

Session 6
The Human Voice and
Fourier Analysis!
Jo (Rut) Karlström, Rudo Römer



Science of Music

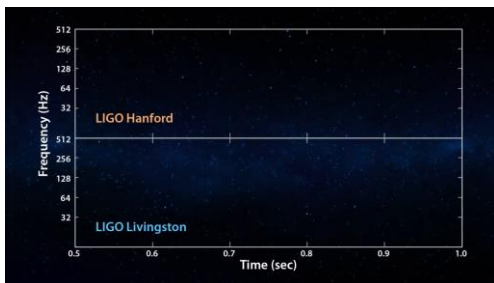


Today's session

We want to learn about

- The acoustics of the human voice instrument
- The analysis of signals via Fourier series
- The appreciation of human singing
- The sound generation in our voice box
- How what we say, determines how we sing

The sound of two colliding black holes



Camille Saint-Saëns' *Samson et Dalila*

- *Samson and Delilah* (French: *Samson et Dalila*), Op. 47, is a [grand opera](#) in three acts and four scenes by [Camille Saint-Saëns](#) to a French [libretto](#) by [Ferdinand Lemaire](#). It was first performed in [Weimar](#) at the Grossherzogliches (Grand Ducal) Theater (now the [Staatskapelle Weimar](#)) on 2 December 1877 in a German translation.
- The opera is based on the [Biblical](#) tale of [Samson and Delilah](#) found in Chapter 16 of the [Book of Judges](#) in the [Old Testament](#). It is the only opera by Saint-Saëns that is regularly performed. The second act love scene in Delilah's tent is one of the set pieces that define [French opera](#). Two of Delilah's [arias](#) are particularly well known: "Printemps qui commence" and "[Mon cœur s'ouvre à ta voix](#)" ("My heart opens itself to your voice", also known as "Softly awakes my heart"), the latter of which is one of the most popular recital pieces in the [mezzo-soprano/contralto](#) repertoire.

From Wikipedia, the free encyclopedia

History of the Singing Voice

- Humans have probably always been singing, earliest recall is from the Bible.
- Important in Ancient Greece, the same as reading poetry.
- Scaled down during early Christianity, instruments, female and popular singing forbidden. Instead, Gregorian chants. <https://youtu.be/D5ubvYqOh1M?t=22m12s>
- Up until 500 A.D. no polyphonic singing, and only after that did profane singing win ground.
- High voice and tenors until Johannes Ockeghem 'invents' basses.
- 1500's: Earliest music still played today, Palestrina Caccini and Monteverdi. (Music clip from The Coronation of Poppea) https://youtu.be/_isL0E-4TsQ?t=13s
- Female voices 'discovered'.
- Four different voice ranges: Soprano, alto, tenor and bass
- Baroque era, Bach and Händel, music is complex and sung like a flute. <https://youtu.be/MUJox9EdSjw?t=2m41s>
- Women was not allowed in churches so instead they castrated boys to keep their high voice, instead of having their voices break in puberty.
- They had bigger lungs and very flexible larynx which made them able to do things we can't replicate today. <https://youtu.be/IQo2PNnwOww?t=16s>



History of the Singing Voice part 2

- **Bel Canto** to sing like an instrument, with precision, high virtuosity, clarity and agility.
- The singer was the creator of emotions and music.
- Vienna Classicism, singers developed bigger range as shown in the Magic Flute. <https://youtu.be/463jDvbw3LQ?t=2m11s>
- With Rossini the relationship between singer and composer starts to change. He writes music hard enough for the singers not to improvise. https://youtu.be/ZbOMO_ifCs?t=5m8s
- 1800's – a new way of singing, biggest change for centuries. The new voice was dramatic. More volume, more emotions, not as precise as Bel Canto.
- 'Projection': a technique which allows singers to unaided be heard over a hundred piece orchestra. <https://youtu.be/N8lD9ZmYHhE?t=2m34s>
- New voice types: Mezzo-soprano and Bass-Baritone.
- The plot of the opera was more important than the individual singers chance to show off. <https://youtu.be/K2snTkaD64U>
- Wagner took their art of singing to new extremes, requiring skills that are on the edge of what the human voice can do. <https://youtu.be/owFDFRoFKHk>
- During early 20th century the microphone was invented, giving space to new ways of singing such as jazz and blues.
- Opera - a dying art form? <https://youtu.be/OV3xp5ZSXSYA?t=26s>

The "voicebox"

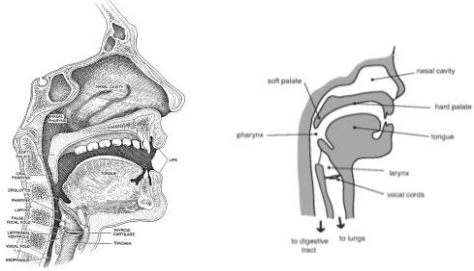
- The voice organ is an instrument consisting of a power supply (the lungs), an oscillator (the vocal folds/chords) and a resonator (the larynx, the pharynx and mouth).
- Singers use the resonator in special ways.

John Sundberg, "Acoustics of the Singing Voice", Scientific American, March 1977



The resonator of the "voicebox": vocal tract

- Vocal tract acts like a resonant chamber such as tube of a horn or body of violin.
- Boundary conditions are set by lips, jaw, tongue, larynx.



Audience participation A

- Finding your own personal singing voice
 - Soprano: C4–C6
 - Mezzo-soprano: A3–A5
 - Contralto: F3–F5
 - Countertenor: E3–E5
 - Tenor: C3–C5
 - Baritone: G2–G4
 - Bass: E2–E4



Fourier analysis: basics

- Nearly any function $f(t)$ can be written as $f(t) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi t}{L} + b_n \sin \frac{n\pi t}{L} \right)$

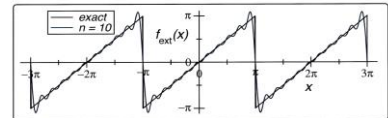


with

$$a_n = \frac{1}{L} \int_{-L}^L dx \cos(\omega_n t) f(t), \quad \omega_n = \frac{n\pi}{L}$$

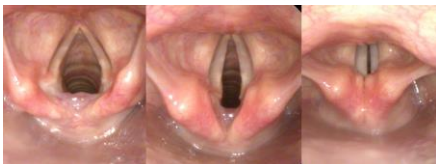
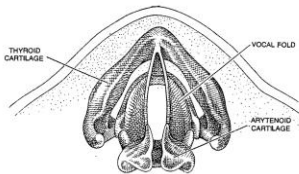
$$b_n = \frac{1}{L} \int_{-L}^L dx \sin(\omega_n t) f(t).$$

$f(x) = x$
 $\forall x \in [-\pi, \pi]$



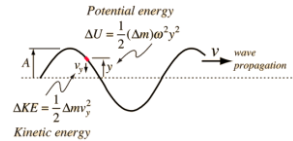
Our vocal cords

- <https://youtu.be/P2pLjFWUjc8>
- Audience participation B: balloons



Fourier analysis: energy of waves and power spectrum

- Energy in (small segment of) string:



$$dU = \frac{1}{2} \mu y^2 \omega^2 dx, \quad \mu = \frac{dm}{dx}$$

$$dT = \frac{1}{2} \mu v_y^2 dx$$

$$y = A \sin(kx - \omega t)$$

$$\frac{dy}{dx} = v_y = A \omega \cos(kx - \omega t)$$

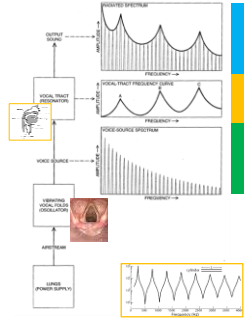
$$P(\omega) = \frac{E}{\tau} = \frac{1}{2} \mu \omega^2 A^2 v$$

$$E = \int_0^\lambda dU + dT = \frac{1}{2} \mu \omega^2 A^2 \int_0^\lambda \sin^2(kx - \omega t) dx + \frac{1}{2} \mu A^2 \omega^2 \int_0^\lambda \cos^2(kx - \omega t) dx$$

$$= \frac{1}{2} \mu \omega^2 A^2 \lambda \quad @ \tau = 0$$

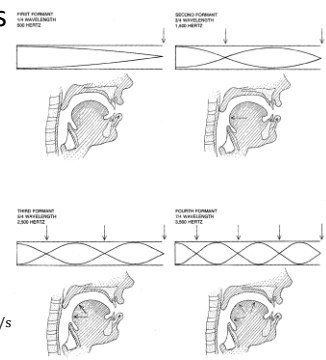
An air column in our throat

- Air stream from lungs is periodically interrupted by the vibrating vocal folds
- Resulting sound has a spectrum, containing a large number of harmonic partials, amplitude of which is decreasing with increasing frequency
- Air column in vocal tract has characteristic modes of vibration (resonances) call formants (A, B, C)
- Sound is modulated according to the distance from formants
- Formant frequencies are peaks in the final output sound



Forming formants

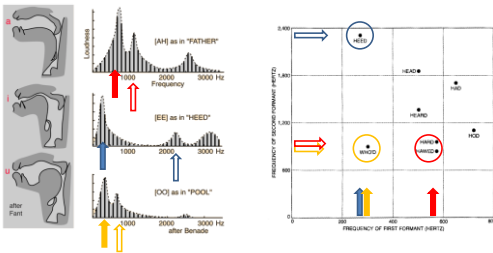
- Formants: standing waves of air-pressure oscillations
- Picture shows vocal tract approximated via cylindrical tube
- pressure waves as in open wind instruments, pressure differential large at vocal fold (glottis), close to zero at lips
- ¼ of wave lengths inside vocal tract for 1st formant
- 250-700Hz for adult male in first formant



$$4 \times 17.5\text{cm} \times 500\text{Hz} = 350 \text{ m/s}$$

Speech formants

- Audience participation C: Aa, ee, oo



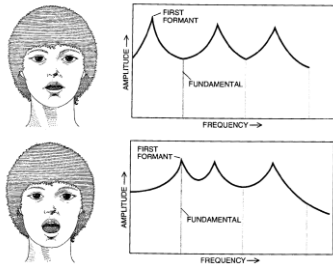
How to sing

- Singers of high pitch (sopranos, etc.) open mouth wider with rising pitch



“How to sing” explained

- High-tone singer often sing at frequencies beyond closed-lips formant, leads to reduction in sound
- Opening lips raises 1st formant to higher frequencies, giving better sound



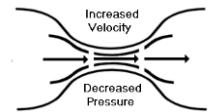
Bernoulli's principle

$$E_{\text{Lungs}} = E_{\text{Larynx}}$$

$$\frac{1}{2} M v_{LU}^2 = \frac{1}{2} M v_{LX}^2 \quad : / V$$

$$\frac{1}{2} \rho_{LU} v_{LU}^2 = \frac{1}{2} \rho_{LX} v_{LX}^2 = \frac{E}{L^3} = \frac{F \cdot L}{L^3} = \frac{F}{L^2} = \frac{F}{A} = P$$

$$P_{LU} + \frac{1}{2} \rho_{LU} v_{LU}^2 = \frac{1}{2} \rho_{LA} v_{LX}^2 + P_{LX}$$

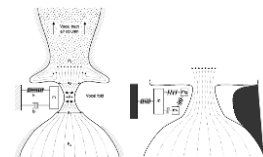


Closed vocal folds:

$$v_{LU} = v_{LX} = 0: P_{LU} > P_{LA}$$

Open vocal folds:

$$v_{LU} < v_{LX}: P_{LU} < P_{LA}$$



Pressure oscillations, aka SOUND!

Fourier analysis:

- Jo's singing:
 - Middle sound
 - Lower sound
 - Higher sound
 - Falsetto
 - Sotto voce

A thousand kinds of singing.

- Yodeling, <https://youtu.be/vQhqikWnQCU>
- Kulning, <https://youtu.be/IJtW8CiGiEk>
- Screamo, <https://youtu.be/lnRMwptcgAo?t=17s>
- Carl Jenkins, Call for Prayer, A Mass for Peace. <https://youtu.be/Dw3kRv6SVN0>

Singing formants

- What to do with a hundred piece orchestra and only your voice?
- Humans can naturally do a very piercing sound – a reflex. Opera singers have learnt to do so at will. They call it projection.
- Instruments and untrained singers decrease in amplitude as frequency increases above 200Hz
- An opera singer has a pronounced peak between 2000-3000Hz. < **The Singing Formant**
- Opera singers access this 'extra volume' by relax in the muscles and lowering the lowering the voice box.
- The negative side effect is that it distorts vowels, to an untrained ear, completely.

Closing: Why I sing!



It ain't over until the fat lady sings!

Soprano/Mezzo-soprano [A3-C6]



Maria Callas



Leona Lewis



Cecilia Bartoli



Renée Fleming



Birgit Nilsson

Contralto/Countertenor [F3-F5]



Marian Anderson



Adele



Philippe Jaroussky

Tenor [C3-C5]



Jussi Björling



Justin Timberlake



Jonas Kaufmann



Luciano Pavarotti

Baritone-Bass [E2-G4]



David Bowie



Peter Mattei



Bryn Terfel