Challenges of Climate change   Seminar   Week 8

This seminar is meant to introduce you to two things:

1. Mathematical modelling of the negotiation process – "Game Theory".

2. Computer simulations, which can explore the behaviour of mathematical models, and thus the consequences of rules governing negotiations, very rapidly and easily.

We begin with a model of climate negotiation with N participants ("players"). The negotiation is conducted over many rounds. At each round players make a bid (an offer to reduce emissions by a certain amount). After the first round, each player's bid is determined by the previous bids of all the other players, according to a rule I'll explain in a moment. In some sense each player's bid is the smallest reduction he can get away with, consonant with having a reasonable chance of achieving agreement.

The N players have sizes (their current emissions levels) $s_1, \ldots, s_N$. Thus the total current emissions is $\sum_i s_i$. The target of the negotiation is to reduce this by some fixed proportion T – in other words, to reduce the total to $T \sum_i s_i$.

Player i bids d_i, some number between 0 and 1; this is the proportion of his emissions he hopes to be able to retain. If, say player 3 bids $d_3 = 1$, he is saying he will make no reduction. If $d_3 = 3/4$, he is offering to reduce his emissions by $1/4$. Taking into acorn the sizes, we see that by bidding d_i, player i is offering to reduce his emissions to $d_i s_i$.

If

$$\sum_i d_i s_i \leq T \sum_i s_i$$

then the negotiation has succeeded and each player receives a "payoff", or benefit: he is able to continue emitting $d_i s_i$ tons per year. If

$$\sum_i d_i s_i > T \sum_i s_i$$

then negotiations have failed, and payoff, in this model, must be less that the payoff in case of success. So at the outset of the simulation, some number δ between 0 and 1 is fixed, and it is agreed that the payoff in case of failed negotiation is just $\delta d_i s_i$. The smaller is $'delta$, the worse the cost of failure. In real life this might come from trade disruption, or wars due to a failure of negotiation, or, in the long term, to climate change itself, and its ensuing disruption.

(*One might query this step in the model. Perhaps it would be more realistic to set the payoff in case of failure as* δs_i.)

We assume each player must make his bids inside some range, his so called *strategy set* or *strategy interval.* In this model, the strategy interval for p[layer i is $[\delta M_i, M_i]$. The set of sensible bids for each player is

$$B_i = \{d_i : \text{payoff with } d_i \geq \text{payoff with } d_i' \neq d_i \text{in strategy interval}\}$$

.

Definition A set of bids $(d_1, \ldots, d_N)$ is *feasible* if

$$\sum_i d_i s_i \leq T \sum_i s_i \quad \text{and for each } i, d_i \text{ is in strategy interval } [\delta M_i, M_i].$$

Definition A set of offers is *in disagreement* if the set cannot be made feasible by just one player changing his offer (within his strategy interval).

Many rounds of negotiation can take place. We denote player i's bid at round r by $d_i^{(r)}$ (the upper index "r" is not a power (as in "d_j to the power r") but simply an index to indicate that it is the r'th bid. At the start of the $r+1$'st round, player i reasons as follows: each other player j will most probably make a new offer of $E_j^{(r+1)} = \frac{1}{r}\sum_{k=1}^{r} d_j^{(k)}$ – the average of all his previous bids. So player i makes the following bid:

$$d_i^{(r+1)} = \begin{cases} \frac{1}{s_i}\left(T\sum_k s_k - \sum_{j\neq i} E_j^{(r+1)} s_j\right) & \text{if this quantity lies in his strategy interval} \\ M_i & \text{otherwise} \end{cases}$$

In other words, he bids the highest value of d_i that will enable the target to be met, assuming that the other players bid as he expects them to.

If, at any point, a feasible bid is reached, then the negotiations have succeeded. Otherwise, after 100 rounds, they are deemed to have failed.

The computer can easily carry this out, through as many rounds as one wishes, and the outcome of different experiments can give us some insight into what strategies may succeed, and how good our model is.

Mathematical notation For brevity we use the notation

$$\sum_{i=1}^{N} d_i$$

as an abbreviation for

$$d_1 + \cdots + d_N.$$

This may be abbreviated to

$$\sum_i d_i,$$

if we know from the context that we are going to sum from 1 to N. The expression

$$\sum_{j\neq i} d_j$$

means "the sum of $d_1, d_2, \ldots, d_N$, omitting d_i" or "the sum of all of the d_j for $j \neq i$". More formally it could be written as

$$\sum_{j=1, j\neq i}^{N} d_j$$

Mathematical Modelling The term "model" refers here simply to the set of rules which govern the behaviour of the automated "players" in the simulation we have described. Mathematical modelling is the process of devising such rules, in an attempt to emulate the behaviour of real negotiators. If we can produce models which convincingly replicate the real negotiation process under different negotiation protocols, then by means of simulations like the ones we are doing with Luke Addison's software, we can investigate the likely effect of varying those protocols. The protocols governing the next set of negotiations, in Paris in December 2015, will be crucial for maximising the chance of success. Mathematical modelling and computer simulation may help to achieve this.