Challenges of Climate Change   Lecture 4: Uncertainty

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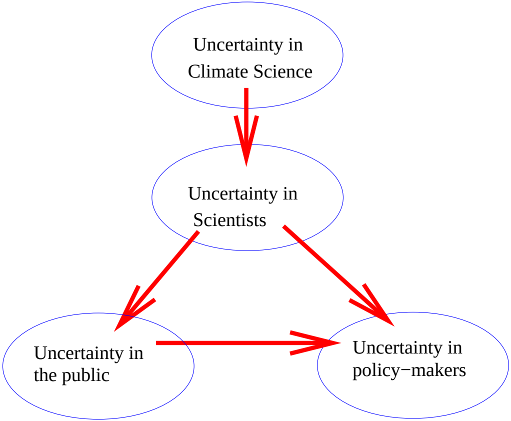
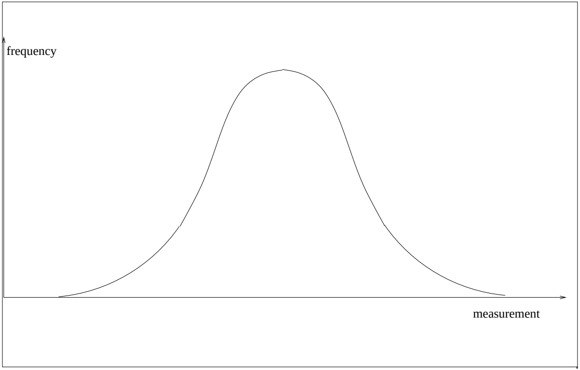
Uncertainty

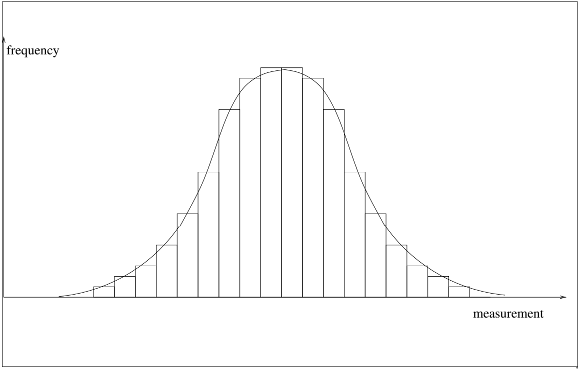


Uncertainty in statistics

Quantities such as temperature, ice cover, rainfall are often found to vary according to a certain familiar pattern, known as a "Gaussian" or "normal" distribution, which is represented graphically by a "bell curve".

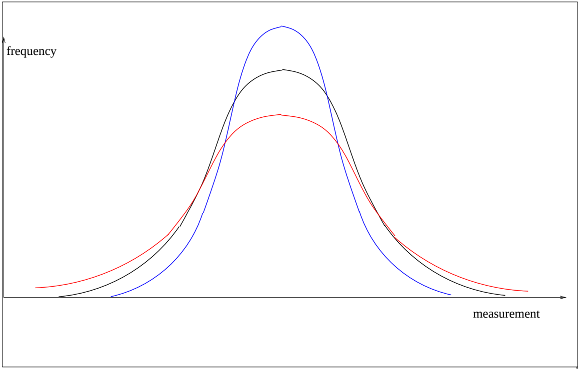


The bell curve 1



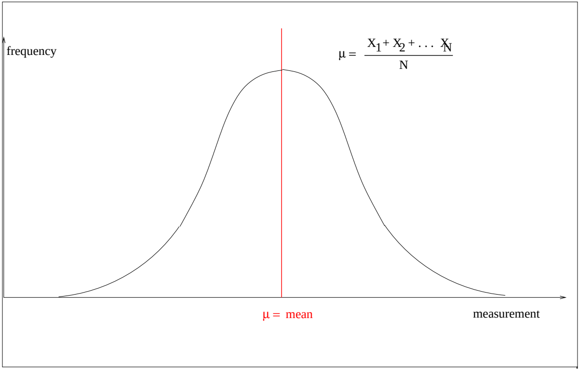
The curve is a smooth approximation to the “staircase” of discrete numerical data. Especially when the number of data points is very large, the curve is a good approximation. In the long run smooth curves are easier to use than discrete data – one can use the methods of calculus to analyse their properties.

The bell curve 2



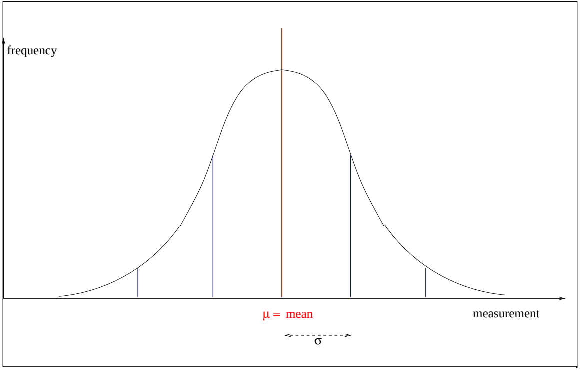
The curves may be short and fat (widely dispersed data) or tall and thin (data concentrated near to the mean).

The bell curve 3



The position of this curve along the measurement axis (more to the right or to the left) is determined by the *mean*, μ.

The bell curve 4



The width of the curve is determined by another number, the *standard deviation*, usually denoted by σ.

The standard deviation of the data is calculated as follows: let $m_1, \ldots, m_N$ be the measurements, and let $\mu = \frac{1}{N}\{m_1 + \cdots + m_N\}$ be the mean. Then we define the Variance of the data by

$$V = \frac{1}{N}\{(m_1 - \mu)^2 + \cdots + (m_N - \mu)^2\}$$

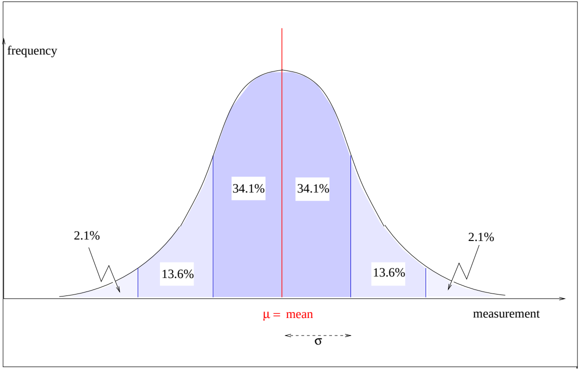
and set $\sigma = \sqrt{V}$.

For example, if the measurements are 2, 3, 4, 5 and 6, then

$$\mu = \frac{2+3+4+5+6}{5} = 4$$

$$\sigma = \sqrt{\frac{(2-4)^2 + (3-4)^2 + (4-4)^2 + (5-4)^2 + (6-4)^2}{5}} = \sqrt{2}$$

The bell curve 5



68.2% of measurements lie within one standard deviation of the mean – between $\mu - \sigma$ and $\mu + \sigma$.

A formula if you want it

Once you specify the mean μ and the standard deviation σ, the bell curve is given by the formula

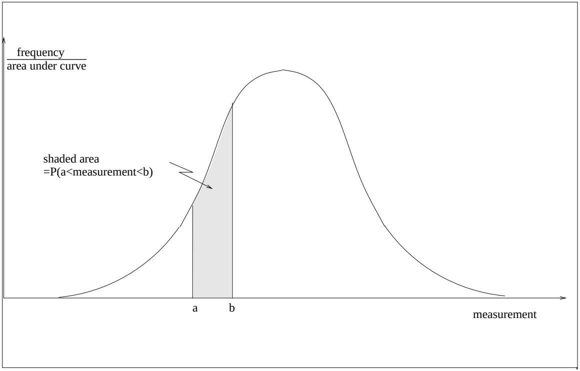
$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Here the axes are calibrated so that the area under the curve is equal to 1. This convention comes from probability theory. Any measurement-frequency curve can be re-cast as a probability curve, by dividing the vertical scale by the total area under the curve. Once that is done, the area under the curve in the re-calibrated graph is equal to 1, and the interpretation as a curve of probabilities is as follows: for each pair of numbers $a < b$ on the measurement axis,

$$P(\text{measurement between } a \text{ and } b\ ) = \text{shaded area under curve}$$

– see the next slide.

The bell curve 6



An example is given below, on the slide “Probabilities”.

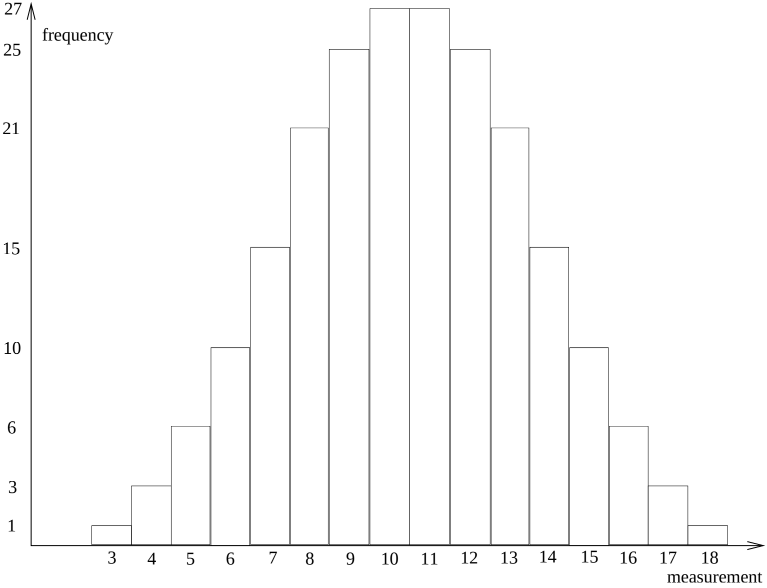
Throwing dice

Imagine throwing two standard dice and measuring the total scored. The possible totals are 2, 3, . . . , 12. Not all are equally likely. We can easily calculate how likely each is, assuming that the dice are fair: each ordered pair below is equally likely.

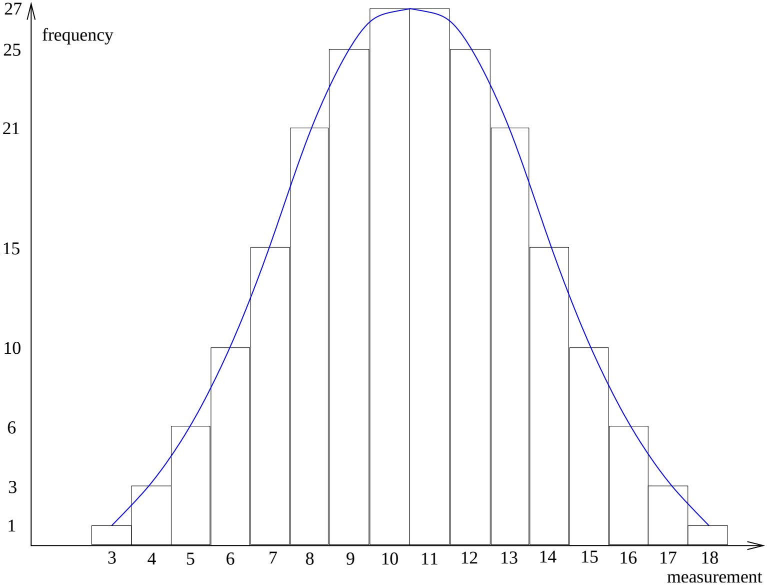
					(6,1)					
				(5,1)	(5,2)	(6,2)				
			(4,1)	(4,2)	(4,3)	(5,3)	(6,3)			
		(3,1)	(3,2)	(3,3)	(3,4)	(4,4)	(5,4)	(6,4)		
	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(3,5)	(4,5)	(5,5)	(6,5)	
(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)	(2,6)	(3,6)	(4,6)	(5,6)	(6,6)
2	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**

The upper profile of the pyramid of outcomes is triangular.

With 3 dice, the possible totals are 3,. . . , 18:



With 3 dice, the possible totals are 3,. . . , 18:



The blue curve begins to resemble the bell curve.
With more dice, the resemblance is stronger, though it is not the standard bell curve of the Gaussian distribution; instead it it is the *binomial distribution*. In the case of the dice, there is a mathematical argument justifying the distribution, and we can be very precise about it.

Complex physical phenomena are often the result of "the throws of very many dice", but because "not all the dice are equal or fair", measurements are more typically distributed according to the bell curve – especially when they involve repeated samples of a population. The standard presumption, if there is no *a priori* reason otherwise, is that a given distribution is Gaussian.
The job of the experimenter/statistician/pollster:

- calculate the parameters of the distribution - its mean μ and its variance σ.
- more generally figure out what the distribution is, if there is some reason to believe it is not Gaussian
- use the distribution to test hypotheses, e.g. that global average temperatures are increasing.

Probabilities

The measurement-frequency graph can be re-cast as a graph of probabilities. In the case of the two dice, the probability that we get a total of n is

$$P(\text{total } = n) = \frac{\text{number of ways of getting } n}{\text{total number of outcomes}}.$$

Total	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
# ways	1	2	3	4	5	6	5	4	3	2	1
probability	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

Observe that probabilities sum to 1.

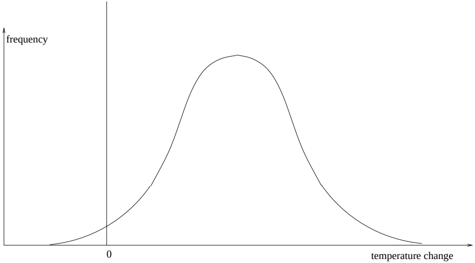
Example of probabilistic statement:
$P(5 \leq t \leq 9) = 24/36 = 2/3 = .66$.
I can be 66% confident that the sum of two dice will be between 5 and 9.

Hypothesis testing

Suppose that we know that the annual growth of 10-year old Warwickshire oak trees has Gaussian distribution with mean 80 kg and standard deviation 10kg. Suppose we fertilise an oak and measure its growth to be 89kg. Is this good evidence that the fertiliser worked? The figure of 89 is less than one standard deviation from the mean of 80, which means, according to the graph, that more than 15.7% of oaks achieve this without fertilisation. One example is poor evidence, and it would be better to try the experiment on a large sample of many oaks. We can calculate the mean of the *sample*, and *its* standard deviation. Standard statistical techniques then allow us to calculate the probability that the fertiliser is actually effective, in more precise terms: e.g. what is the probability that applying the fertiliser will increase annual growth by at least 10kg.

Global mean temperature

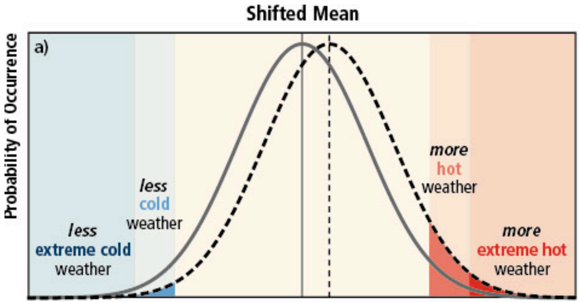
Mean temperatures are measured at a vast number of sites, and the results, when plotted on a graph, give something like this:

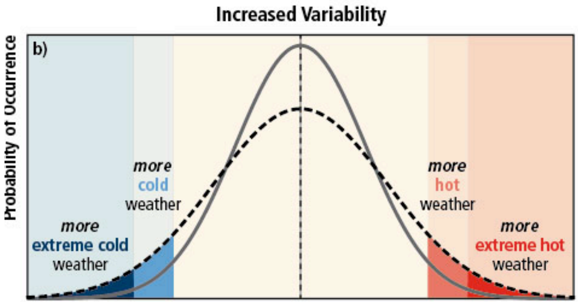


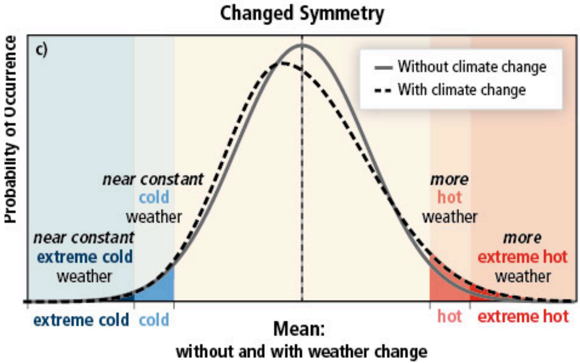
How strong is this as evidence that temperatures have changed? In this graph, some temperatures have gone down, while it seems that more have gone up. It is conventional to quantify the strength of evidence for warming provided by a graph of this kind as

$$\frac{\text{Area under the graph to the right of 0}}{\text{total area under graph}}$$

The next three slides show some cartoon graphs from the IPCC illustrating some consequences of possible changes to the distribution of temperatures. In each, the current distribution is shown as Gaussian, with a solid grey line, while a possible future distribution is shown with a black dashed line. The possible future distribution in each case is Gaussian or nearly Gaussian. But changes in its shape reflect important differences in climate.

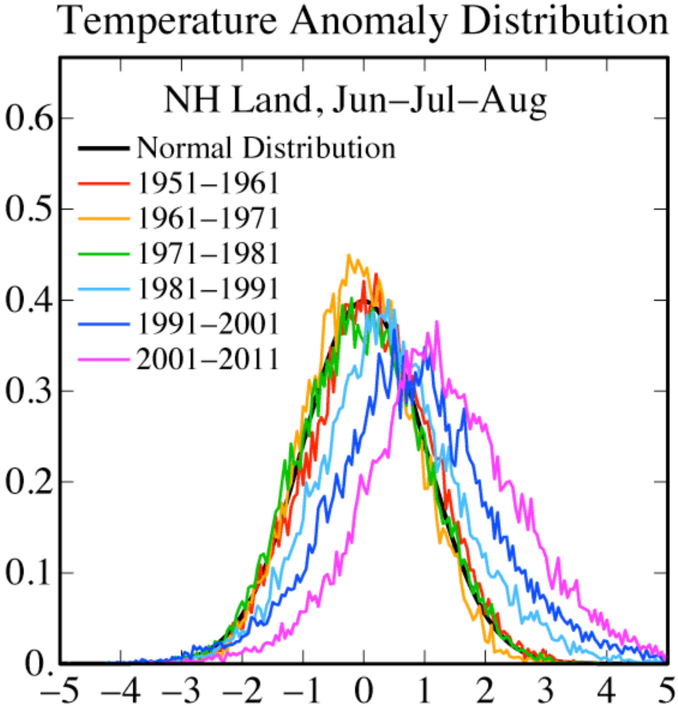

Shifted Mean
Probability of Occurrence
a)
less
extreme cold
weather
less
cold
weather
more
hot
weather
more
extreme hot
weather


Increased Variability
b)
Probability of Occurrence
more
cold
weather
more
extreme cold
weather
more
hot
weather
more
extreme hot
weather



Changed Symmetry
c)
Probability of Occurrence
Without climate change
With climate change
near constant
cold
weather
near constant
extreme cold
weather
more
hot
weather
more
extreme hot
weather
extreme cold
cold
hot
extreme hot
Mean:
without and with weather change

The next slide shows measurement-frequency graphs of some real data, from a NASA briefing paper by James Hansen, Makiko Sato and Reto Ruedy, at www.giss.nasa.gov/research/briefs/hansen_17. Hansen is the American climate scientist who, more than anyone else, has helped to draw attention to the dangers of climate change. The graph shows six graphs of temperature distributions obtained by taking measurements over many Northern Hemisphere (NH) land sites in the three summer months. For comparison, it shows a normal distribution, approximating the Northern Hemisphere summer land distribution for 1950-80. The vertical scale shows frequency, re-scaled so that the total area under each graph is 1. The horizontal axis is calibrated by standard deviations of the black normal distribution. Which of the three possible distributions of the last three slides does the sequence of distributions appear to tend towards?


Temperature Anomaly Distribution
NH Land, Jun–Jul–Aug
Normal Distribution
1951–1961
1961–1971
1971–1981
1981–1991
1991–2001
2001–2011
0.6
0.5
0.4
0.3
0.2
0.1
0.
–5
–4
–3
–2
–1
0
1
2
3
4
5

Doubt on the data

The debate on global warming (though not its causes) might well be resolved by a graph of the kind just shown, assuming its measurements were trusted. Unfortunately the data is subject to a great deal of doubt. Hansen's graphs show only data from a relatively short period, and sceptics argue that climate always shows "natural variability" over short periods.

The willingness to dismiss changes as "natural variation" is an expression of ignorance. It relies on the idea that we cannot account for the weather; that it is "random" and meaningless; that it does not have proper causes. (Parenthesis: what does "random" mean? Is it possible that physical events happen without proper causes?) While it is true that much in the weather cannot be predicted, longer term fluctuations in climate are increasingly well understood. For example, the correlation between climate and the Milankovich cycles described in Lecture 1 is increasingly well-established by early proxy measurements from tree rings and ice cores.

Even so, proxy data rely on the hypothesis that "all other things were equal", and so are themselves the subject of debate. And measurements by early scientists may be unreliable: old thermometers may have been inaccurate.

Even contemporary measurements may be affected by local increases in temperature due to urbanisation. Some measurements from China have been cast into doubt for this reason - information on some geographical locations was lost, and it is not known whether they have urbanised.

What is much less in doubt is the change in the total energy received from the sun and retained due to the increase in greenhouse gases (CO_2, methane and CFCs).

Part II: Science is not the only source of uncertainty

Despite the reasons for uncertainty in science, there is an increasing, and now practically universal consensus among climate scientists that the climate is changing and that carbon emissions from burning fossil fuels are the cause.

IPCC reports say this more and more strongly; the 2013 report has 95% confidence that at least half of the temperature changes are due to human agency, up from 90% in the previous report.

The media do not always reflect this consensus. When BBC Radio 4 discussed the IPCC report on Friday September 27, their Today item at 8.00am reported that they couldn't find a climate scientist in the UK who would dispute the findings of the IPCC. For their World at One news programme, they did find a sceptic: a retired geologist from Australia, Bob Carter, who leads the Nongovernmental International Panel on Climate Change, funded by US conservatives and in particular by the Heartland Institute.

Carter's interview dominated the World at One coverage of the IPCC report, and reporting in several later bulletins.

The BBC argues the need for "fairness and balance".

It has been criticised for "false balance" by, among others, Professor Steve Jones, a biologist from University College London, who it had commissioned to review its science reporting.

Other media outlets with less commitment to public service are influenced by commercial imperatives: controversy is exciting, agreement is dull. But why don't they give air time to the scientists who argue that the IPCC findings are too cautious?

Do we jeopardise our future by confusing information with entertainment?

Merchants of Doubt

There is something more organised going on. A number of politically conservative foundations in the US promote denial, and not just of climate change. Naomi Oreskes and Erik Conway show in "Merchants of Doubt" (2010) how some of the same people and organisations have repeatedly promoted doubt

- That smoking causes cancer
- That sulphur emissions in smoke cause acid rain
- That CFCs cause ozone depletion
- That passive smoking damages health, and now
- That carbon from burning fossil fuels is causing dangerous climate change

Oreskes and Conway follow four distinguished physicists, including Frederick Seitz, a former president of the National Academy of Science of the USA, who have been involved in creating uncertainty on all of these issues.

What are the common features of the scientific debates in which they intervened?

- In every case, the science suggested a need for government regulation. Is it opposition to regulation that drives them? Evidence for this: funding comes from large corporations which may stand to suffer economically in case regulations are imposed. (We stray into politics.)
- In each case, the science asserts that our doing A will cause B, which will harm us. In each case, B has several possible causes, and it is possible to increase doubt about the extent to which A is to blame.

Attributing blame

Climate change is a particularly extreme case. Direct evidence that it is already happening is very hard to assess. Are the events shown in next two slides the consequences of climate change?

Refugees from Somalia gathered on the border with Kenya. Are they refugees from drought, or from war? Does drought cause war?

Cabs submerged after Hurricane Sandy stuck the East coast of the USA in late October 2012. Sandy was unusually strong, northerly and late in the year.

Attributing blame

Is climate change to blame for these events?

In a complex system, events may have many causes.

If I say "I'm certain this hurricane was caused by climate change"
I mean "Without climate change it would not have happened"
- something impossible to prove.

Probabilistic version:

in 1974-1993, $P(\text{hurricane in late October}) = p_1$.

in 1994-2013, $P(\text{hurricane in late October}) = p_2$

We need to understand the language of probability theory, but . . .

In a survey carried out by the Royal Statistical Society in 2011

> *a total of 97 UK MPs were asked this probability problem: if you spin a coin twice, what is the probability of getting two heads?*
>
> *Only 38 of the 97 replied correctly, although 72 said they felt confident when dealing with numbers.*

Challenge for all of us: increase our own understanding of probability and uncertainty, and share it.
We need our legislators to understand probabilities.

Successes of the deniers

Another feature of the denial promoted by the George C Marshall Institute, the Heartland Institute, and others:

their remarkable success in getting the ear of government. Oreskes and Conway document how, repeatedly, when mainstream scientific bodies came to worrying conclusions about acid rain, or the danger of tobacco smoke, the US administration initially paid as much or more attention to the deniers, and only after much delay acted on the scientific consensus.

- Is this market fundamentalism?
- Is it a hangover from the Cold War – that averting deaths through passive smoking, or reducing environmental damage caused by acid rain, or mitigating climate change, all require coordinated action by governments, anathema to the victors of the cold war ? Green politics seem currently to belong to the left more than the right. Why? Historically this has not always been the case.

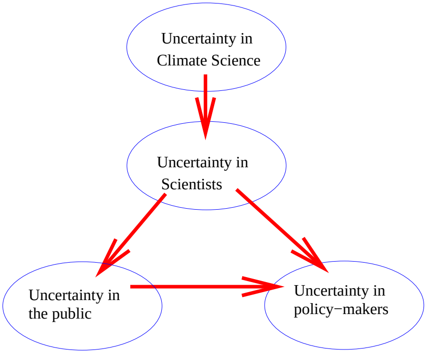
Successes of the deniers

- Is it because government is responding to the interests of large corporations? These interests may well be different from those of the public at large. Isaac Azimov's three laws of robotics unfortunately omitted a definition of "robot". If it is "human-created device capable of independent action", perhaps we should think of corporations as robots. In this case it is worth noting that maximising shareholder value may be incompatible with the three laws.
- Is it related to the anti-science movement, and in particular the denial of evolution? This last is something the four physicists never did, incidentally. But the groups promoting anti-science and climate-change denial certainly overlap.

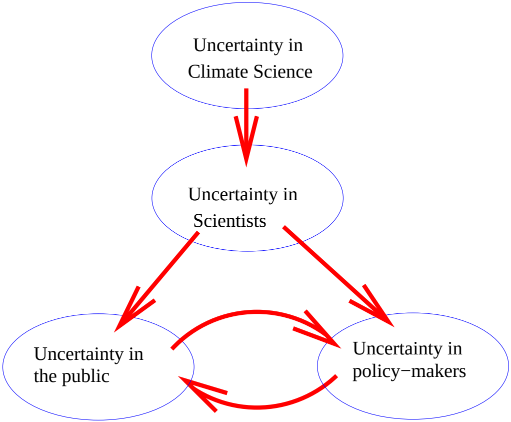
Is denial convenient for governments?

Do governments have other reasons to embrace denial?

Party funding? Elections?



This diagram has no loops. But when we add an arrow to show governments influencing the public, a loop appears, making feedback possible. This could be dangerous!



In a democratic system, political parties compete for votes by offering better living standards. These are threatened in the near term by green levies and emissions controls – while the threat to living standards from climate change lies over the electoral horizon. By embracing denial, political parties in government are able to justify their (self-interested) failure to take action.

Governments pay lip service to the need to act, but their body language conveys the opposite. There is no urgency. Our political parties hope to appease climate change. And the public is happy to be reassured.

Conspiracy Theories?

In the climate debate, both sides accuse the other of "conspiracy".

"Climate change is just a hoax to get funding for scientists"

"The fossil fuel industry funds climate denial in order to safeguard the worth of its assets".

The science is so complex, and involves so many fields, that no-one can understand it all. Therefore it belongs to large organisations – whose conclusions individuals are hard-pressed to assess. Fossil fuel extraction is also carried out by very large, powerful and opaque organisations.

The superabundance of "information" on the internet may lead to the creation of opposing tribes more than to the better instruction of the public. Is adopting a view about climate change more a matter of tribal loyalty than a rational appraisal of the arguments?

Further reading

1. Nate Silver, “The Signal and the Noise”, Penguin Books, 2013.
2. Mike Hulme, “Why we disagree about climate change”, Cambridge University Press, 2009
3. Naomi Oreskes and Erik Conway, “Merchants of Doubt”, Bloomsbury, 2011. See also YouTube video of Naomi Oreskes’s talk, linked from the book’s website.
4. Naomi Oreskes and Erik Conway, “The collapse of western civilisation: a view from the future”, Daedalus, the Journal of the American Academy of Arts and Sciences, available free via link from Reading List
5. The New Climate Dice: Public Perception of Climate Change By James Hansen, Makiko Sato, Reto Ruedy August 2012, available online at http://www.giss.nasa.gov/research/briefs/hansen_17/