A new short proof of a theorem of Ahlswede and Khachatrian

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Definition 1. An intersecting family is a collection of sets, every two of which have a point in common. A family of sets is trivial if there is a fixed element that lies in all of its sets, otherwise it is nontrivial.

Definition 2. A family $F$ is $t$-intersecting if for every $F, H \in F$ we have $|F \cap H| \geq t$; if in addition $|\cap_{F \in F} F| < t$, then $F$ is nontrivial $t$-intersecting, otherwise it is a trivial $t$-intersecting family.

Theorem 1. Let $1 \leq t < k \leq 2t + 1$ and $n \geq (t + 1)(k - t + 1)$. Suppose that $F$ is a nontrivial $t$-intersecting family of k-sets of $[n]$. Then $|F| \leq (t + 2) \binom{n-t+2}{k-t-1} + \binom{n-t+2}{k-t-2}$ and this is best possible.

Ahlswede and Khachatrian proved this theorem in 1996, which answered a question of Frankl and Furedi in 1986, their proof requires the entire machinery of the proof of the complete intersection theorem and later in 2007 Balogh and Mubayi gave a new, short proof, that I present it.