Overflow

Contents

- 8 Source cards
- 8 Sink cards
- 20 Node cards
- 52 Edge cards which form the Edge deck

You may also want a piece of paper for notes but this is not required.

Players

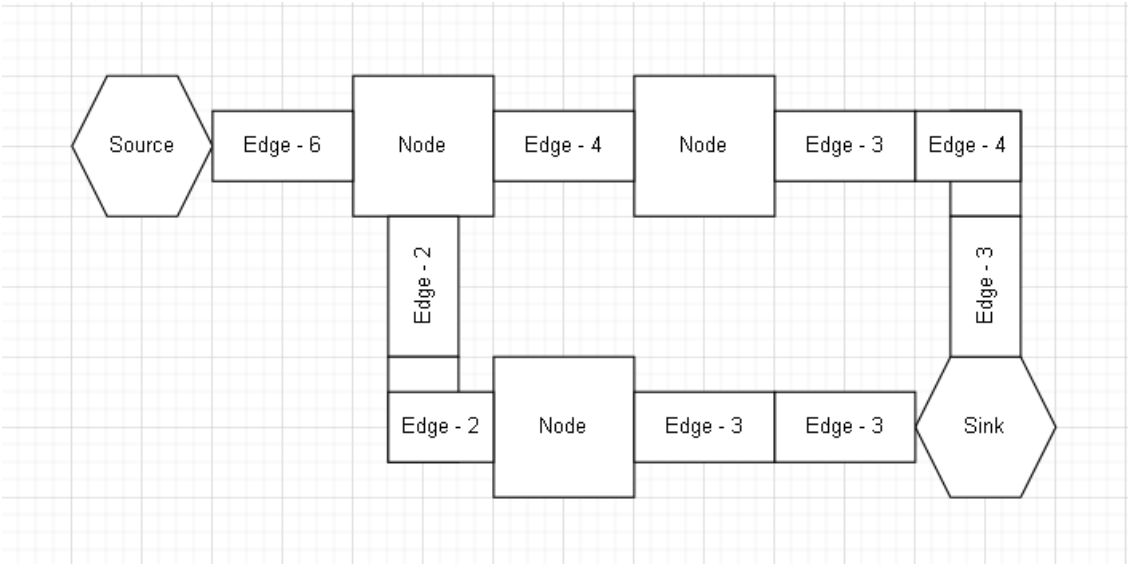
There is no player limit, but groups of 1-4 work best, for ages 10 and up!

Learning Objectives

- To understand the basics of directed, weighted graphs.
- To be able to think critically when constructing efficient networks.
- To understand and utilise the max flow/min cut theorem.

Rules

- Flow is the core concept of Overflow, in Overflow this is represented by putting water through aqueducts (Edge cards), each of which has a maximum amount of water (this is called Flow) it can carry, called its capacity.
- Node cards can be connected to Sources, Sinks, and other Nodes utilising Edge cards. Sources, Sinks and Nodes MUST be connected with Edge cards, they cannot be connected without them. You may use multiple Edge cards to connect two in any combination of Sources, Sinks and Nodes. Determine this new Edge's capacity as the minimum capacity of all Edge cards which comprise it.
- Some notes on Flow:
    - Sources produce Flow, Sinks consume it. Calculate the amount of Flow you have in your network by adding the Flow in the Edges connected to the Source together.
    - Every Node must have an equal amount of Flow into and out of it.
    - Each Edge card has a small number in a corner which represents its capacity, you cannot put more Flow through an Edge if it already is at its capacity.
    - To add more Flow to a network, find a route between a Source and Sink which has all of its Edges below maximum capacity, add 1 Flow to each Edge along that route. Repeat this with the route you've found until an Edge reaches its capacity. Then try to find another route.



To put a flow through this system above, we can see the route from the source to sink by the top path. This route can have 3 Flow added to it before it reaches its maximum capacity (when the lowest value edge tile is full). Then we can add another 2 Flow to the bottom route, giving us a total Flow of 5 for this network.

Campaign Mode

Usually a game of Overflow Campaign Mode lasts about 15-30 minutes.
The Campaign Mode of Overflow presents a never-ending test of futureproofing networks as they are designed, and finding optimal Flow through networks. This mode gives you a limited pool of resources to build a network, which grows as your network achieves certain goals. However, once you create a network, obtain your Flow goal, and gain your new resources, then the cards you have played already are stuck in place! So make your decisions wisely and don't be afraid to save resources for later on. You may move Flow around though, feel free to reset the network to 0 Flow after obtaining each goal.

Setup

- Take a Source, Sink and Node card. Shuffle the Edge deck and draw 6 Edge cards.
- Place the Source in the centre of the play space, place as many Edge and Node cards as you would like, and place the Sink card connected to the rest of the network. For this to be a valid network you must be able to create a Flow of 4.

Playing the Game

Now you have a network which has a Flow of 4, draw 2 additional Edge cards. Your next challenge is to construct a network with a Flow of 6 with the available resources. If you succeed, repeat this process of drawing 2 Edge cards and attempting to find a Flow which is 2 higher than the previous goal.

At additional points you gain more resources too, see below for a full guide as to what resources are gained at which point, this table includes the standard 2 Edge cards gained at every step.

Flow Achieved	Edges Gained	Nodes Gained	Sources/Sinks Gained
4	3	1	0
6	2	0	0
8	5	1	1 of each
10	3	0	0
12	4	1	0
14	3	0	0
16	6	1	1 of each
18	3	0	0
20	5	1	0
22	4	0	0
24	7	1	1 of each
26	4	0	0
28	5	1	0
30	5	0	0

Once you are unable to find a network for your next goal, sadly you lose the game. Note down the Flow you did achieve to beat it next time!

If you reach a Flow of 30 then read the rules below and figure out what needs to be gained at each step yourself for further goals:

- Whenever you reach a goal which is a multiple of 4 (8, 12, 14, etc), take an additional Node card and draw an additional Edge card.
- Whenever you reach a goal which is a multiple of 8 (8, 16, 24, etc), take an additional Source card, Sink card, and draw 2 additional Edge cards. You must place both the Source and Sink cards when constructing the network for your next goal.
- Once you reach a Flow of with a multiple of 10 (10, 20, 30), increase the number of Edge cards drawn at each Flow goal by 1.

Optional Rule

Each round, use the max flow/min cut theorem (found in Puzzle Mode) to prove that the Flow you have found is optimal!

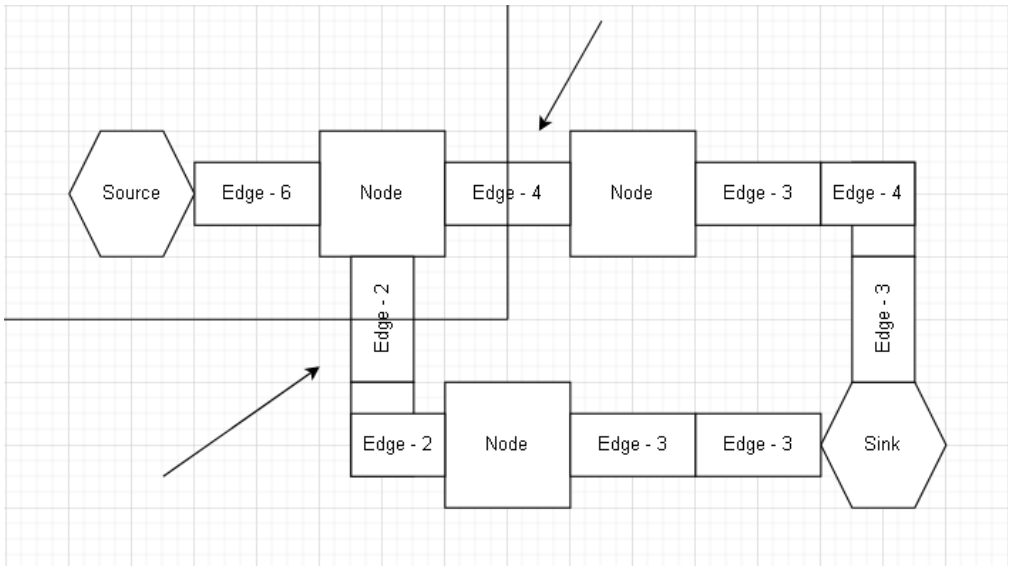
Puzzle Mode

1-10 Minute Playtime

Instead of tackling Campaign Mode, another way to play the game is to solve puzzles with your fellow players. You can also try creating puzzles for other players too! Puzzles may not just be placing tiles and getting a certain flow, they may require you to re-direct flow or other tasks such as finding cuts for the network. With these puzzles, card placement follows the same rules as usual, however instead of drawing from the edge deck, a pre-arranged set of edges and a set number of nodes are available for use. A new rule is also in place, face down node cards are "dead-zones", these are locations which cannot have a card placed. They just have no effect and are completely and utterly inconvenient.

Network cuts

A network cut is a way of splitting a network into two parts, Sources cannot be in the same part as Sinks. They are usually represented either by a line (not necessarily straight) through a network going through only edges or by labelling the Source, Sink and Nodes of the network and writing which ones are in which part. Any given cut has an associated capacity which is found by adding up the capacity of the edges which pass through it heading in the direction of the sink. Example:

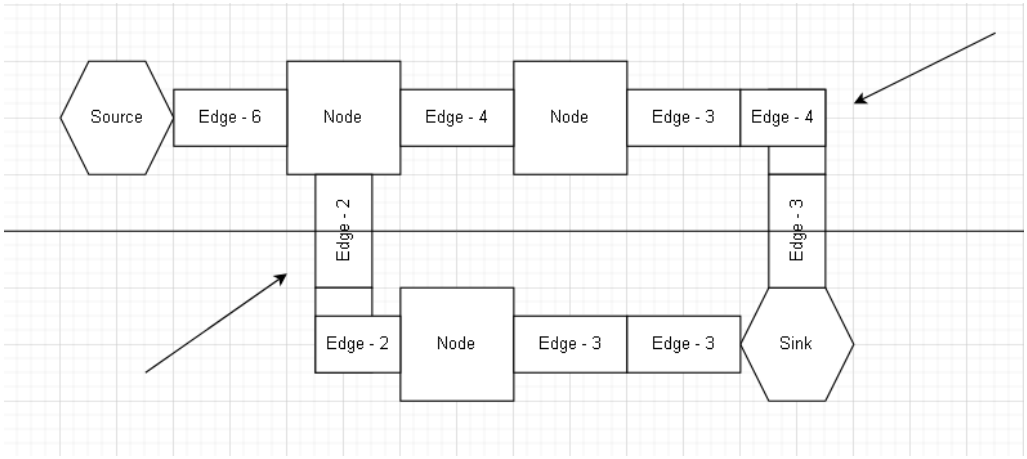


The above network has a cut across itself marked by the black line. This separates the source from the sink through the two edges indicated by the arrows. The capacities of these edges are 2 (left) and 4 (top), so the cut has a capacity of 2+4 = 6.

Max flow/min cut theorem

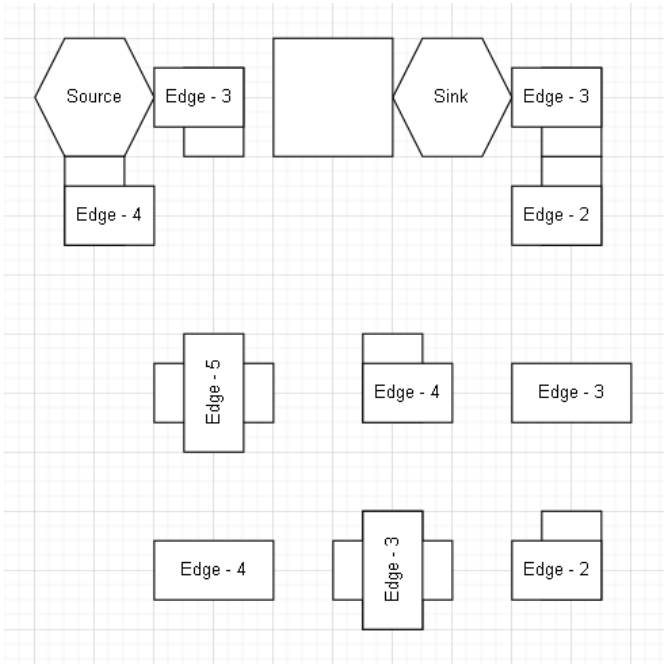
The max flow/min cut theorem is a theorem in network theory, and it states that the minimum cut of a network is equal to the maximum flow through the network. This means that if we can find a Flow of a given value, and a cut of the same value, then the Flow we have found must be a maximum.

Example:

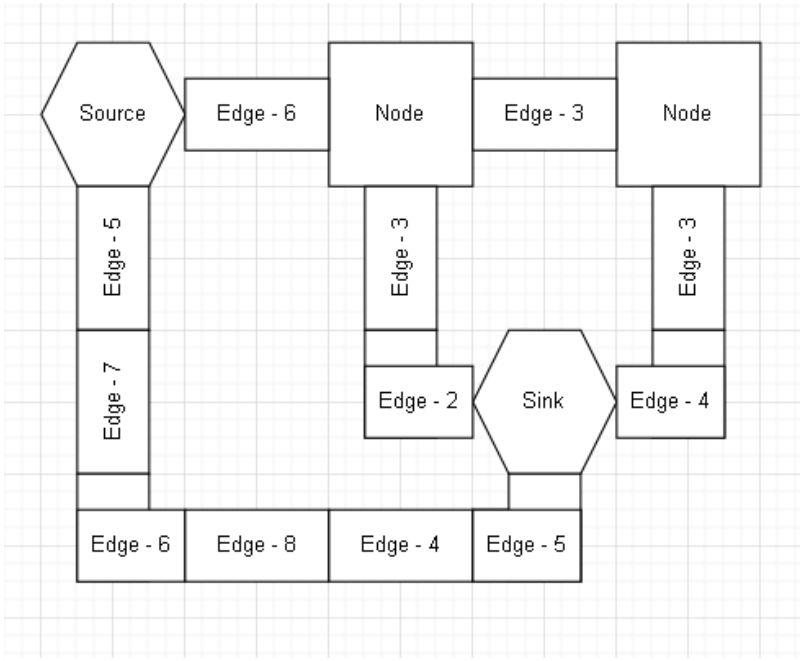


This cut on the same network as above has a capacity of 2+3 = 5, this is in fact the smallest cut we can find through this network; a flow of 5 is also easy to achieve. By the max flow/min cut theorem, we can conclude that 5 is the maximum flow that we can achieve through this network.

Example puzzles:



With the partially constructed network above, use the resources in the bottom half of the image to create a network with a flow of 4 or higher.



Find the maximum flow for this network, use the max flow/min cut theorem to verify that it is indeed the maximum flow.