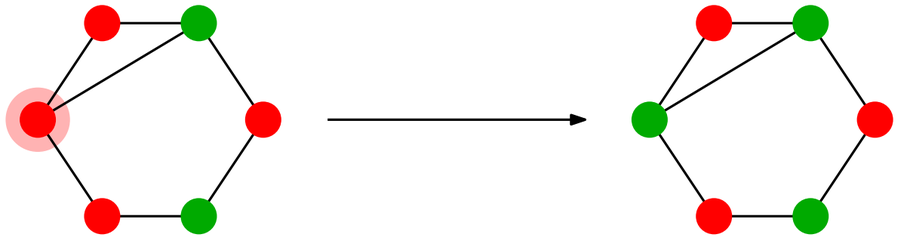

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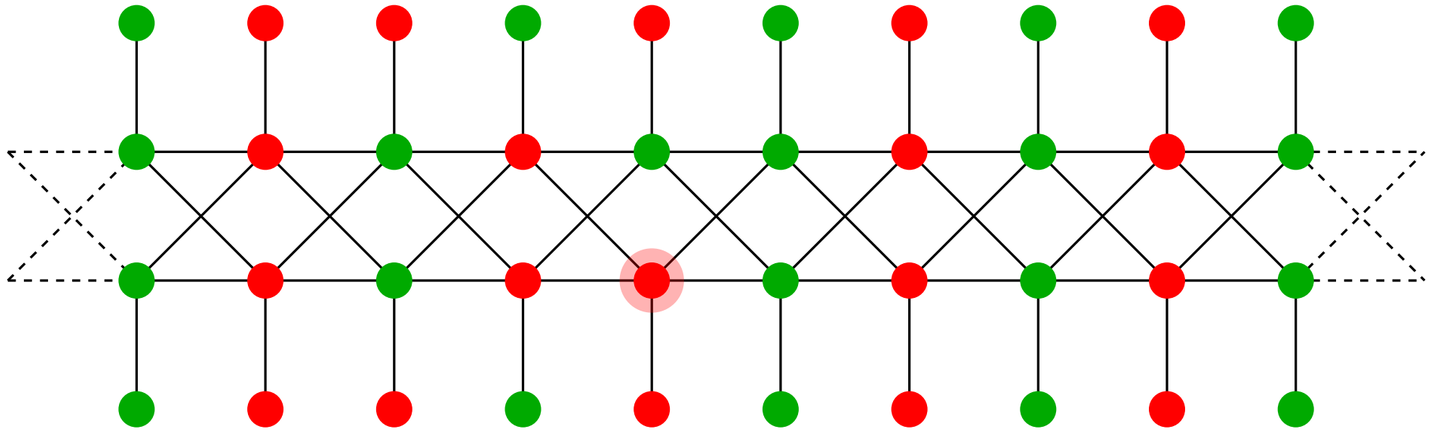

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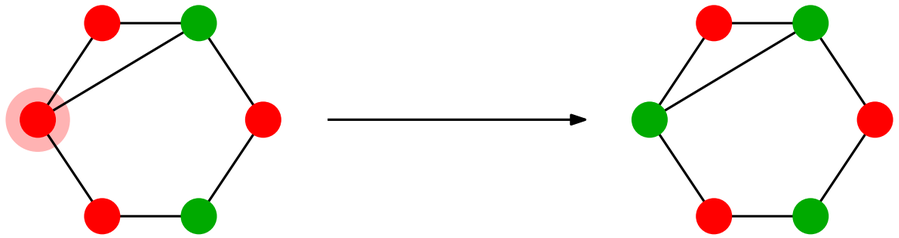

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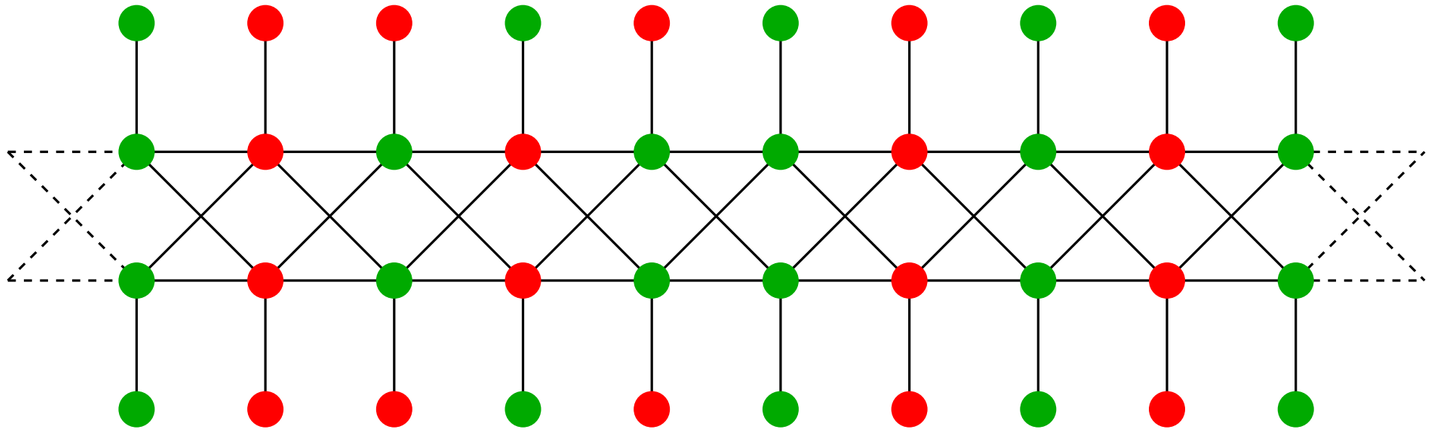

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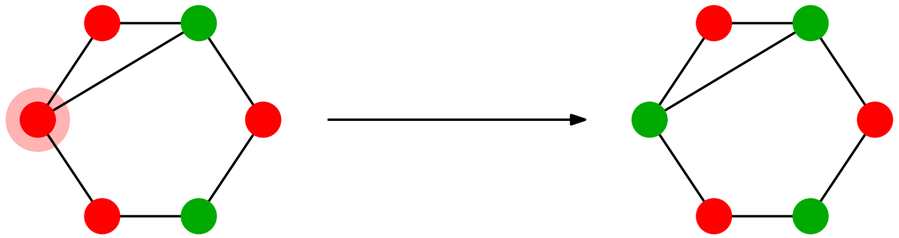

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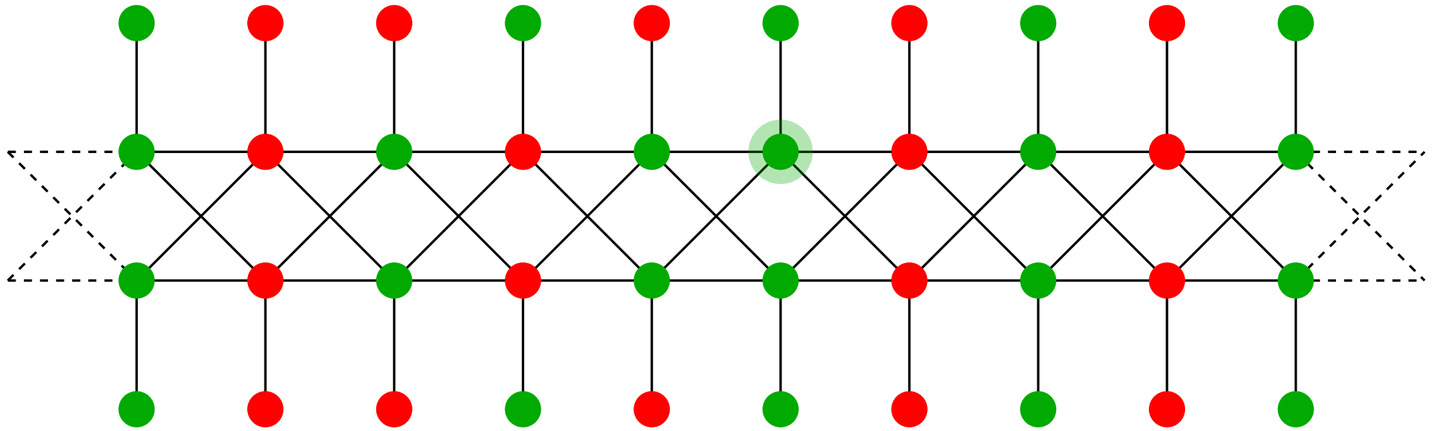

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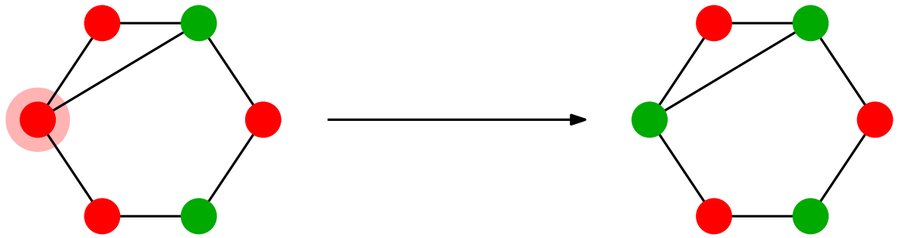

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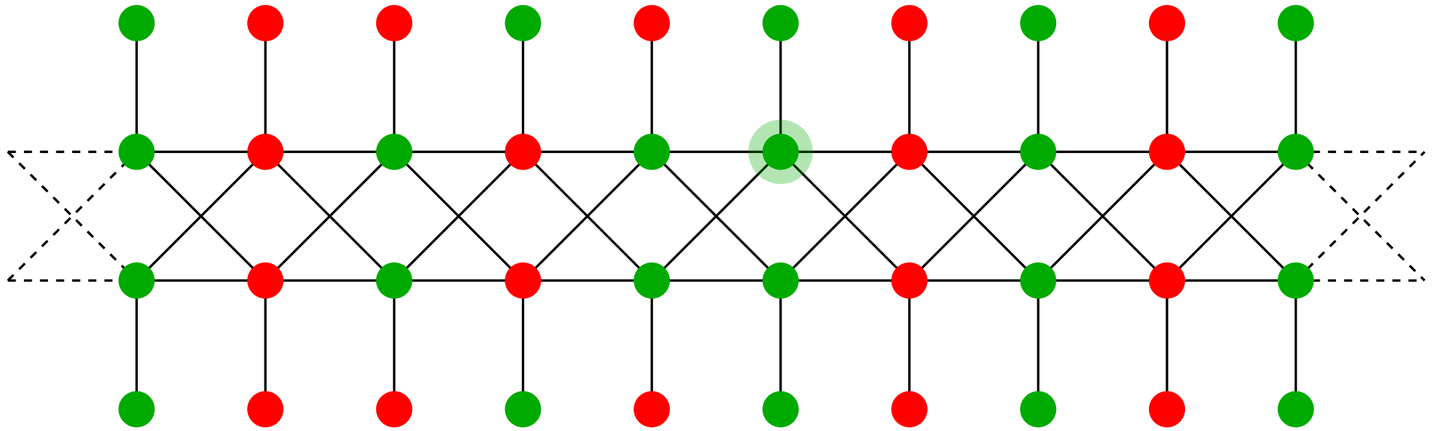

- **Convergence?**

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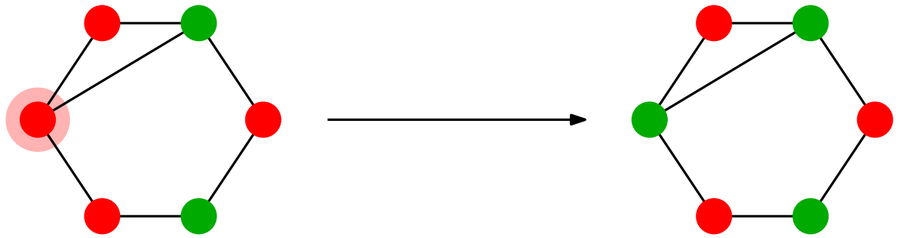

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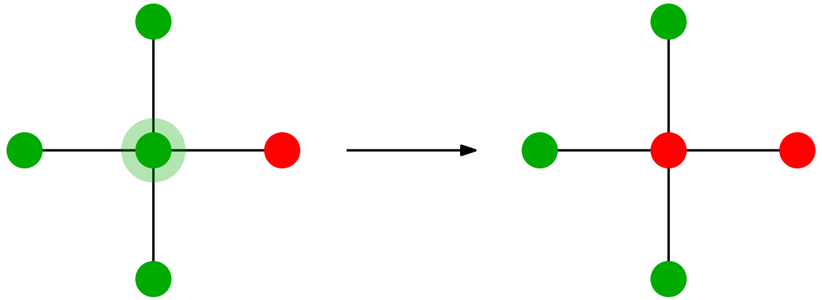

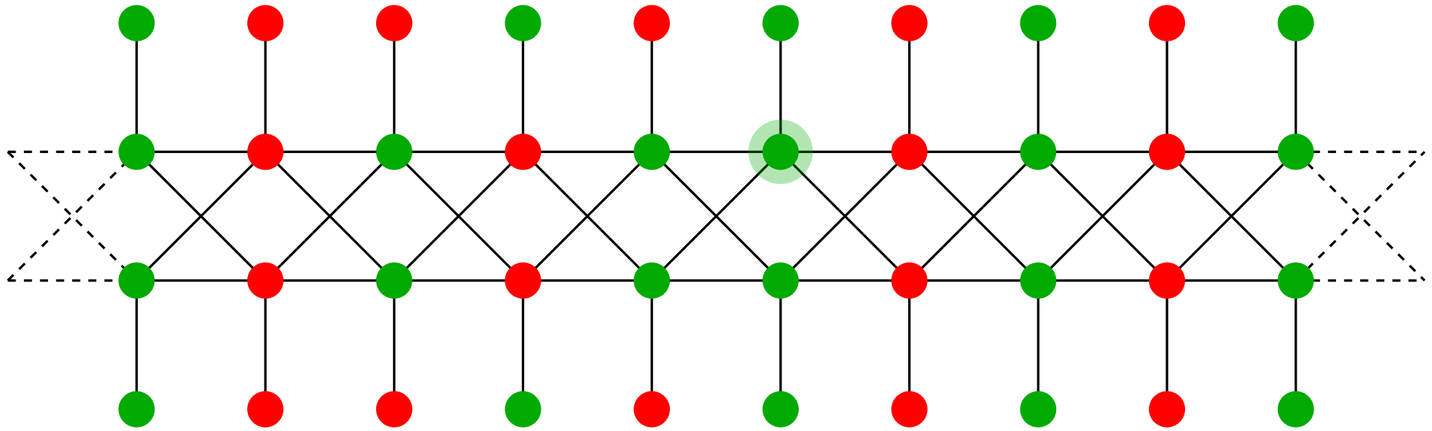

- **Convergence?** Potential function!

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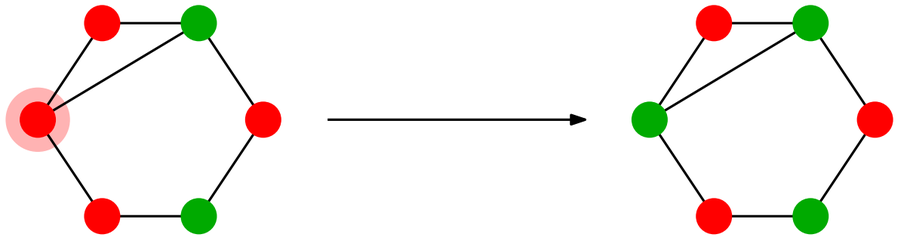

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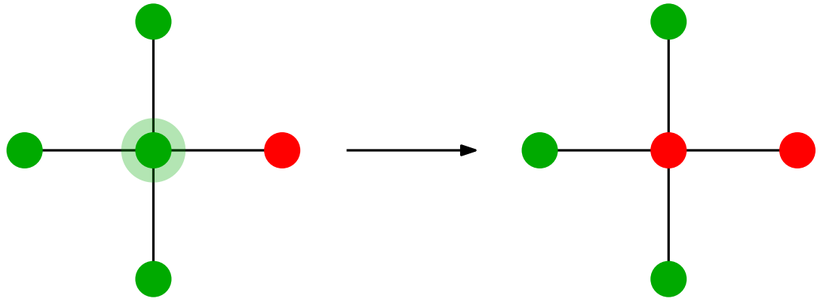

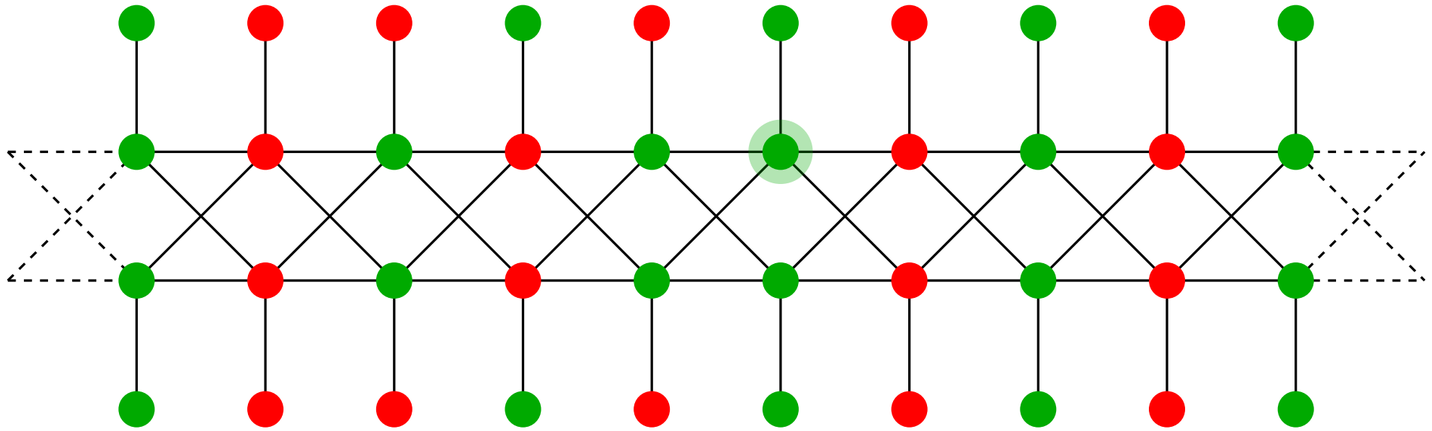

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- Possibly, very long chains…
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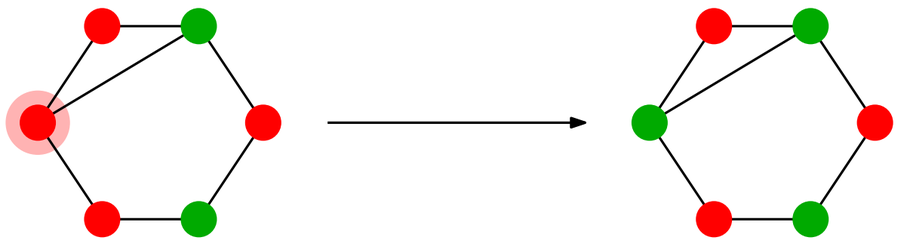

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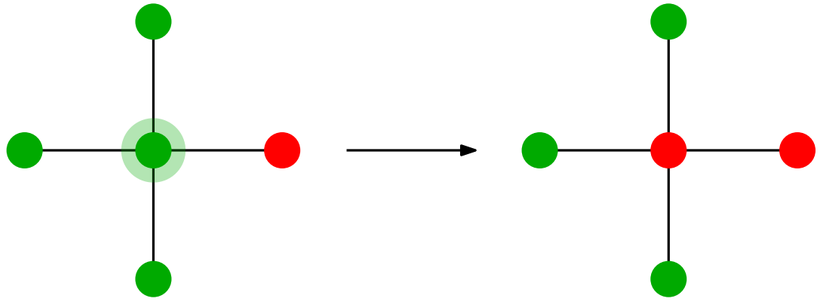

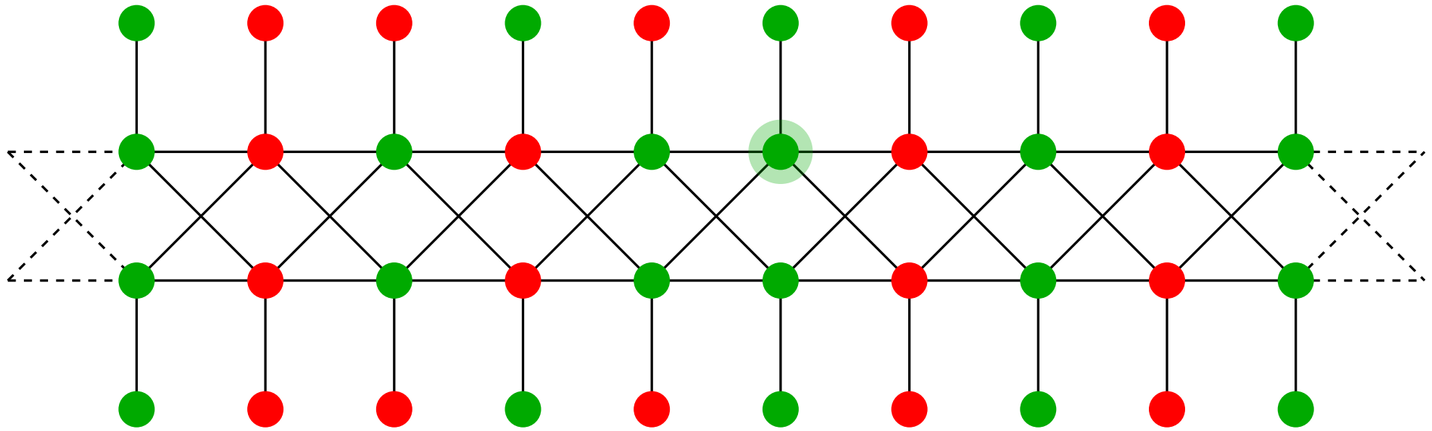

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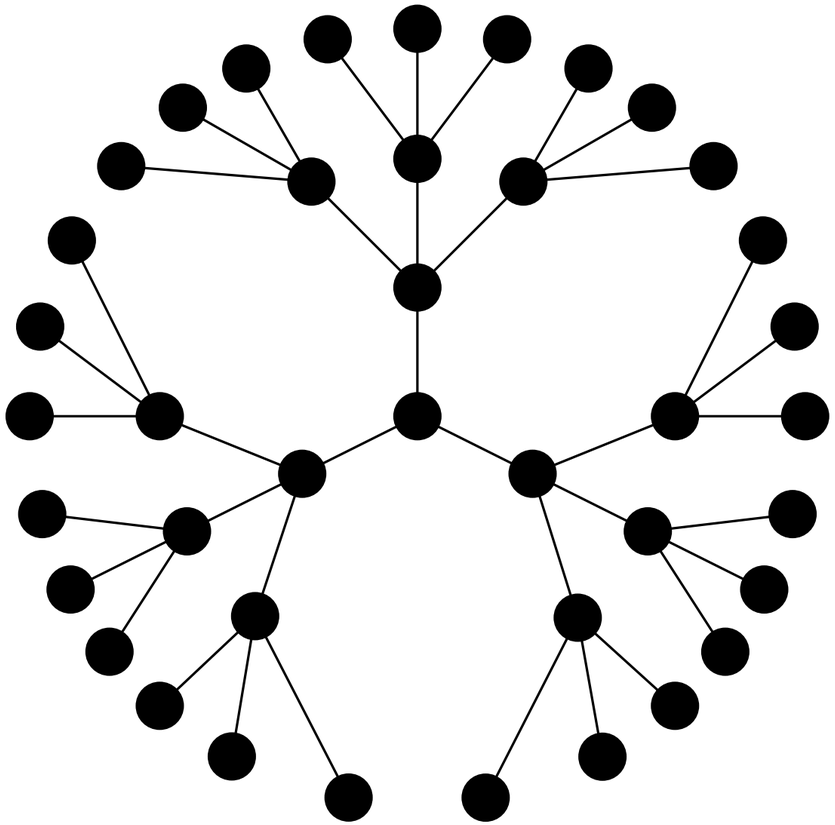
- With distributed algorithms?

The LOCAL model

[Linial FOCS ’87 & SICOMP ’92]

- **Distributed network** of n processors/nodes
  - graph $G = (V, E)$ with $|V| = n$
  - E: communication links
  - each node in V runs the same algorithm



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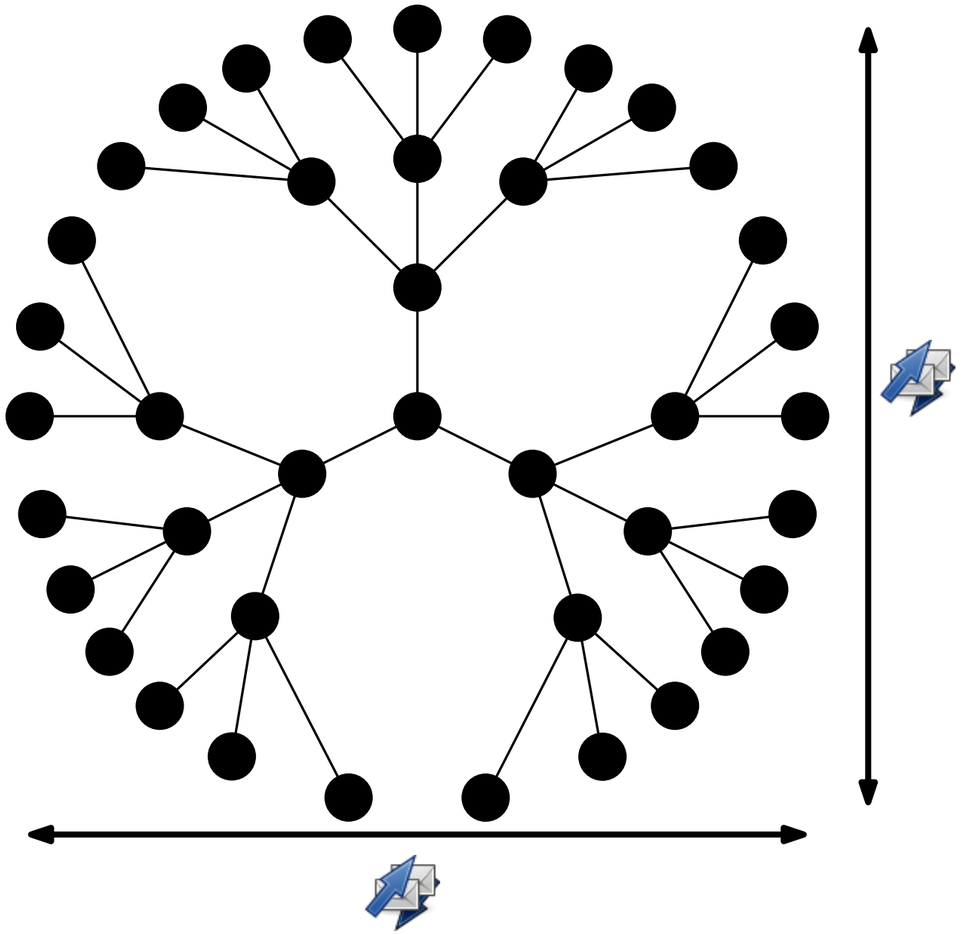

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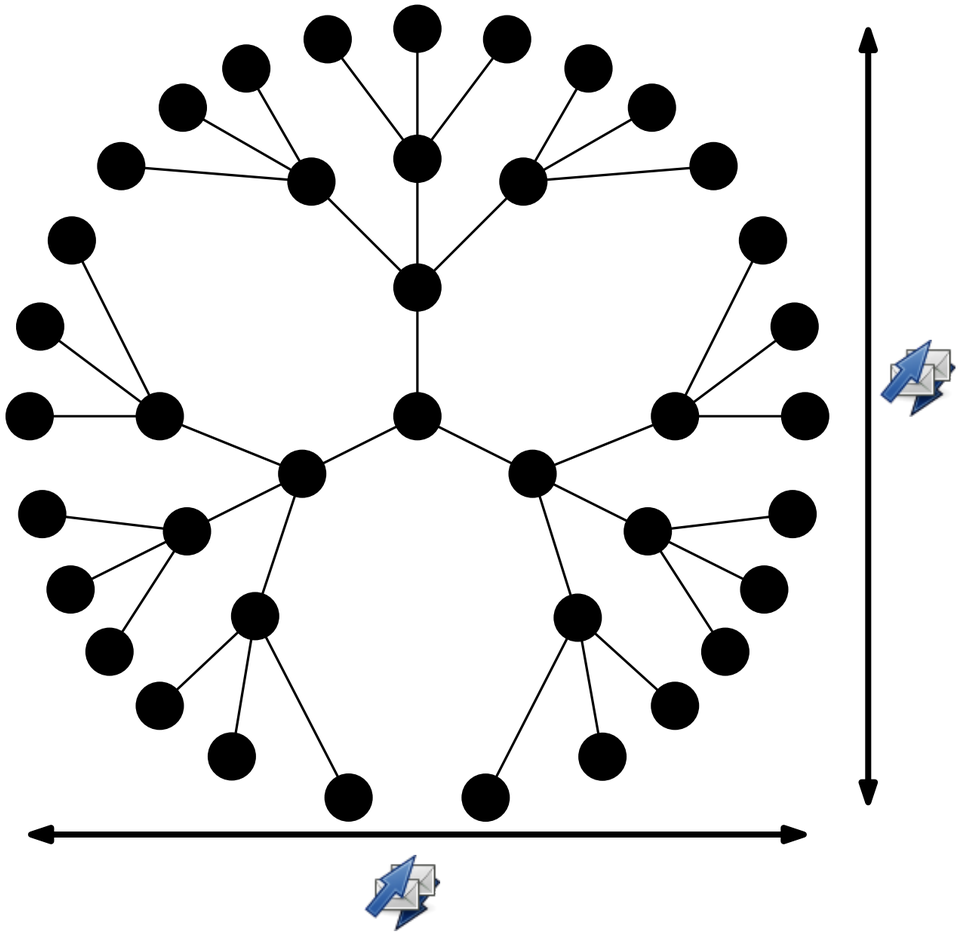

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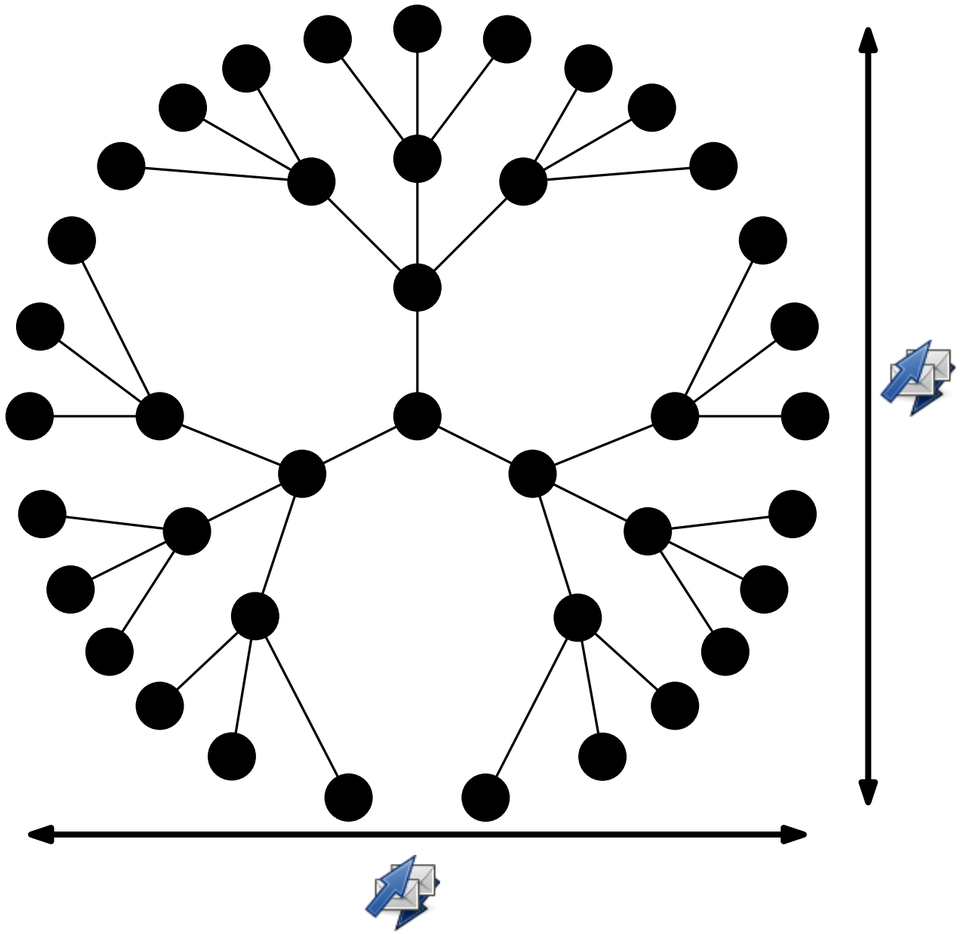
- **Complexity measure**: number of communication rounds



Previous results about LOC

Lower bound: - $\Omega(\log n)$-rounds in deterministic LOCAL (in *bounded-degree* trees)

- $\Omega(\log\log n)$-rounds in randomized LOCAL (in *bounded-degree* trees)

- reduction from Sinkless Orientation [Balliu, Hirvonen, Lenzen, Olivetti, and Suomela, SIROCCO '19]
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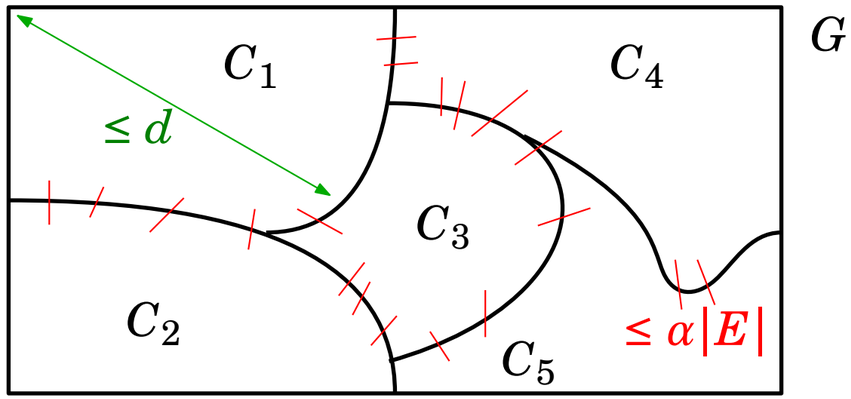
HUGE GAP!

Let's find a *better* distributed algorithm...

Network decomposition

(α, d)-**small boundary decomposition** of a graph $G = (V, E)$:

- partition of V into clusters (sets) $C_1, \ldots, C_k$
- $\text{diam}(C_i) \leq d$ for all i
- # inter-clusters edges $\leq \alpha |E|$



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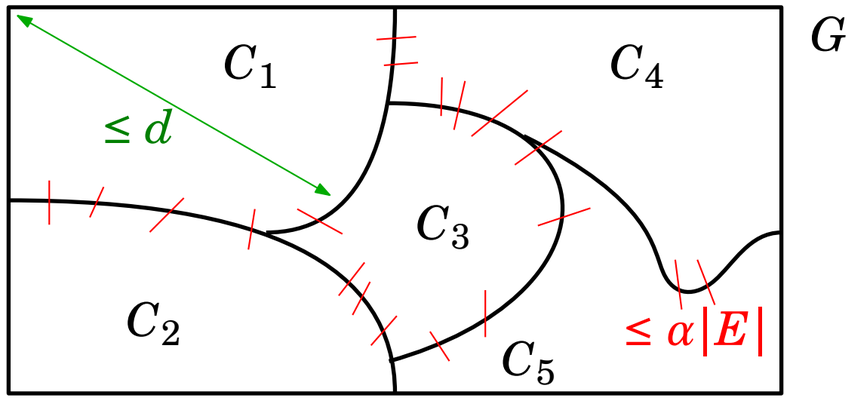
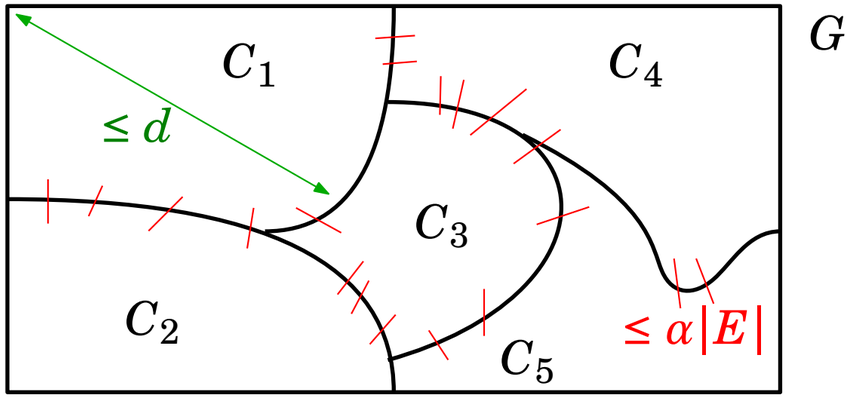


Theorem (adaptation of [Miller, Peng, and Xu, SPAA '13]):

There exists a randomized LOCAL algorithm $\mathcal{MPX}$ that computes an (α, d)-small boundary decomposition of a graph $G = (V, E)$ with the following properties:

- *Running time $O(\log n / \alpha)$.*
- *UB on the clusters' diameter is $d = O(\log n / \alpha)$.*

A “simple” algorithm



- *Brute-force solution with minimum potential in each cluster (time $O(d)$)*

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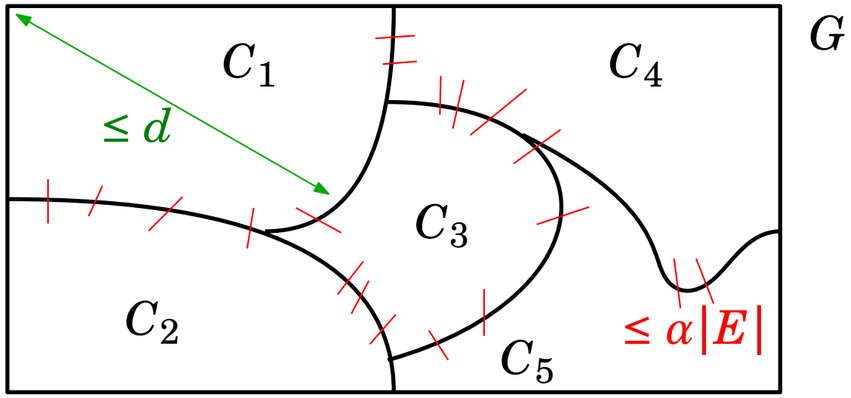

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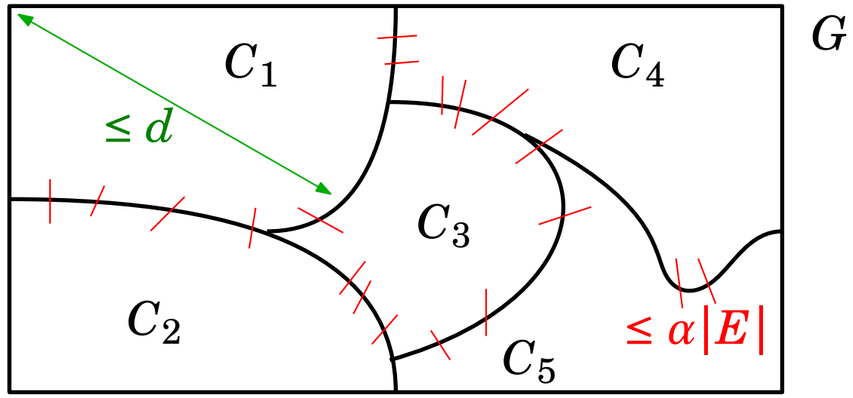

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Overall running time: cost($\mathcal{MPX}$) + $O(d) + O(\alpha|E|) = O(\log n/\alpha + \alpha|E|)$

- **max-degree** Δ: running time $O(\sqrt{\Delta n \log n})$ (minimized by $\alpha = \sqrt{\log n/(\Delta n)}$)

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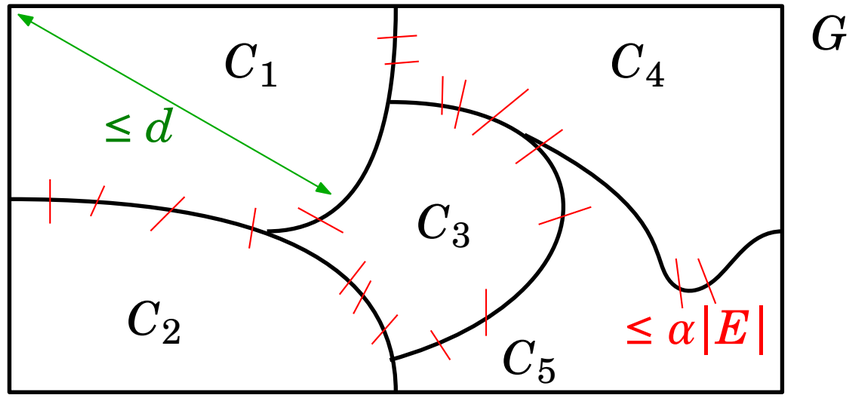

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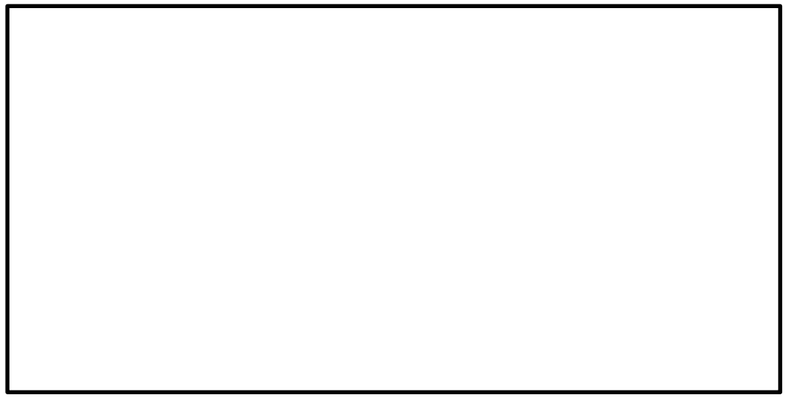
- still far from the lower bounds … How to do better?

Repeating “brute-force” does not work



- *Repeat:*

  - *Run $\mathcal{MPX}$ to get (α, d)-network decomposition (time $O(d)$)*

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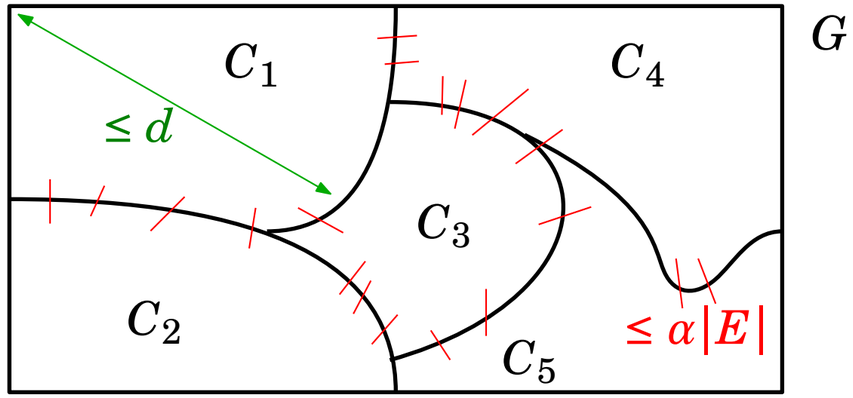

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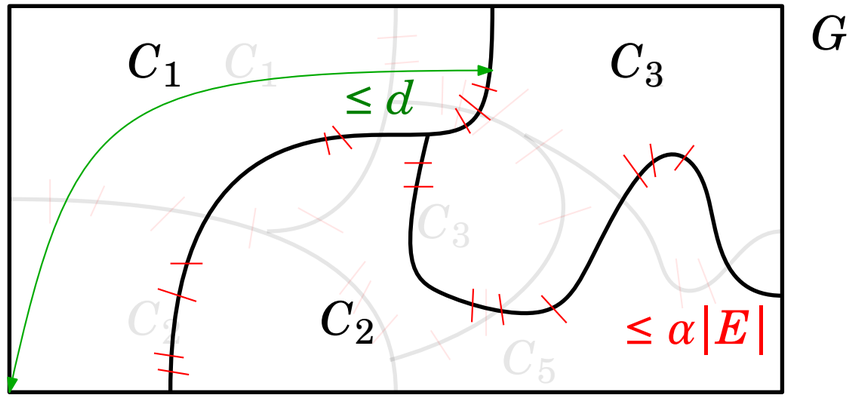

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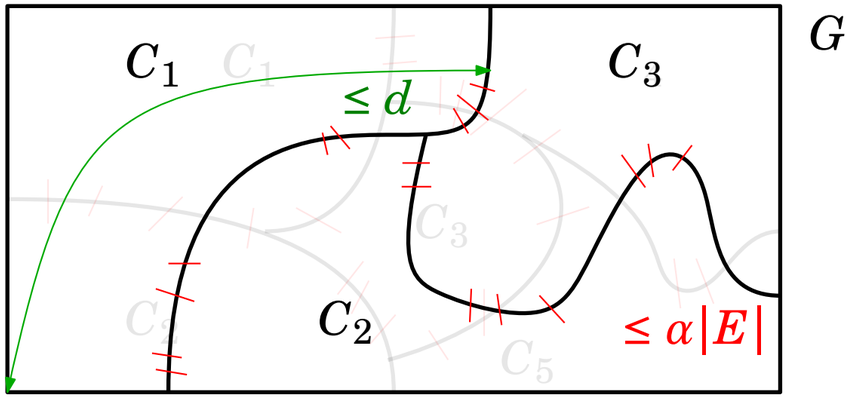

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(c,d)-**colored decomposition** of a graph $G = (V,E)$:

- partition of V into clusters (sets) $C_1,\dots,C_k$
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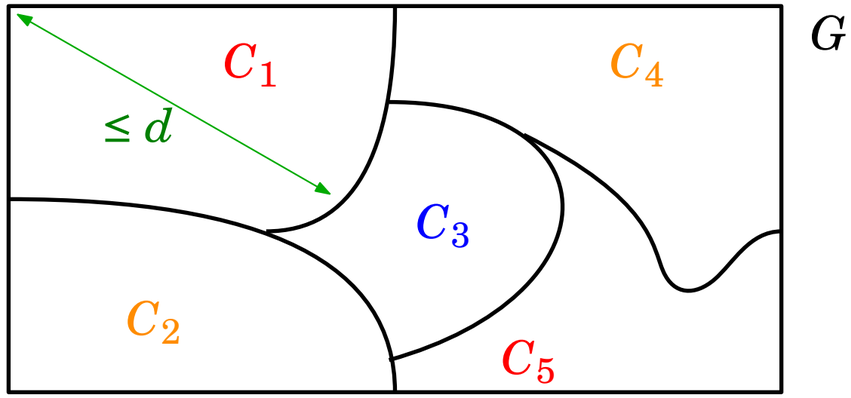

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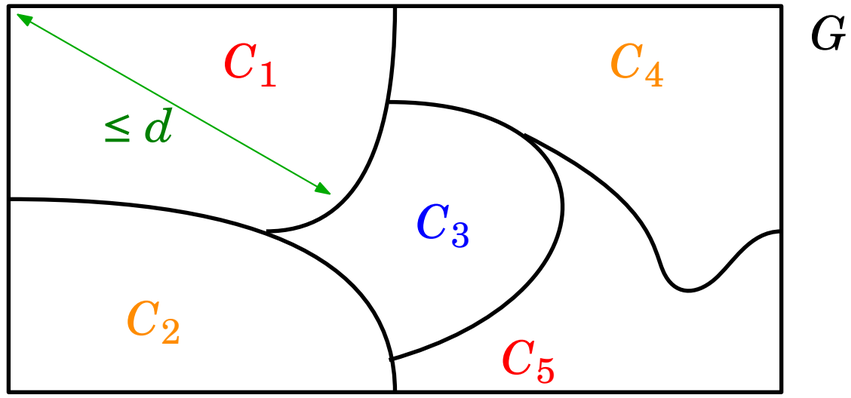

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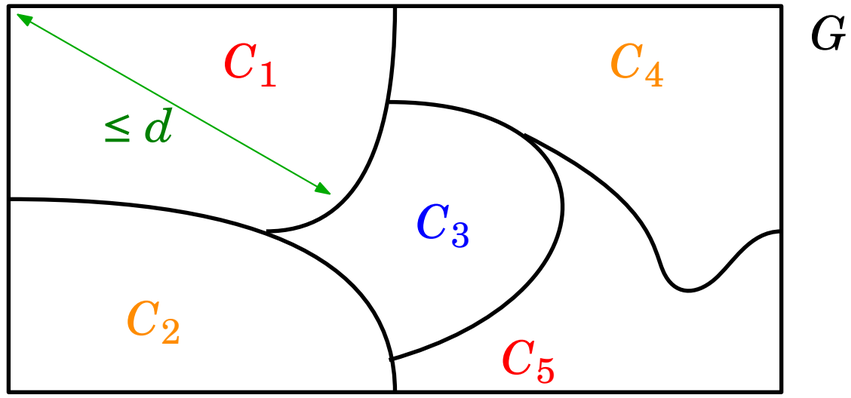

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(c,d)-**colored decomposition** of power graph G^r:

- $\text{diam}(C_i) \le rd$ for all i
- if C_i, C_j same color, $\text{dist}(C_i, C_j) \ge r$ *# guarantee that clusters of the same color are far enough*
- running time multiplied by r

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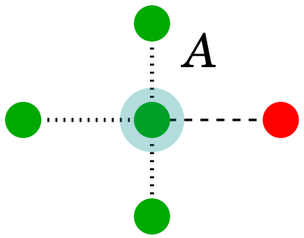

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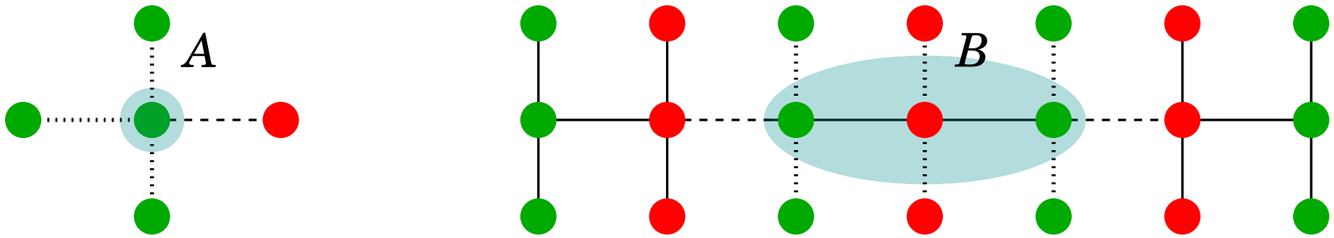

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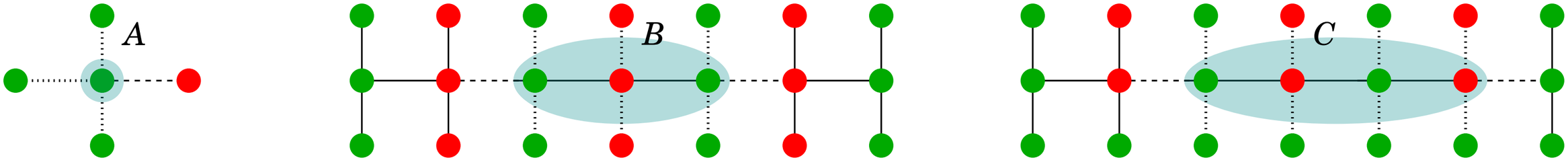

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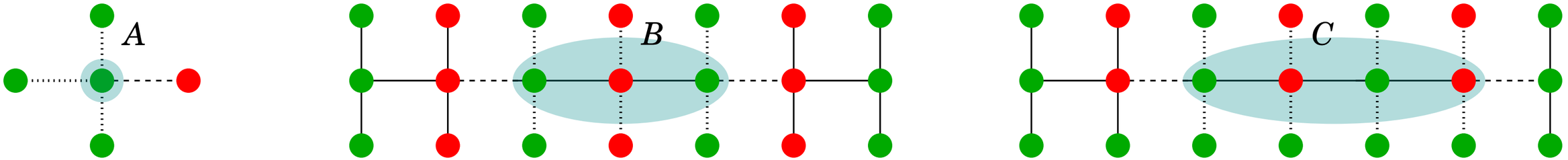

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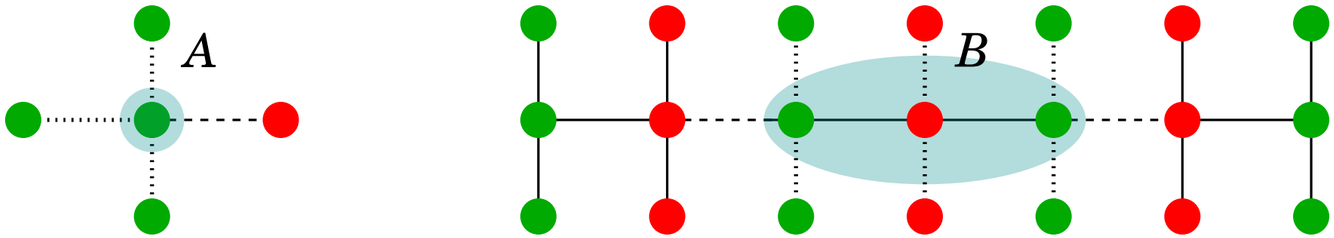

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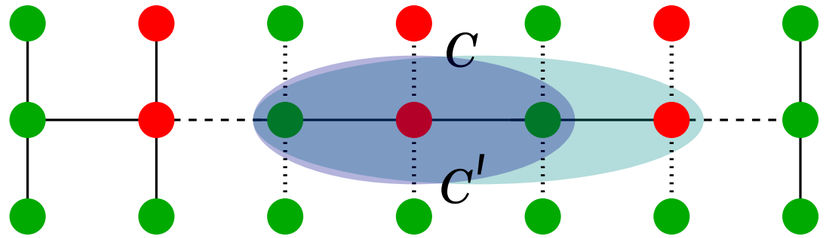

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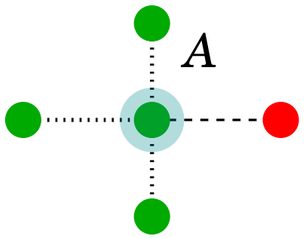

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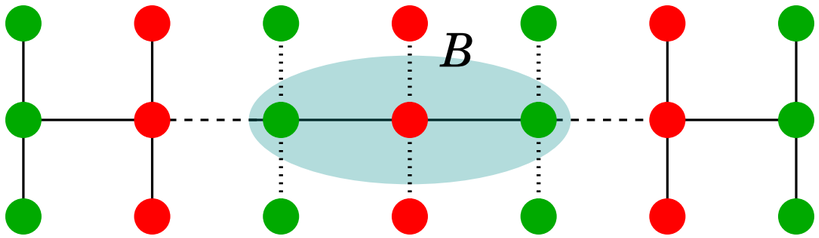

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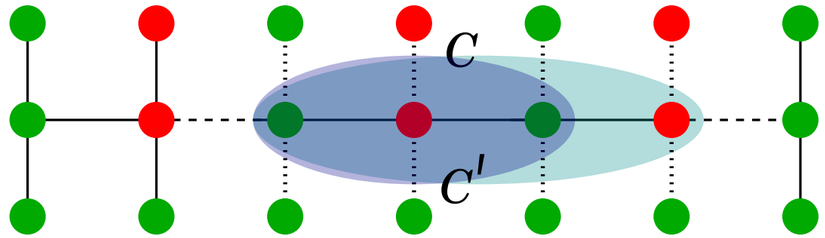

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Error: minimal improving set of $\text{IR} \geq 1$

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Property 1: **local witness** of **minimal improving sets**

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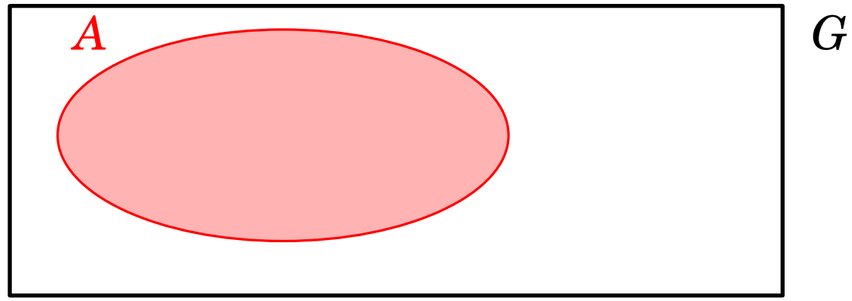

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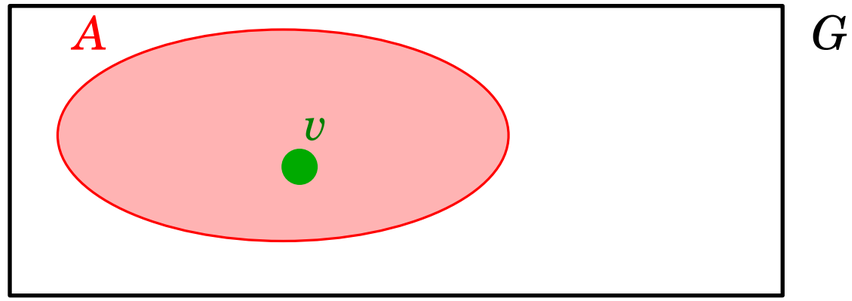

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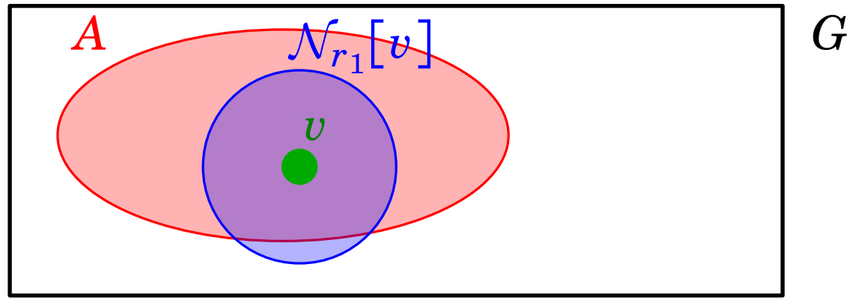

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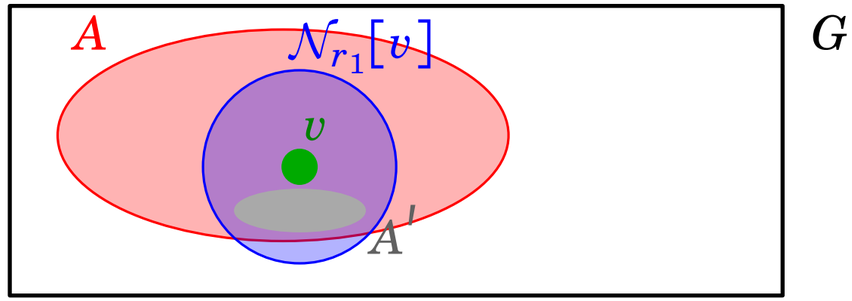

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Property 1: **local witness** of **minimal improving sets**

- $A \subseteq V$ minimal improving set
- $\mathrm{IR}(A) \geq x$
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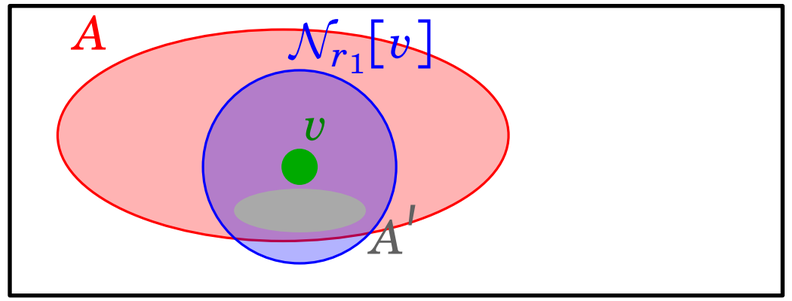

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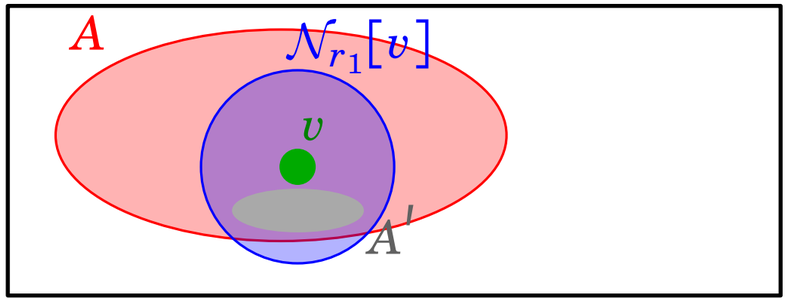

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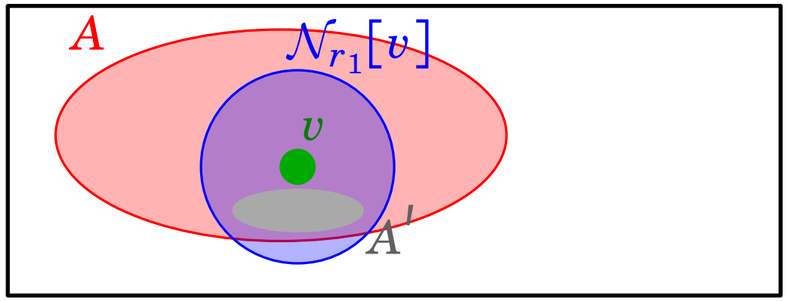

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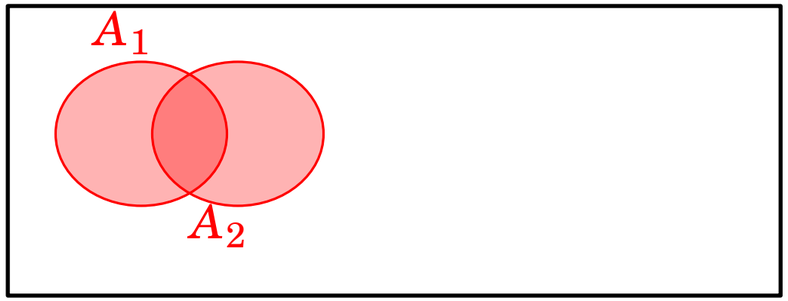

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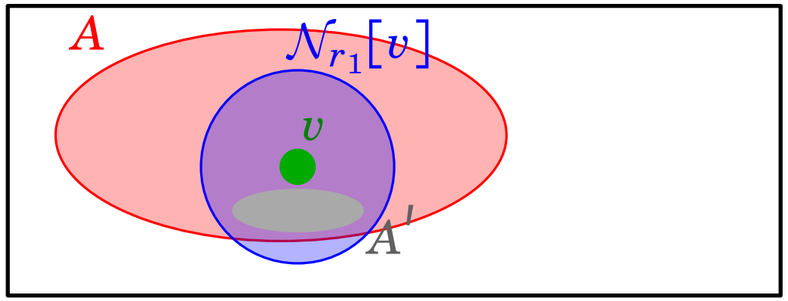

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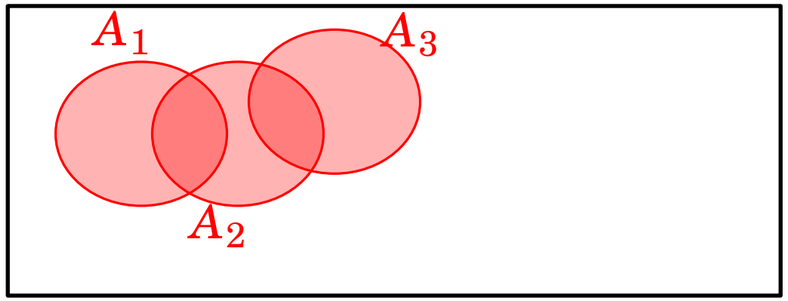

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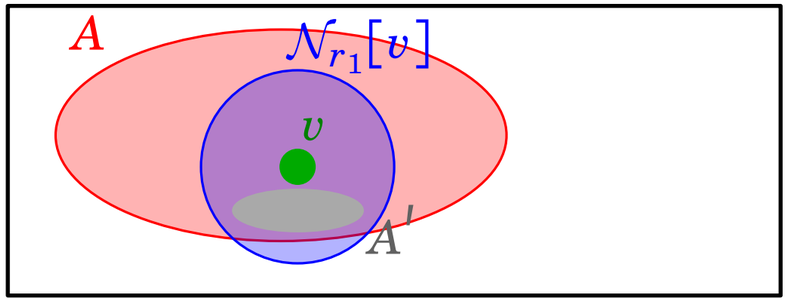

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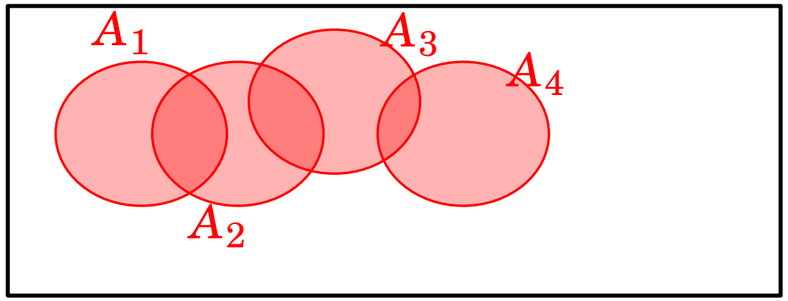

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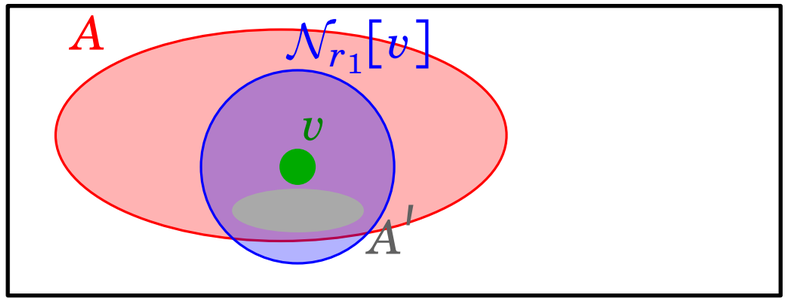

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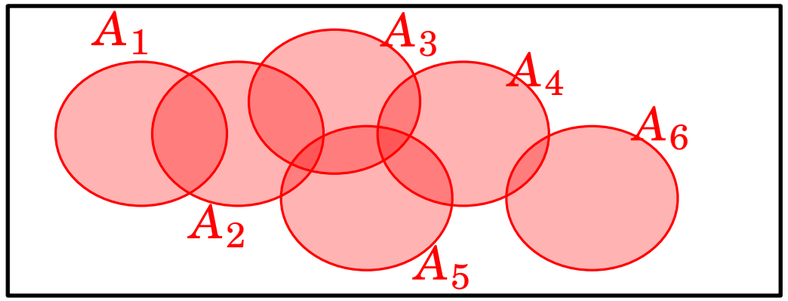

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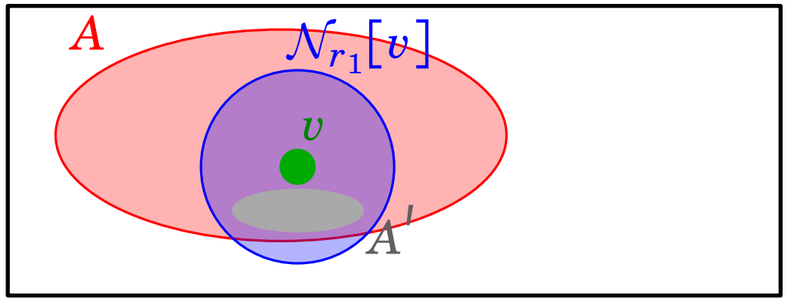

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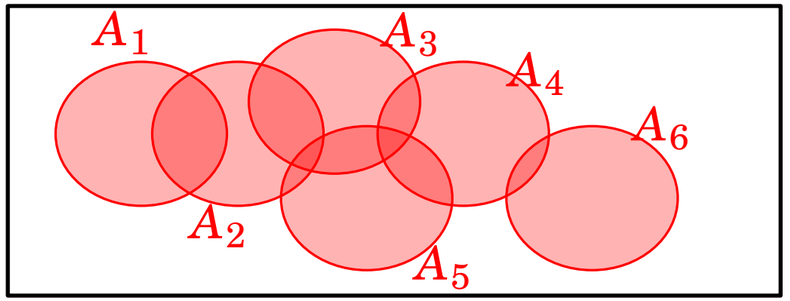

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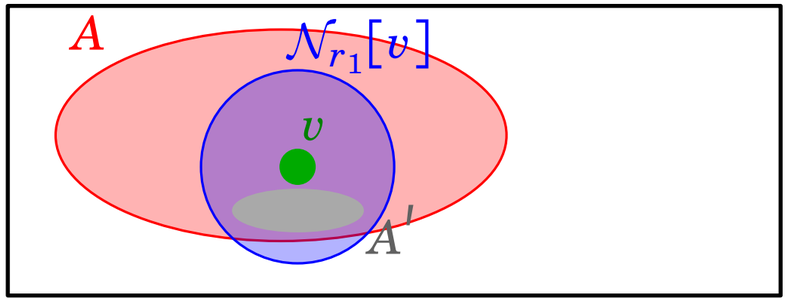

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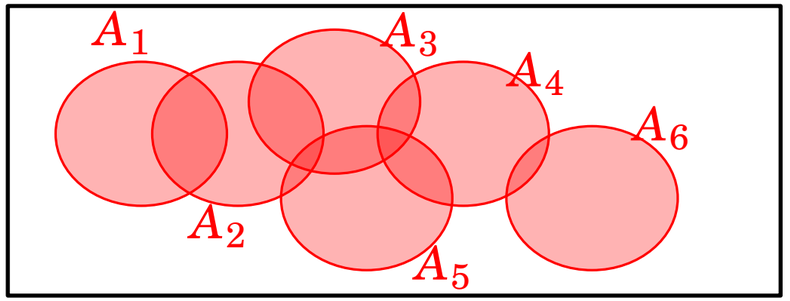

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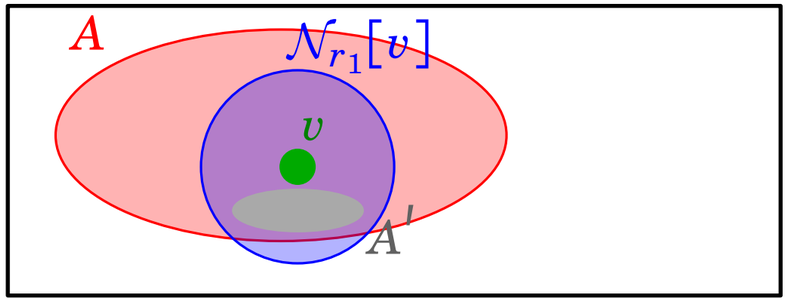

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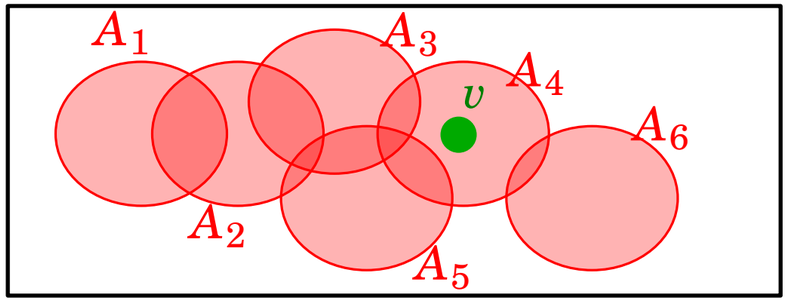

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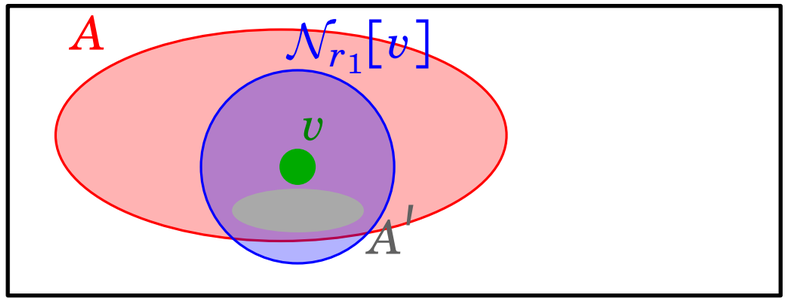

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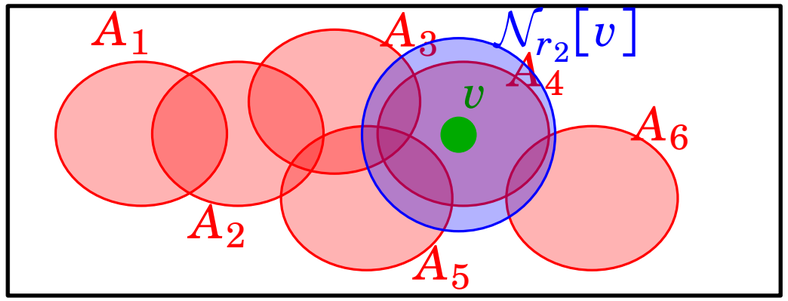

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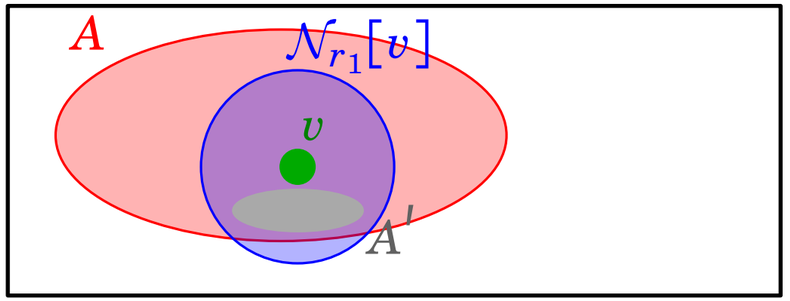

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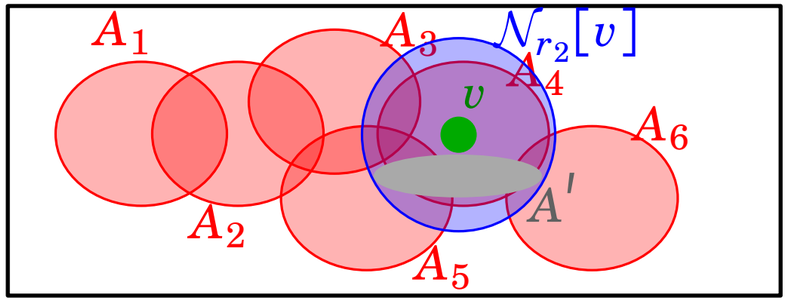

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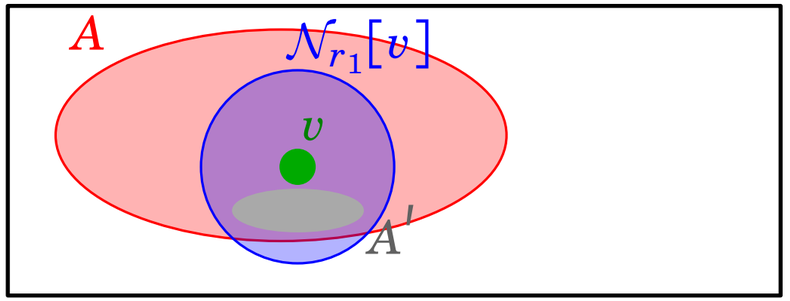

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- $A_1, \ldots, A_k \subseteq V$ sequence of x-improving sets
- $\varepsilon < x$
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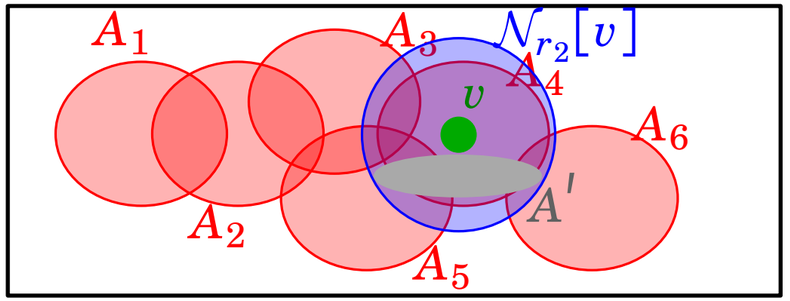

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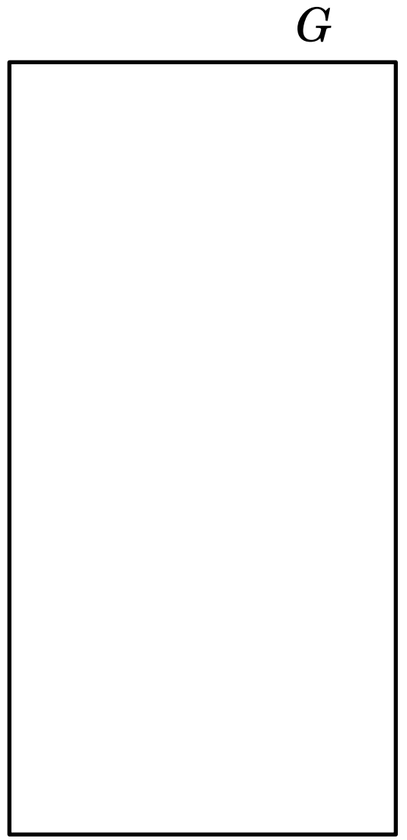

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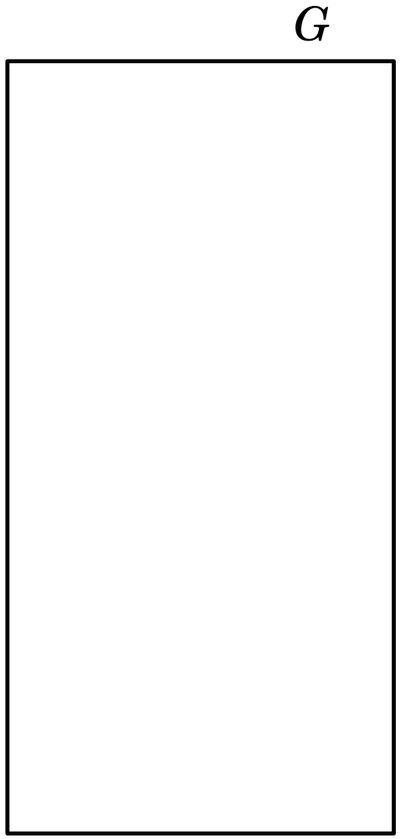
- IR:

- [Ghaffari and Grunau, FOCS '24]:
  $(C_1 \log n, C_2 \log n)$-colored decomposition



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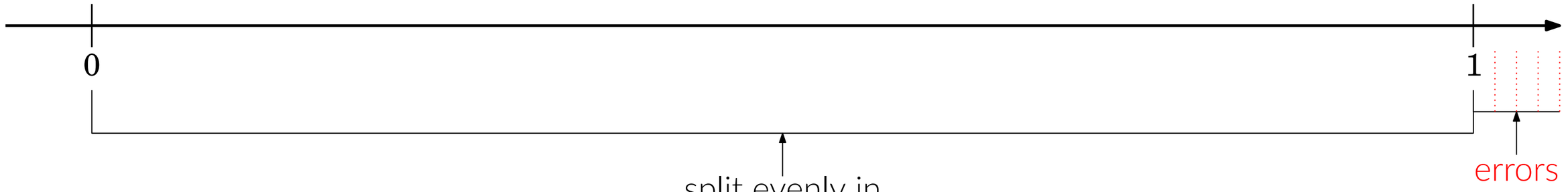

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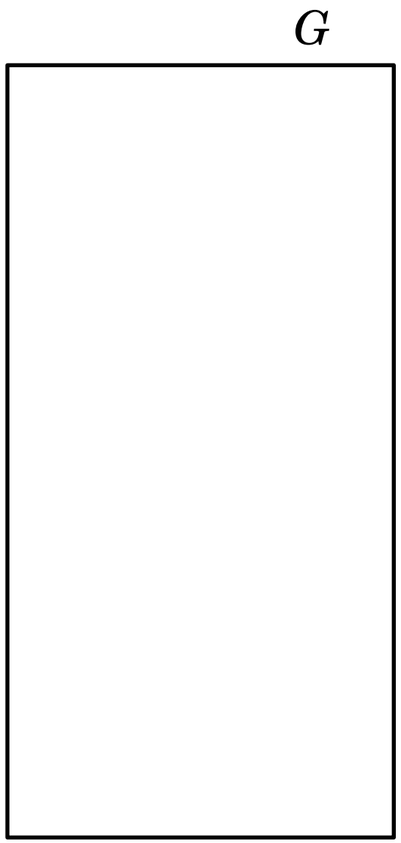

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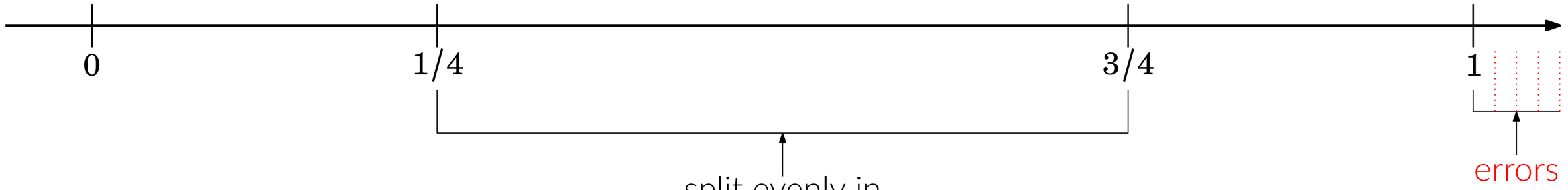

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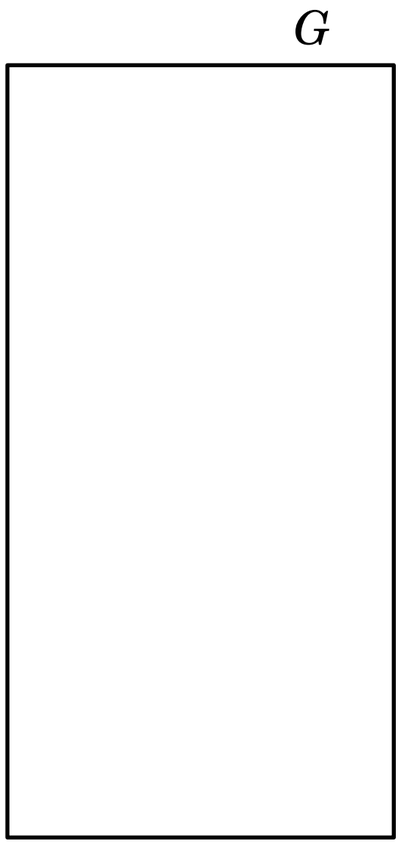

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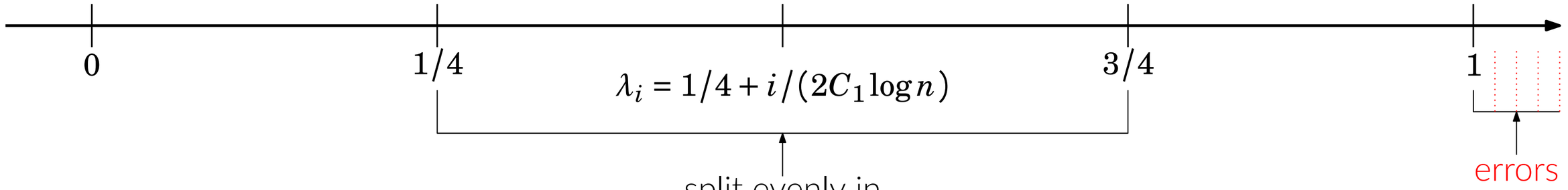

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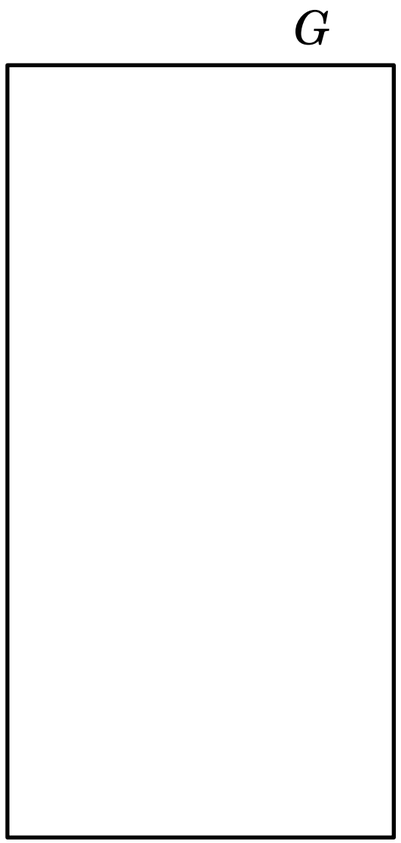

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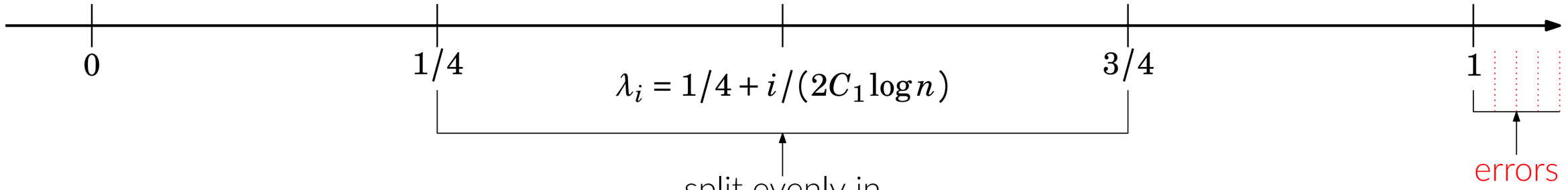

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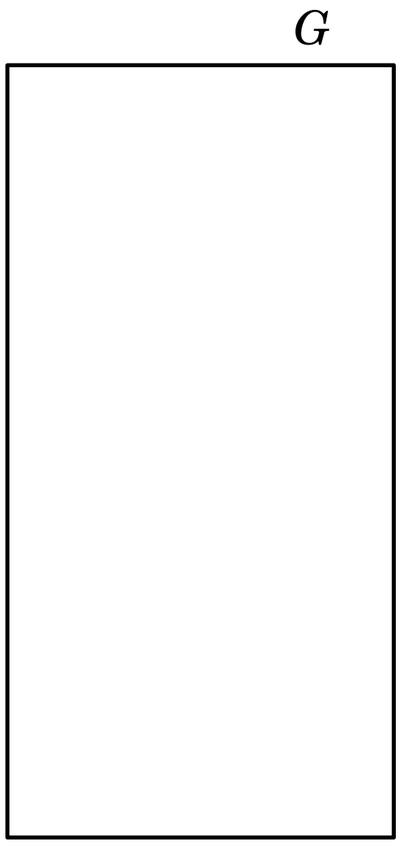
- in phase i, we will **flip improving sets** with IR ≥ λ_i *around* **class color** i



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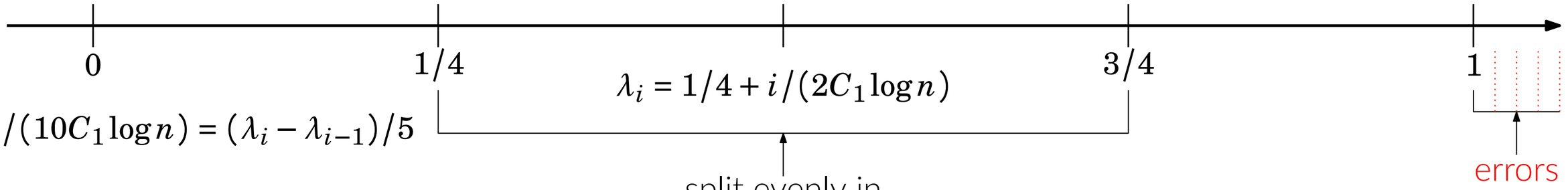

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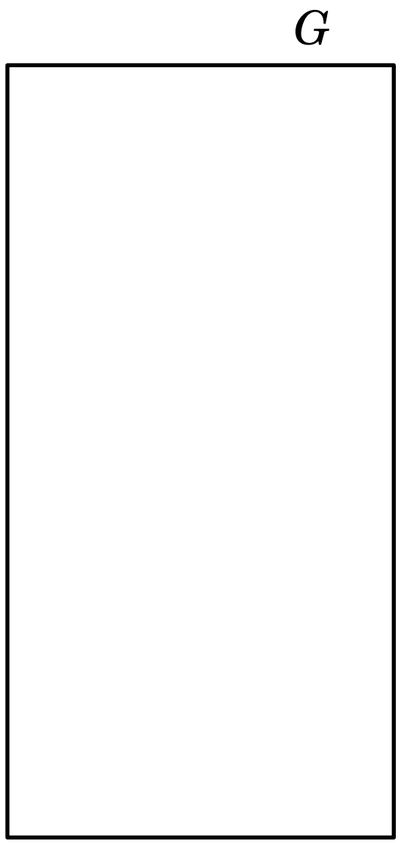

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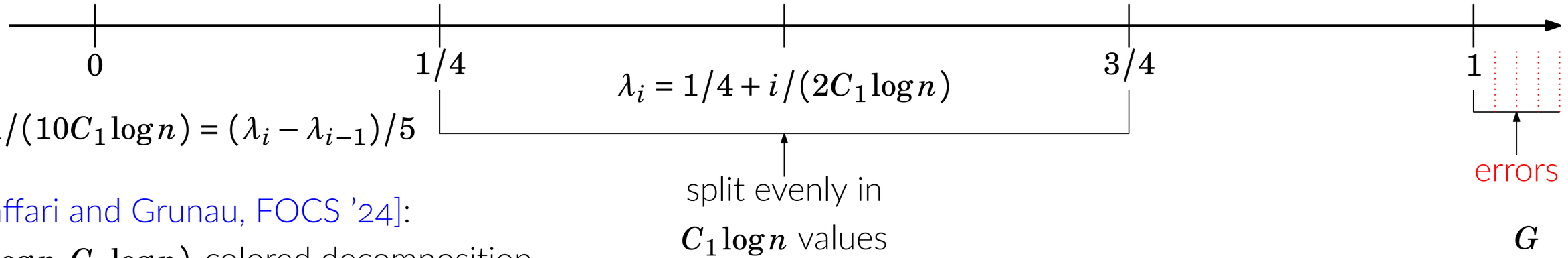

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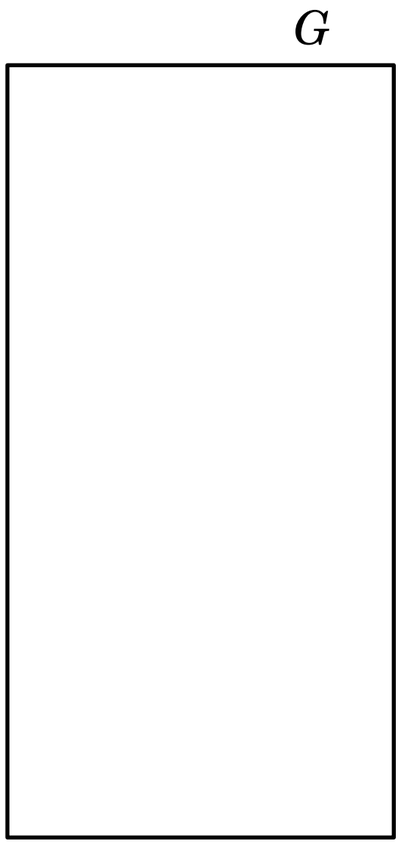

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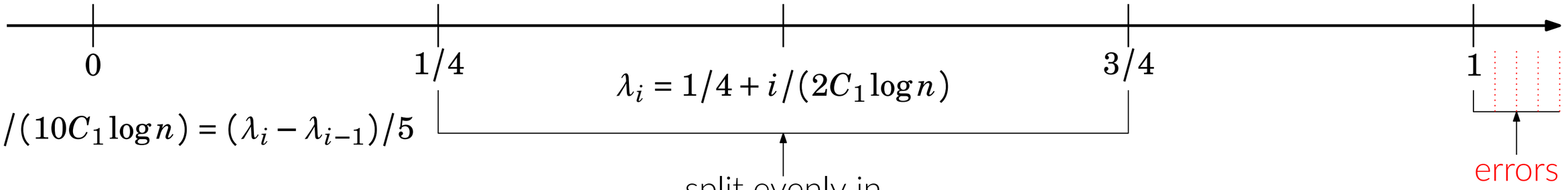

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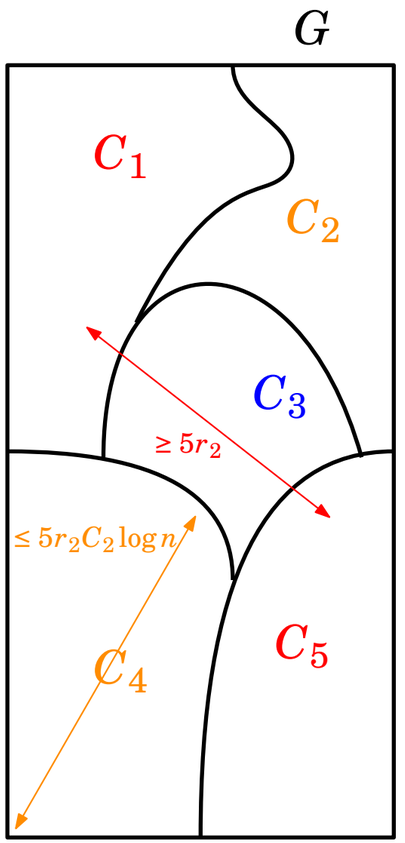
1. **Compute** $(C_1 \log n, C_2 \log n)$-colored decomposition of G^{5r_2}
   - clusters of same color have pairwise distance $\geq 5r_2$



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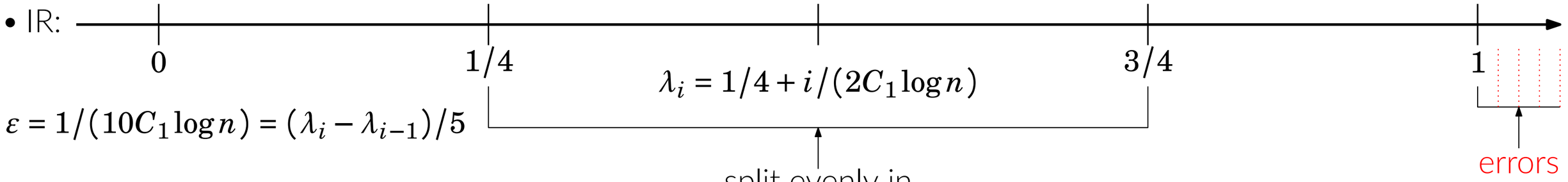

- $\varepsilon = 1/(10C_1 \log n) = (\lambda_i - \lambda_{i-1})/5$

- [Ghaffari and Grunau, FOCS '24]:
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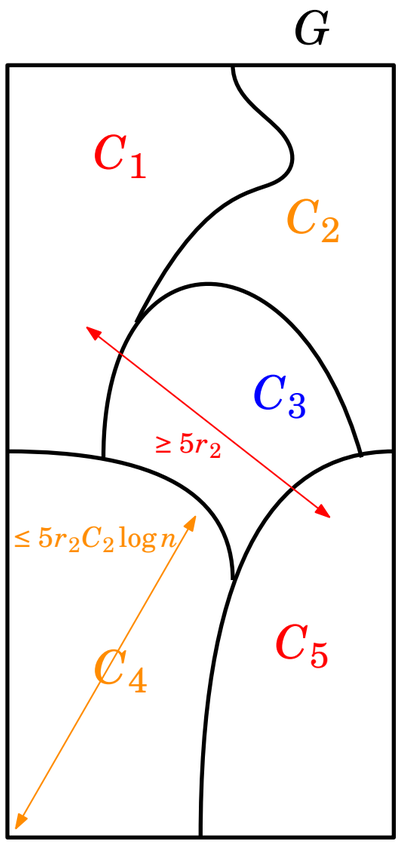

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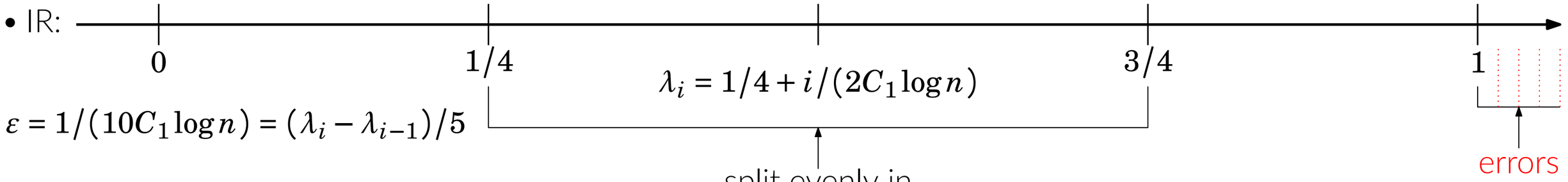

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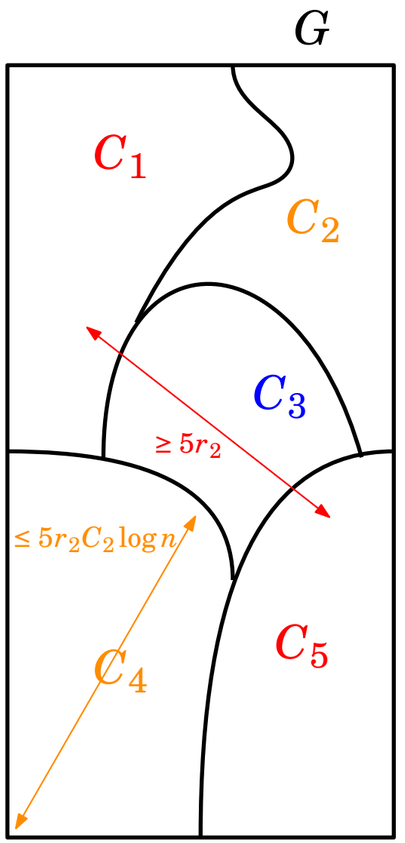

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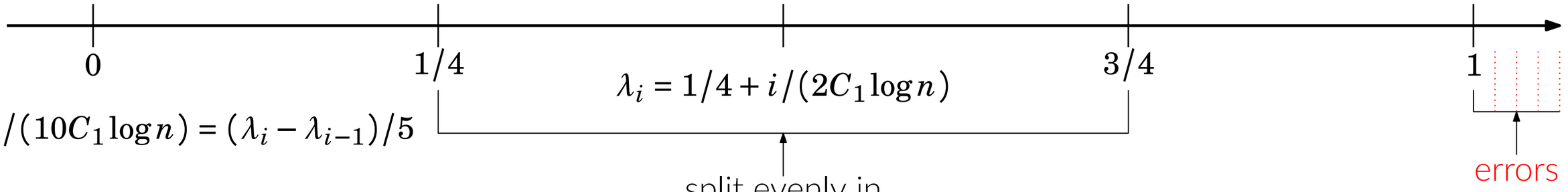

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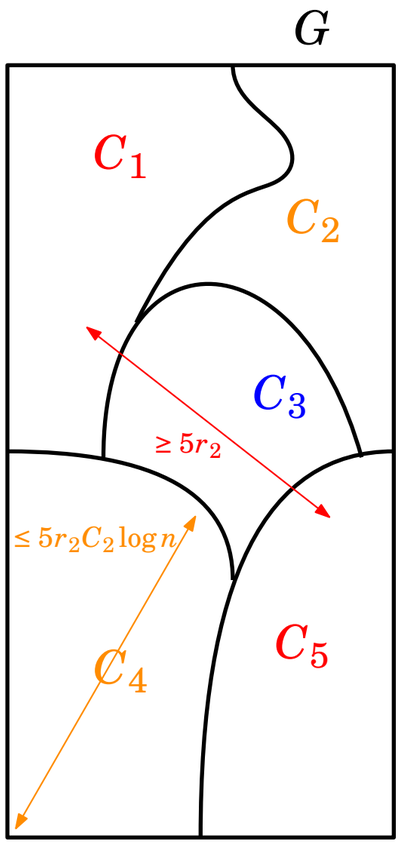

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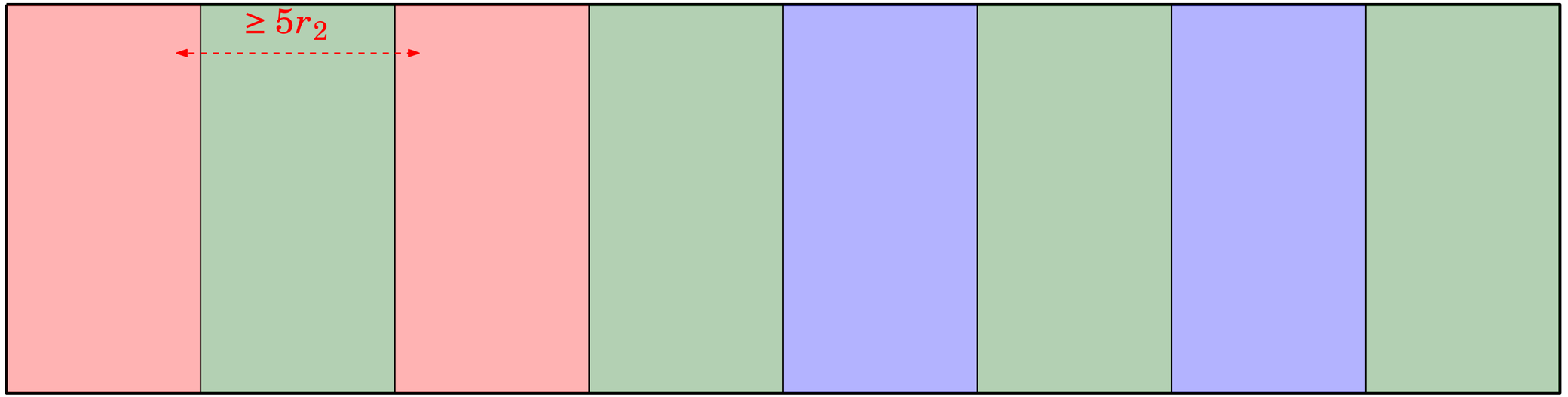

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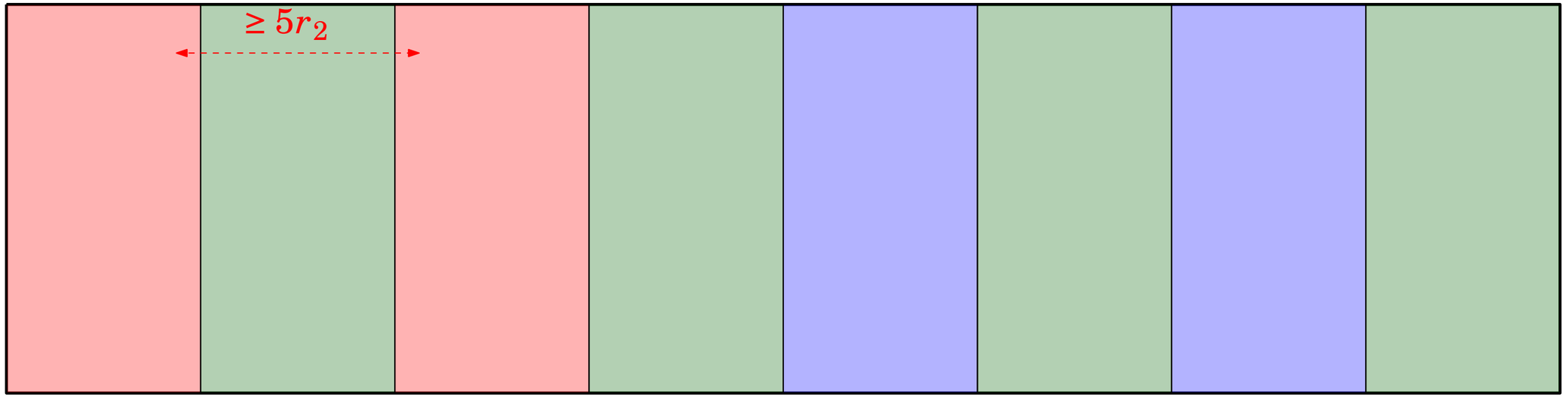

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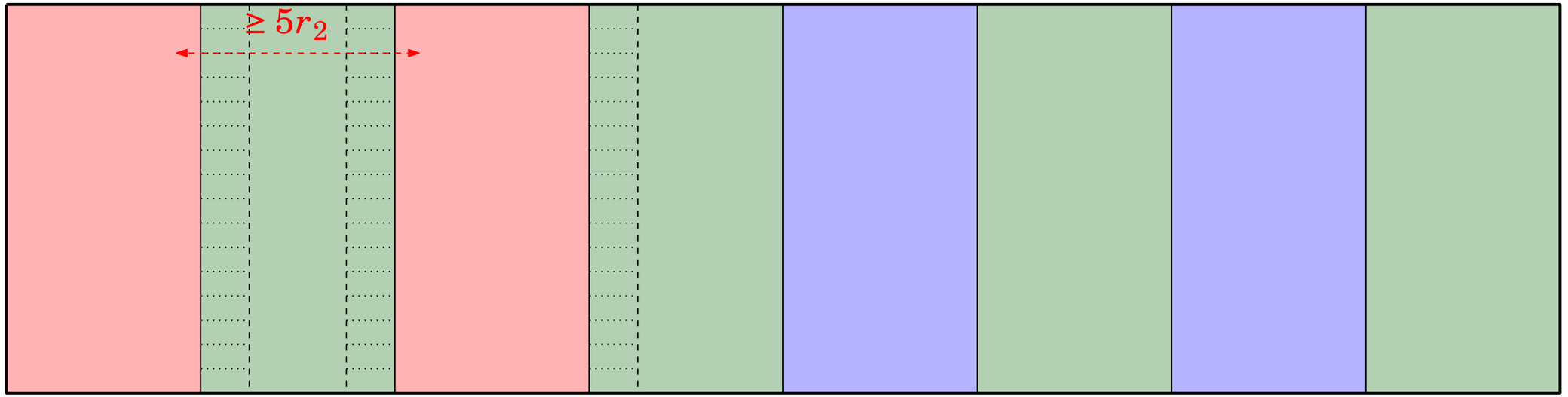

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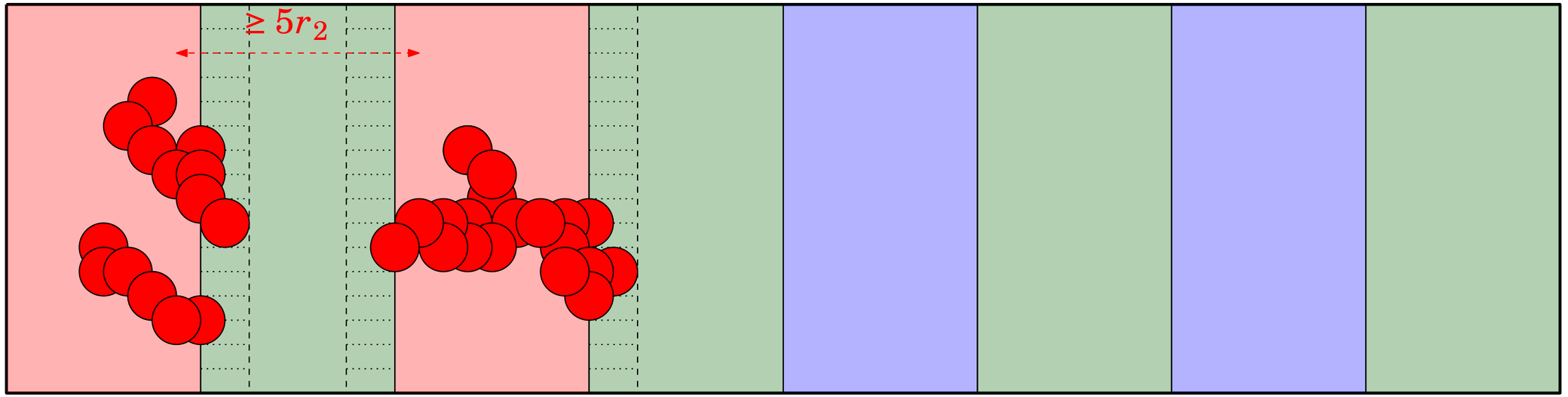

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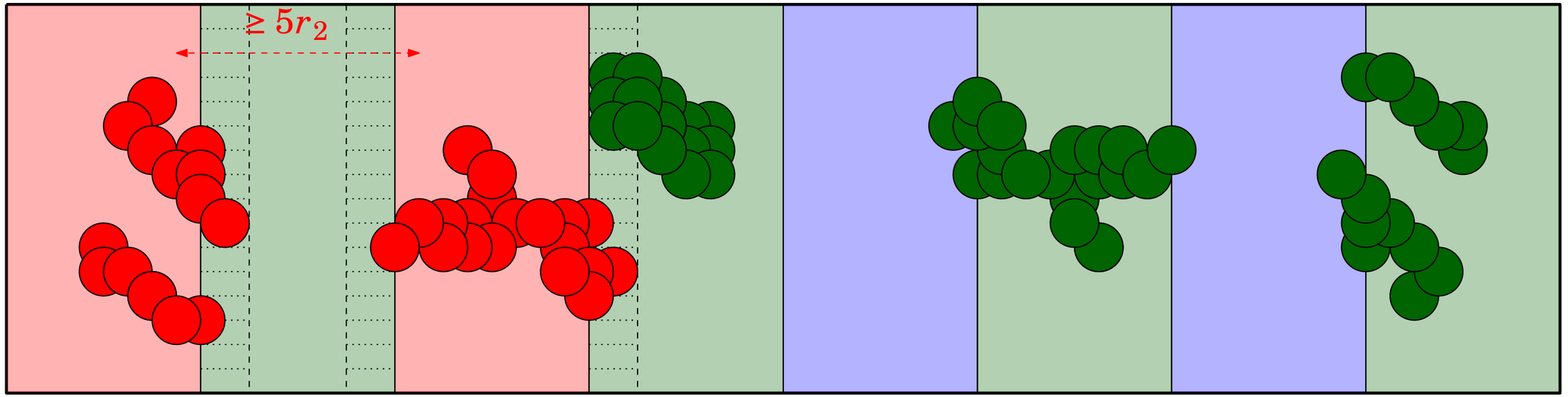

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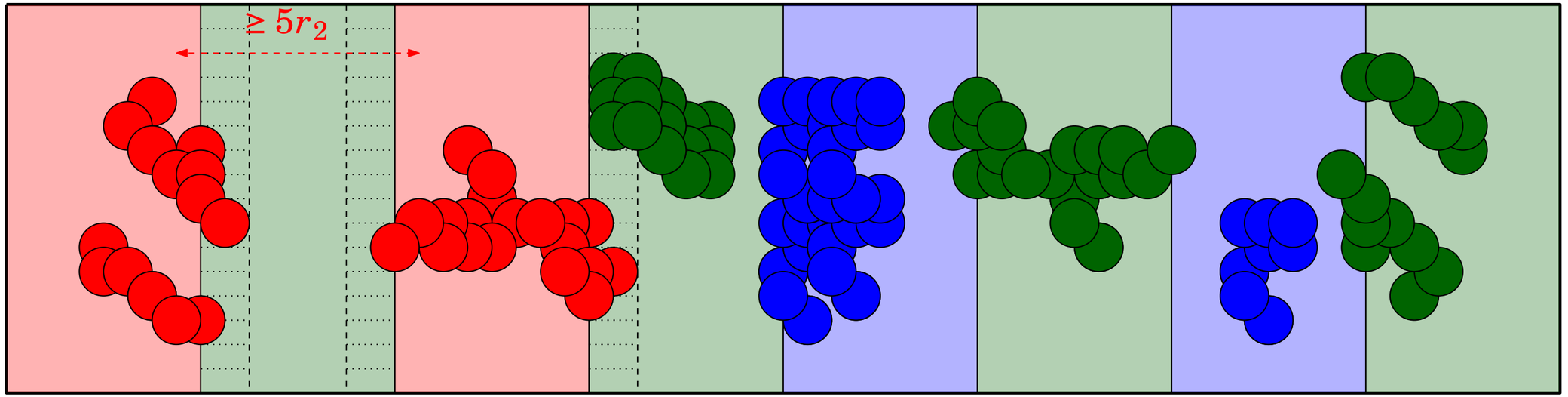

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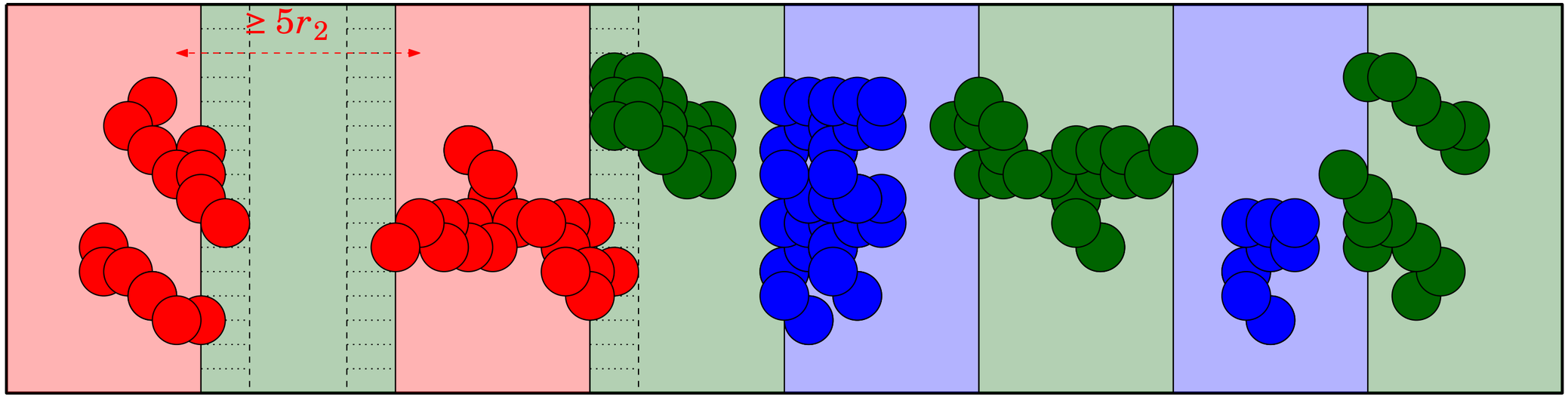

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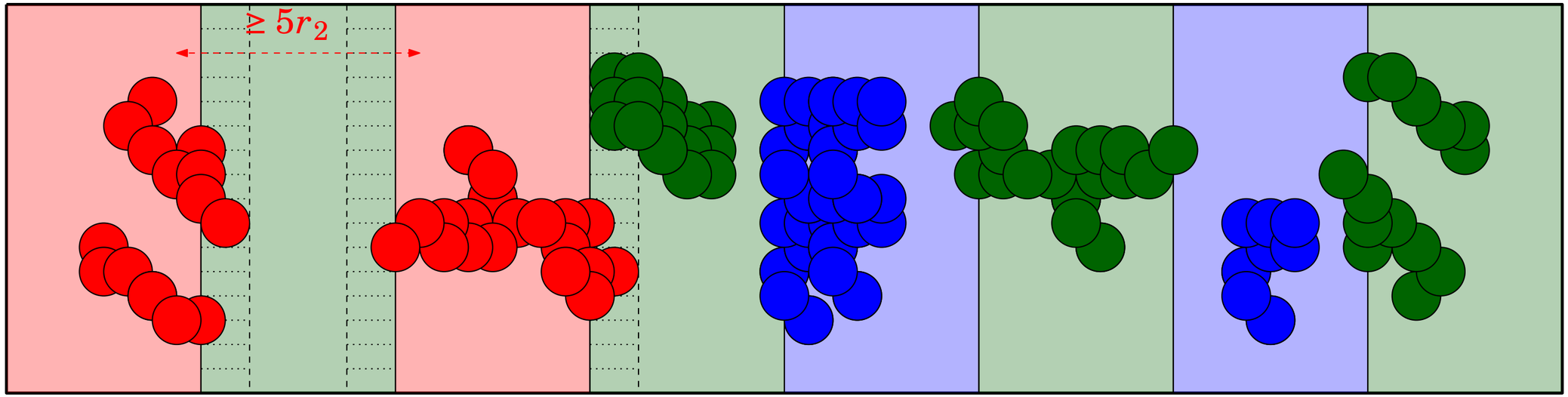

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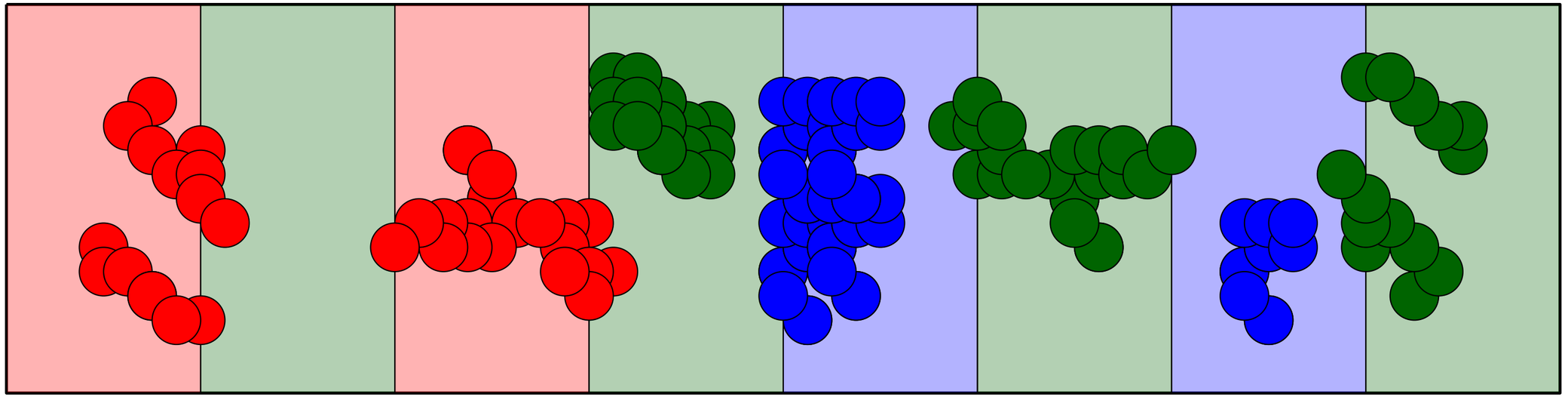

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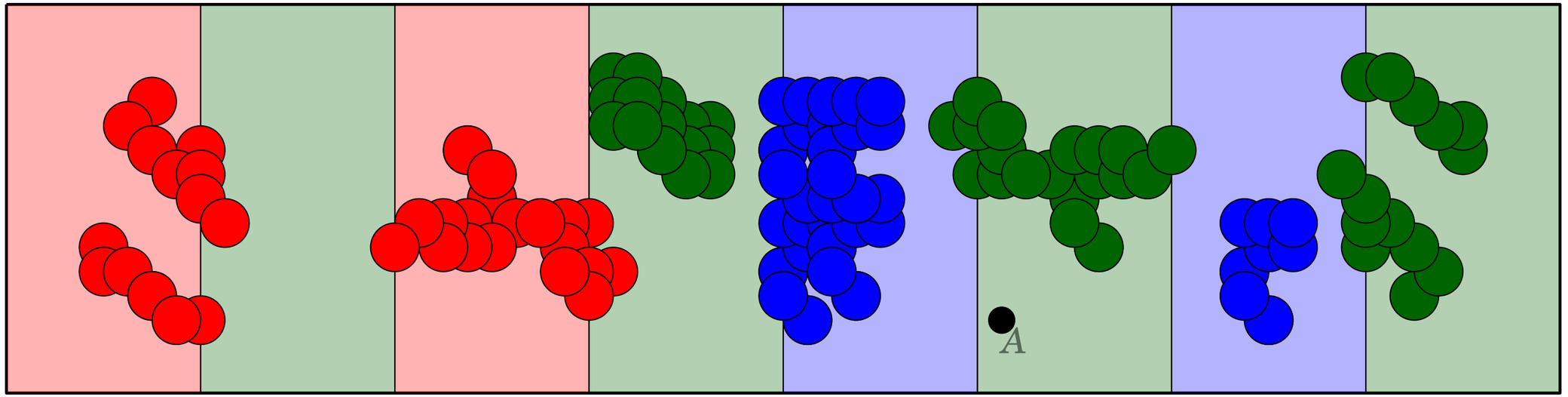

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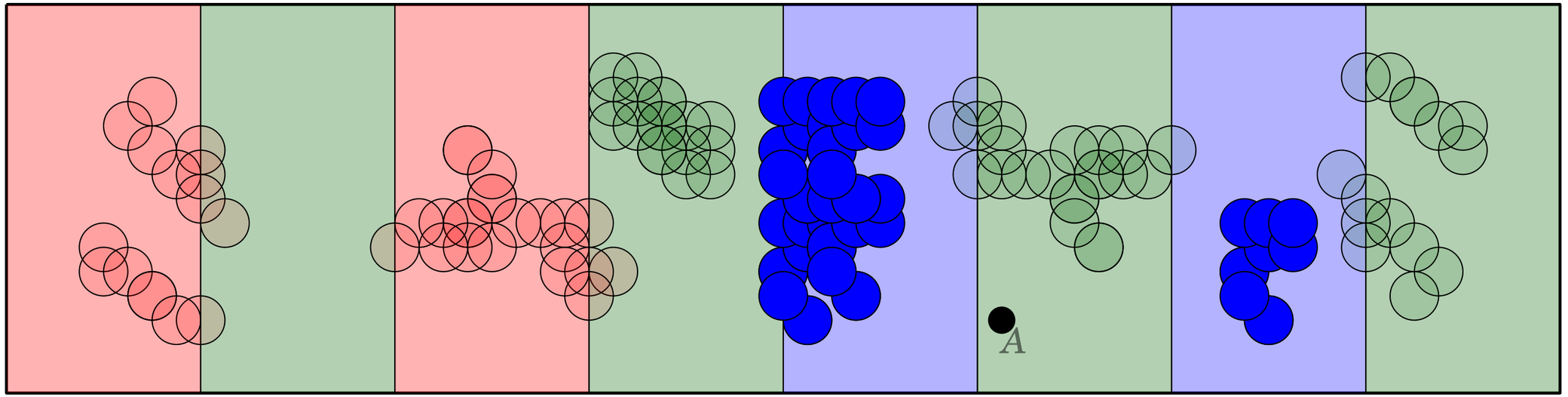

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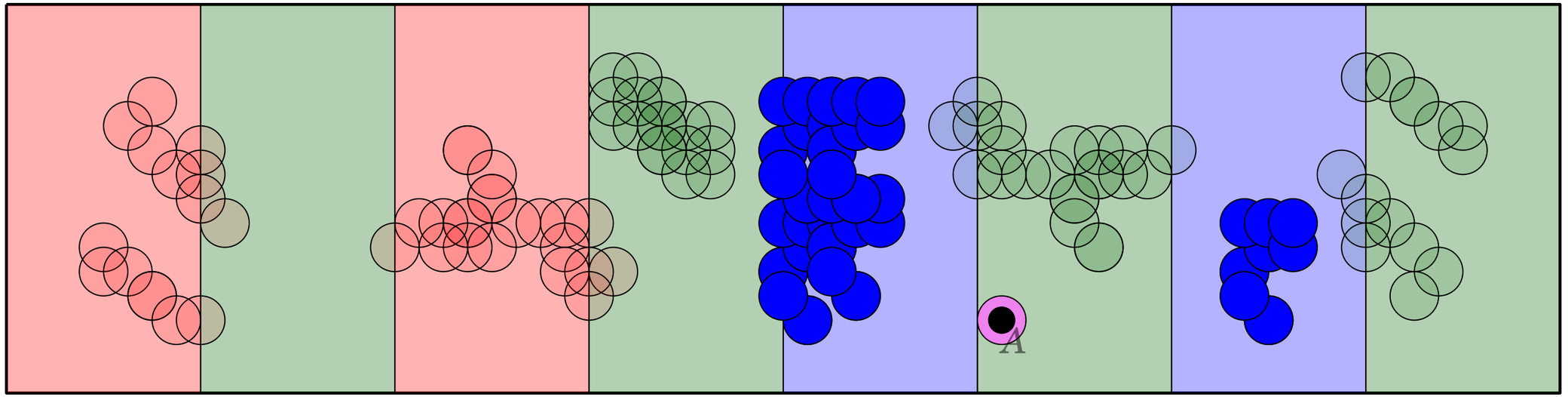

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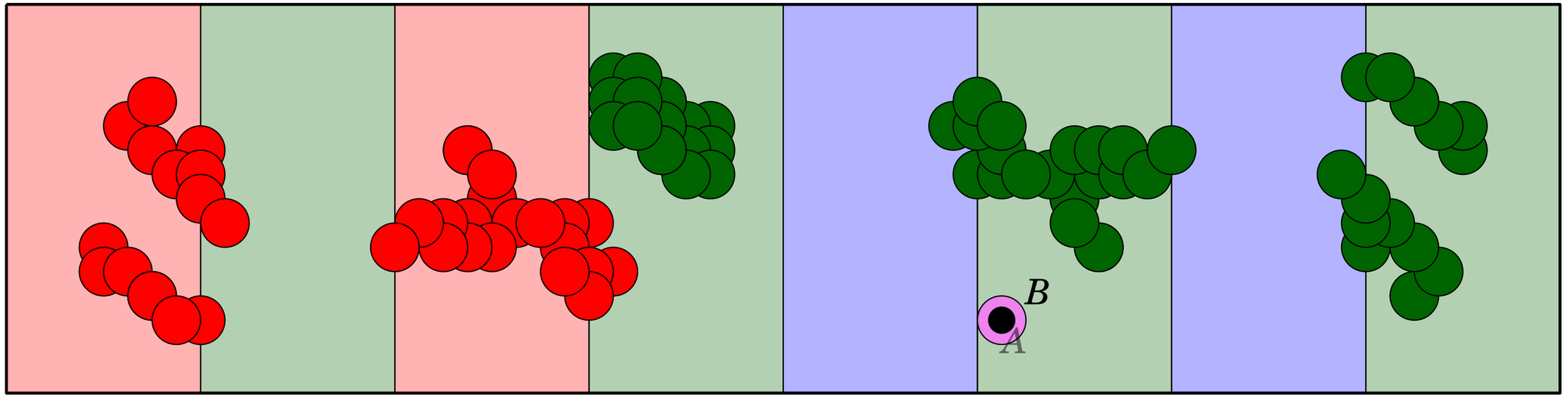

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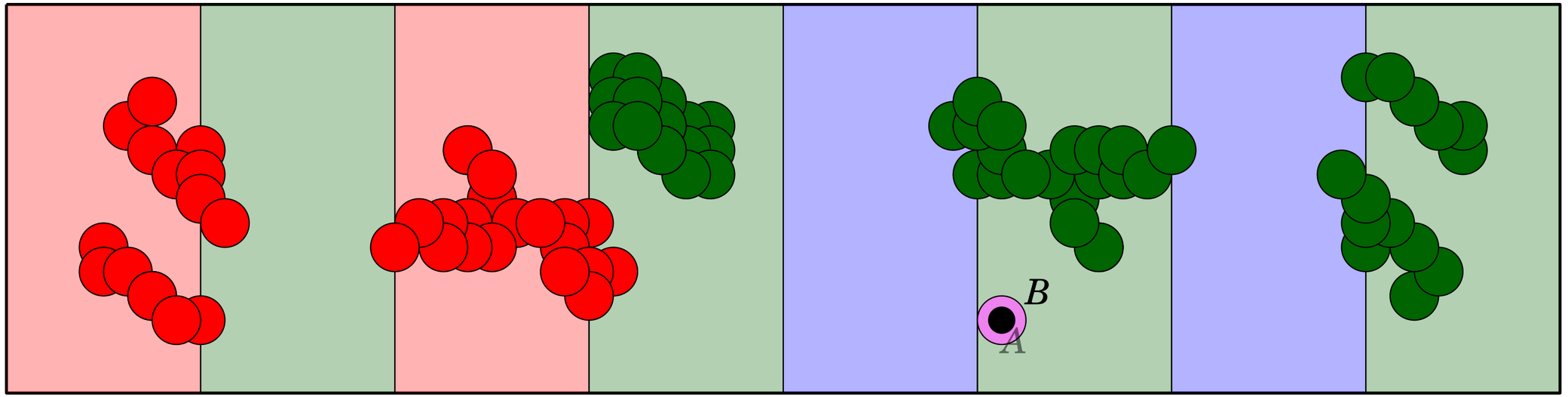

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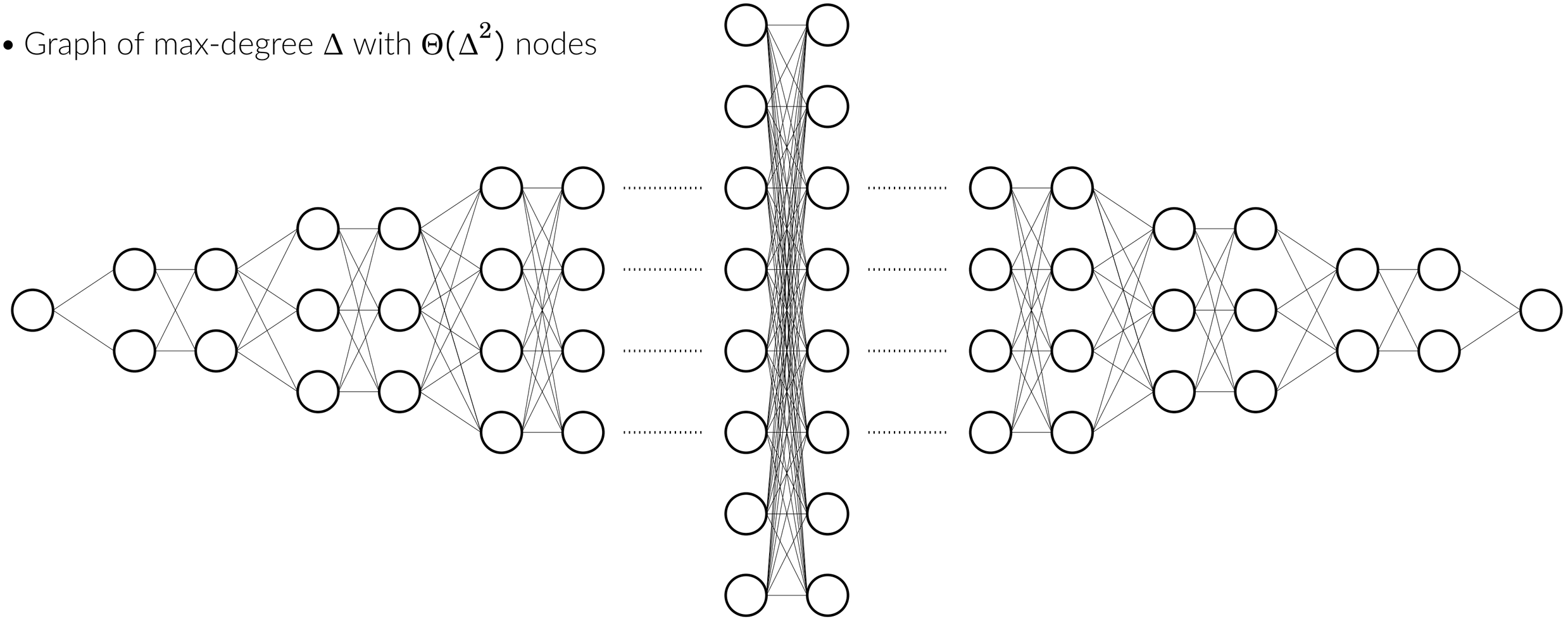
Lower bound

Idea: construct a graph that *forces* propagation

- Graph of max-degree Δ with $\Theta(\Delta^2)$ nodes



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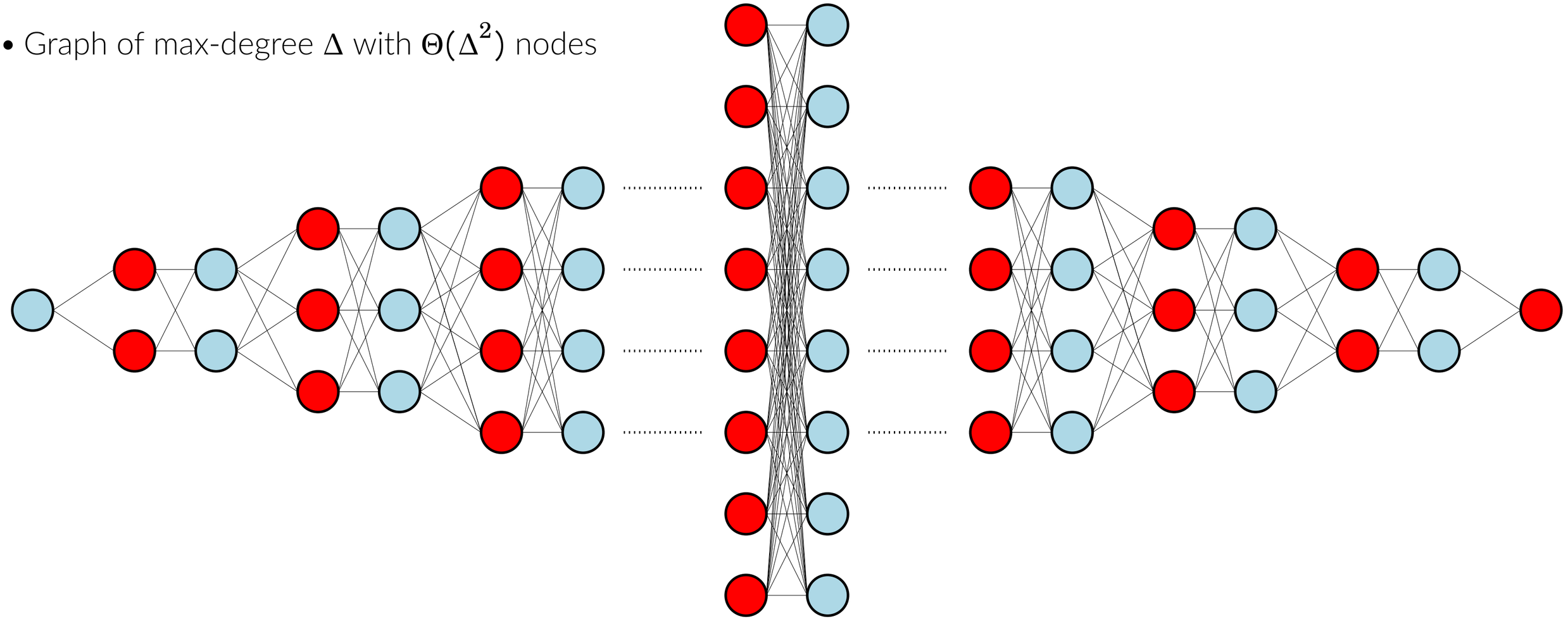

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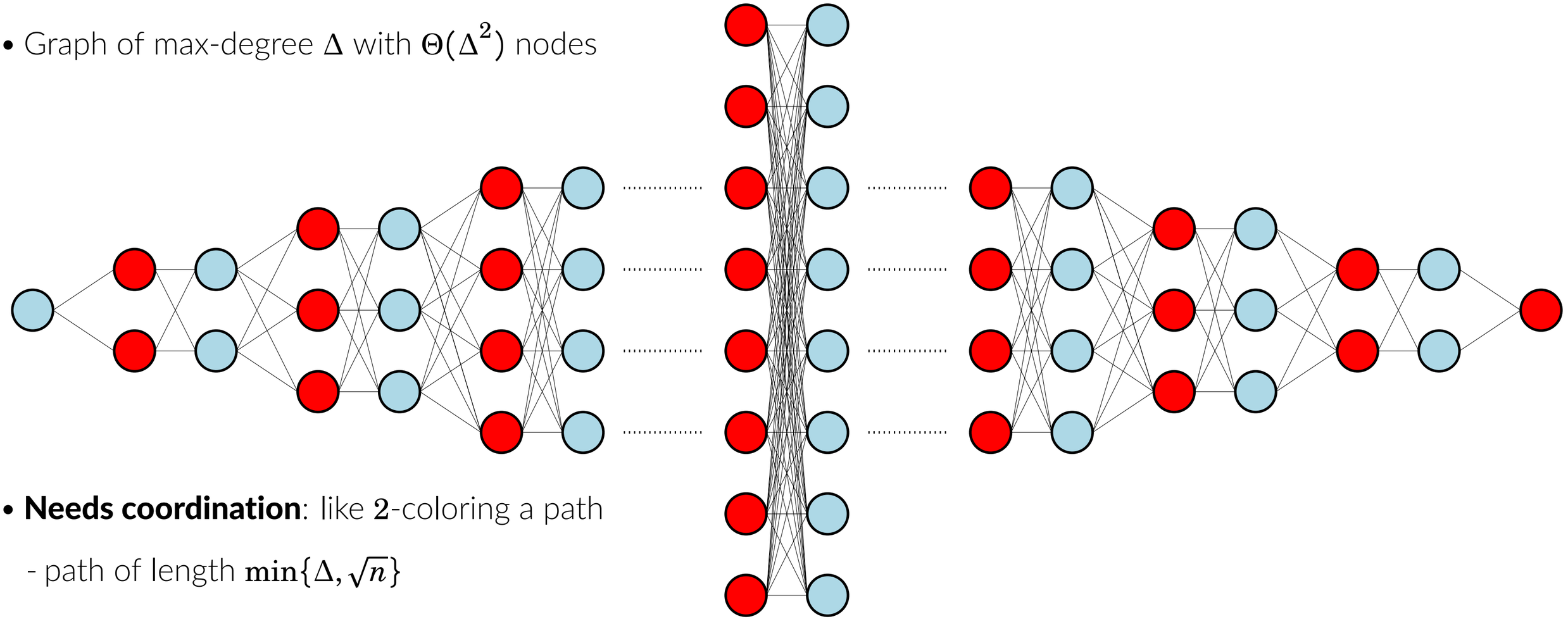

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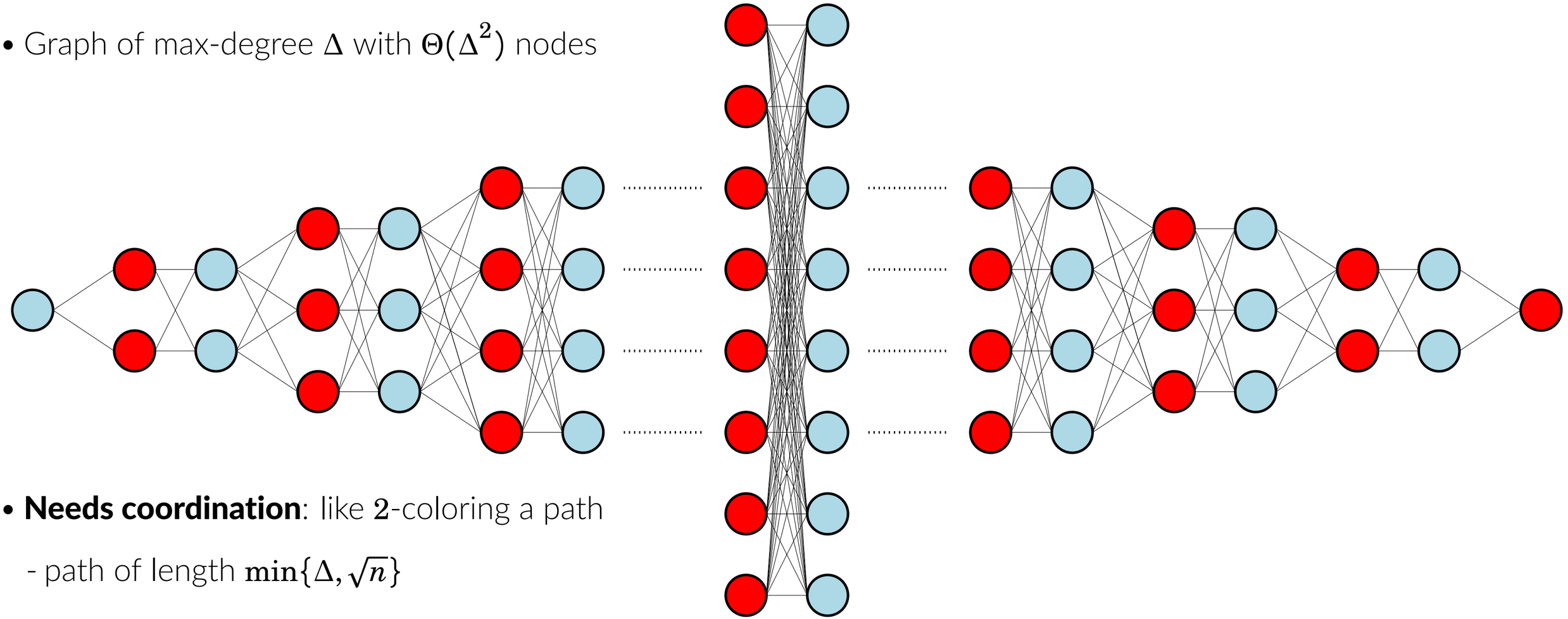

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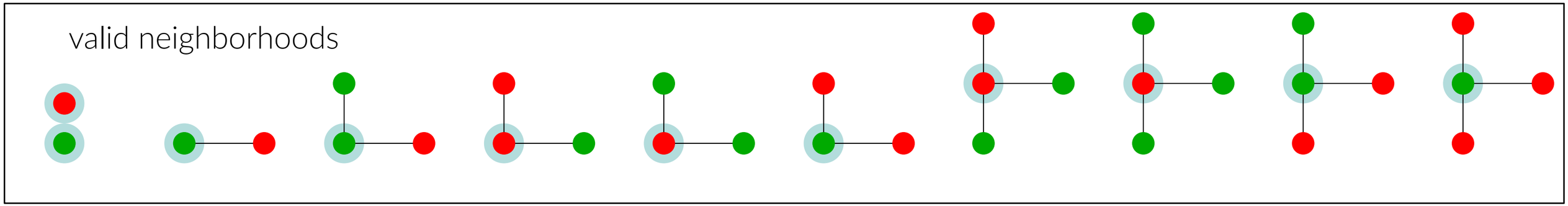

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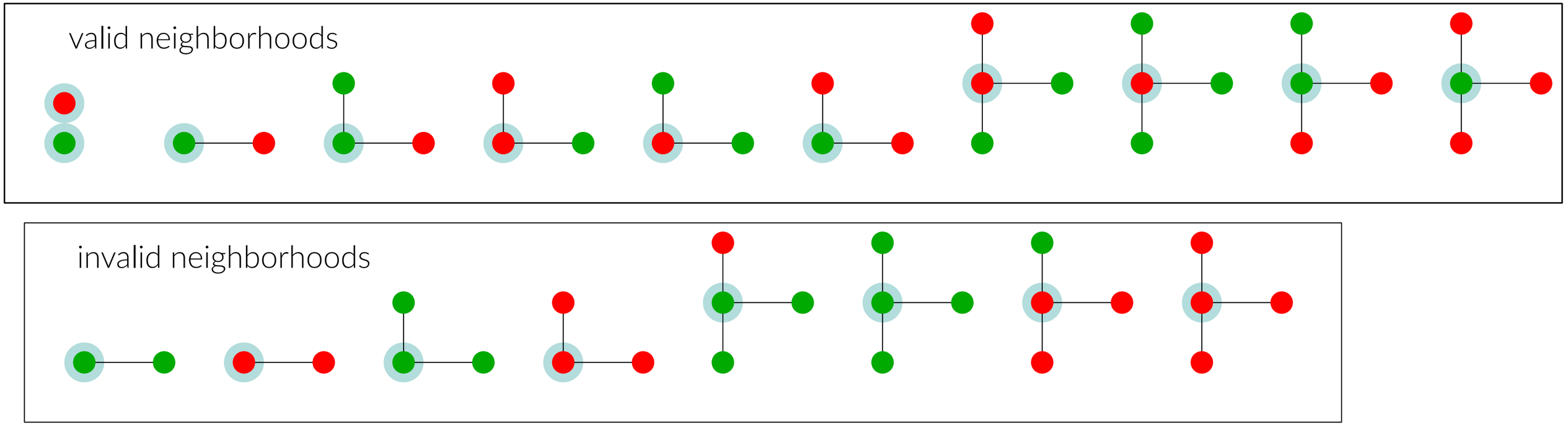

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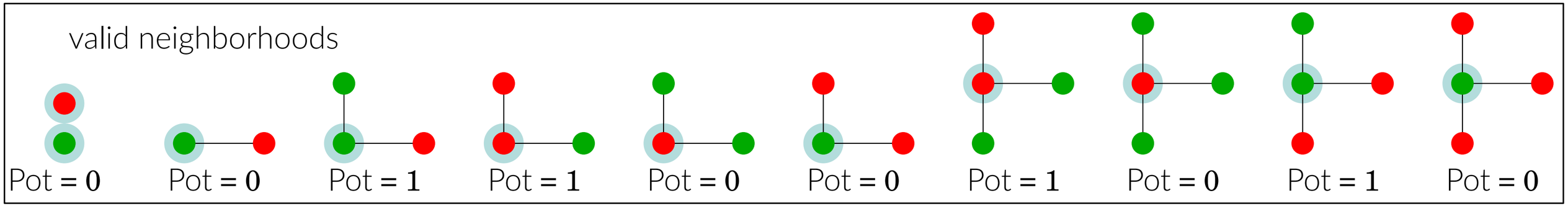

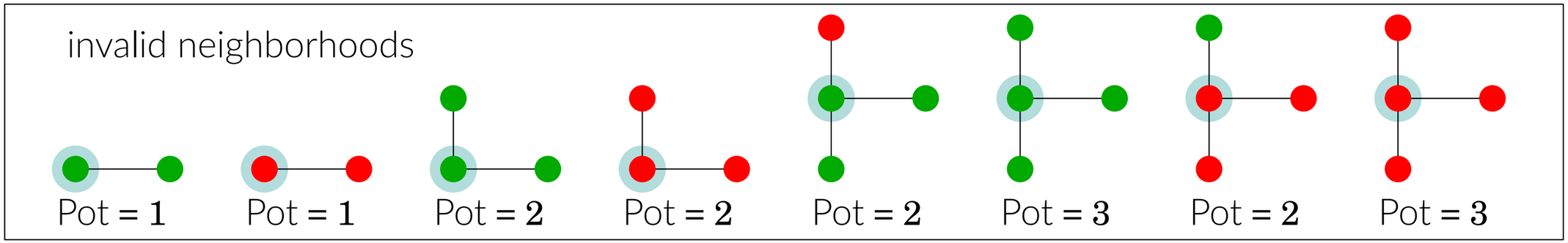

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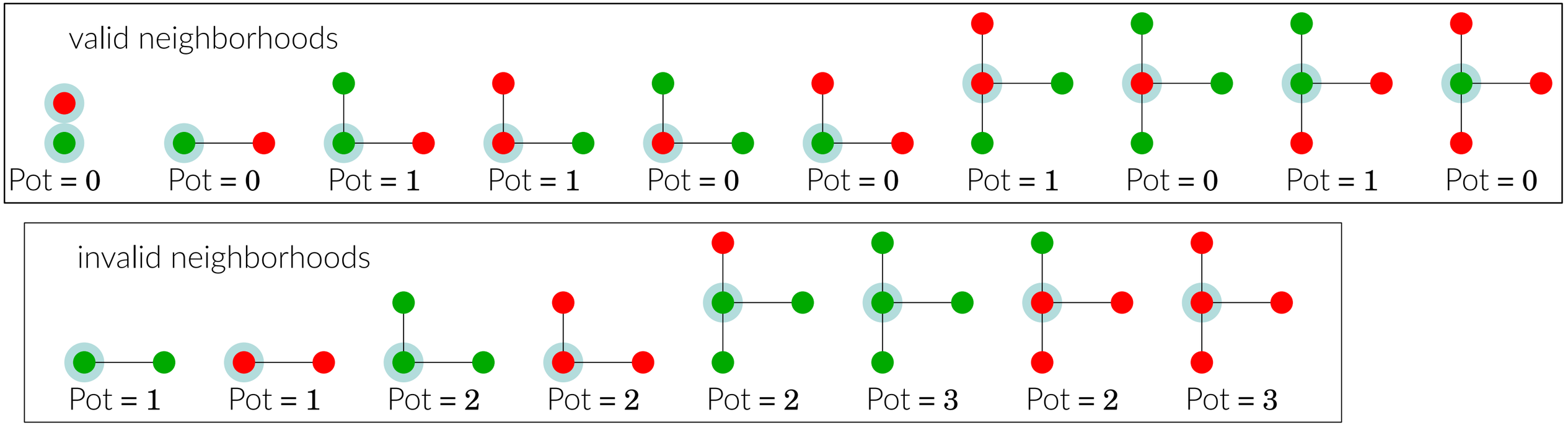

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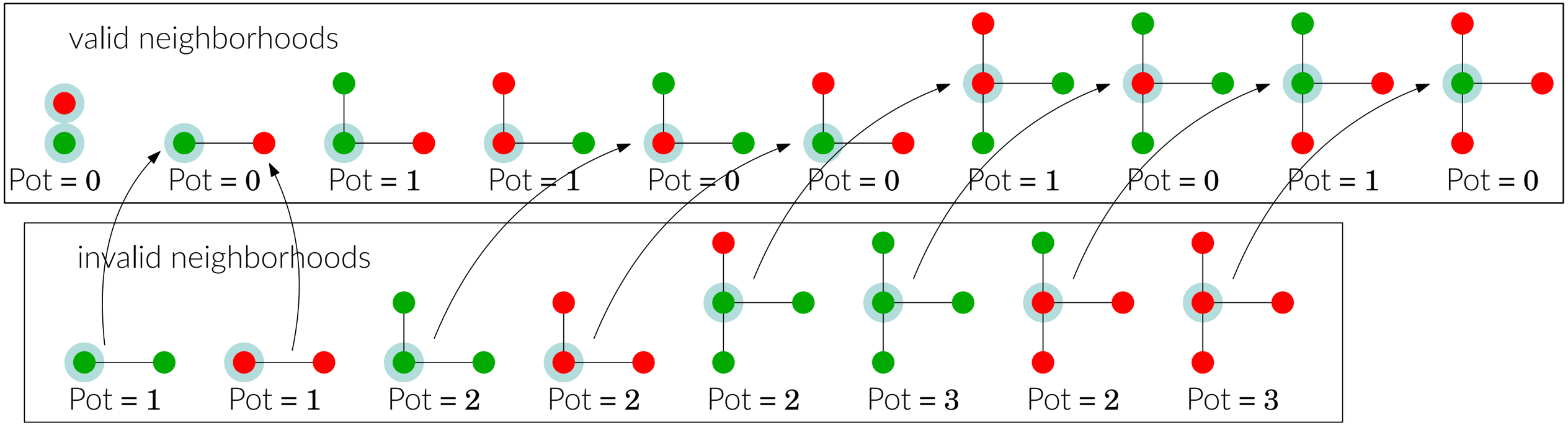

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Lower bound: - $\Omega(\min\{\Delta, \sqrt{n}\})$-rounds (even in quantum or non-signaling models)

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THANKS