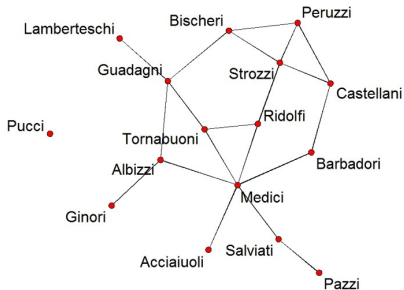


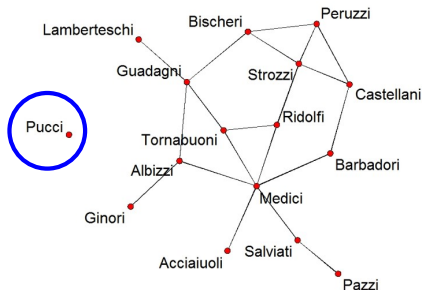
Intro to Social Network Analysis

Lorien Jasny¹

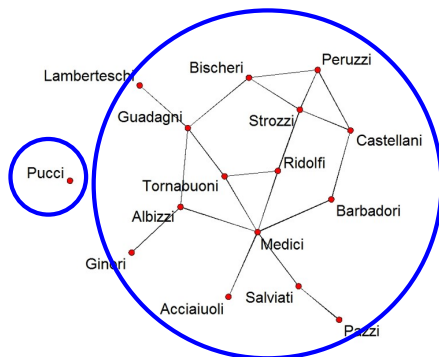
¹University of Exeter
`L.Jasny@exeter.ac.uk`

Online Workshop
September 2026

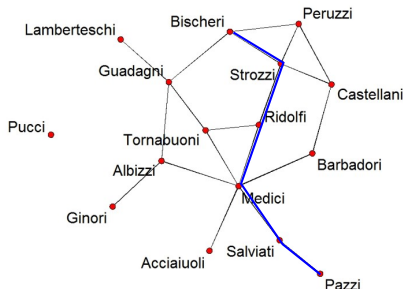




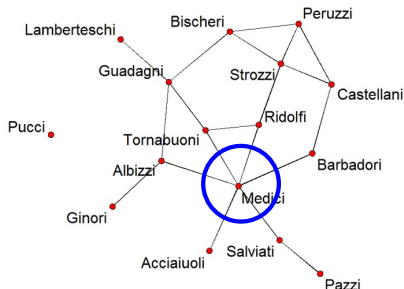
- One isolate



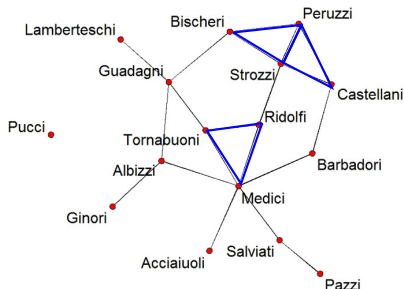
- One isolate
- Two components



- One isolate
- Two components
- Diameter is 5



- One isolate
- Two components
- Diameter is 5
- Medici is most popular



- One isolate
- Two components
- Diameter is 5
- Medici is most popular
- Three triads

For each node, its degree is

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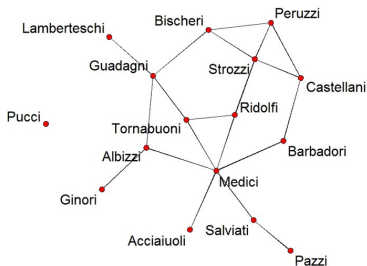
- the number of nodes adjacent to it

For each node, its degree is

- the number of nodes adjacent to it
- or, the number of lines incident with it

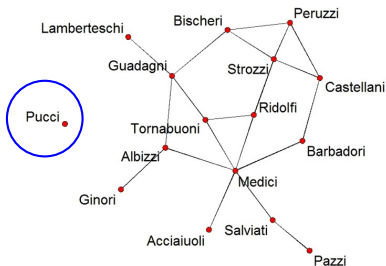
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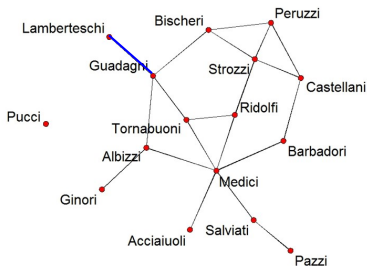
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- Pucci has degree 0

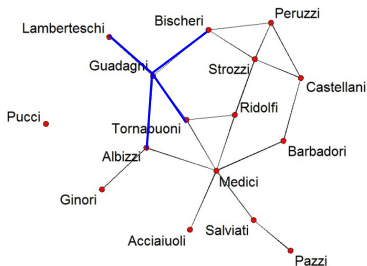
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- Pucci has degree 0
- Lamberteschi has degree 1

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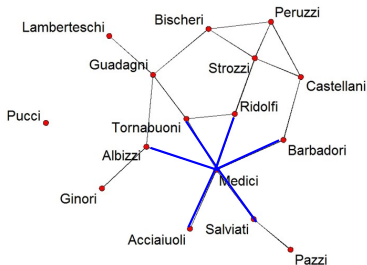
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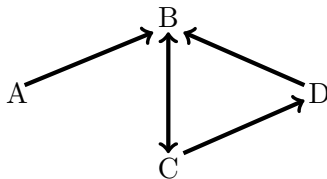
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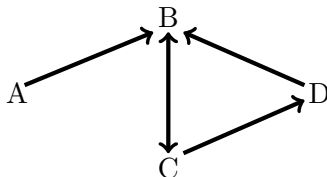
- Pucci has degree 0
- Lamberteschi has degree 1
- Guadagni has degree 4
- Medici has degree 6

In directed graphs,



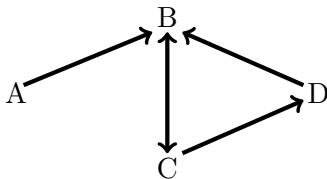
In directed graphs,

- *Indegree* indicates the number of received ties



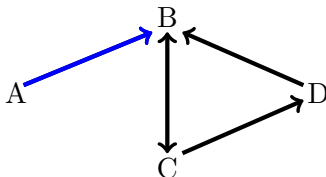
In directed graphs,

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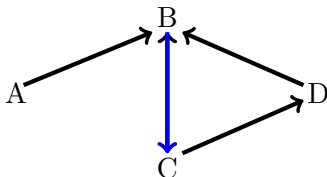
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-
- A has 1 **outdegree**



In directed graphs,

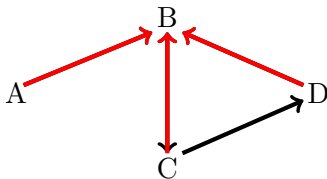
- *Indegree* indicates the number of received ties
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- A has 1 **outdegree**
- B has 1 **outdegree**

In directed graphs,

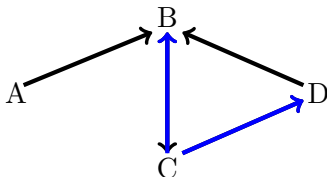
- *Indegree* indicates the number of received ties
- *Outdegree* indicates the number of sent ties



- A has 1 **outdegree**
- B has 1 **outdegree** and 3 **indegree**

In directed graphs,

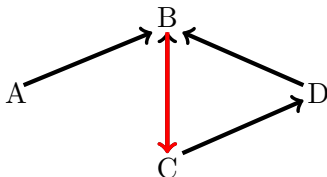
- *Indegree* indicates the number of received ties
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- A has 1 **outdegree**
- B has 1 **outdegree** and 3 **indegree**
- C has 2 **outdegree**

In directed graphs,

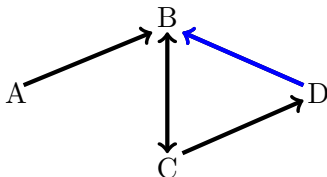
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In directed graphs,

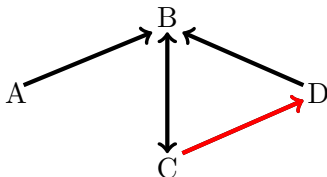
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- A has 1 **outdegree**
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- D has 1 **outdegree**

In directed graphs,

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- A has 1 **outdegree**
- B has 1 **outdegree** and 3 **indegree**
- C has 2 **outdegree** and 1 **indegree**
- D has 1 **outdegree** and 1 **indegree**

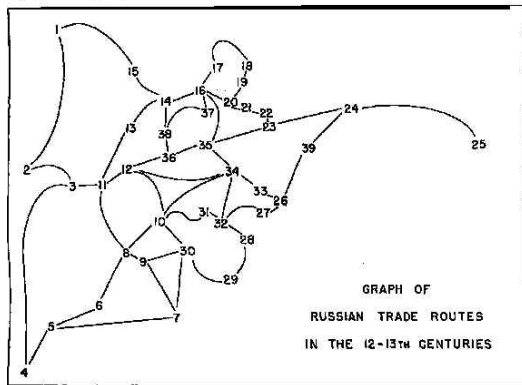
Betweenness Centrality

Proportion of
shortest paths
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Betweenness Centrality

Figure 2. *Graph of Russian trade routes in the 12th - 13th centuries.*

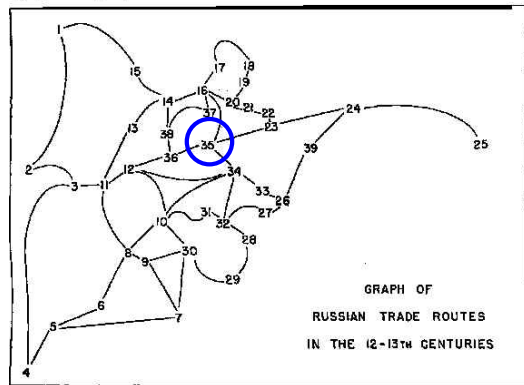


Forrest Pitts (1978)”

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Betweenness Centrality

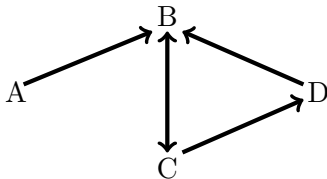
Figure 2. *Graph of Russian trade routes in the 12th - 13th centuries.*



Forrest Pitts (1978)”

Closeness Centrality – Alternate Measure

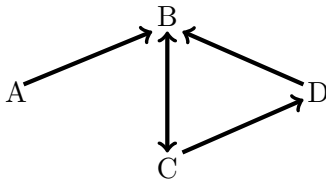
How far is each node from the other nodes in the graph



- $C^2(v) = \frac{\sum \frac{1}{d(v,i)}}{|V|-1}$
- B's Distance to
 - C = 1
 - D = 2
 - A = inf

Closeness Centrality – Alternate Measure

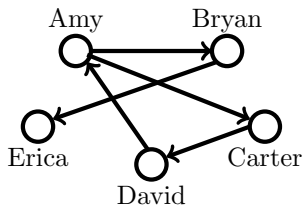
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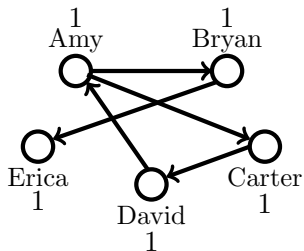


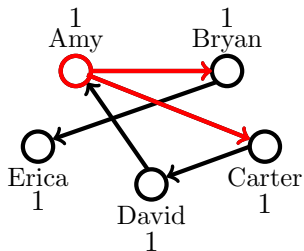
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- B's Closeness Centrality

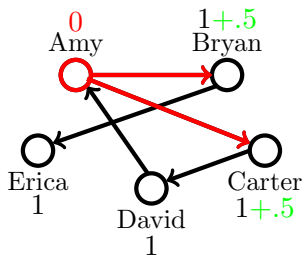
$$= \frac{\frac{1}{inf} + \frac{1}{1} + \frac{1}{2}}{3} = \frac{1}{2}$$

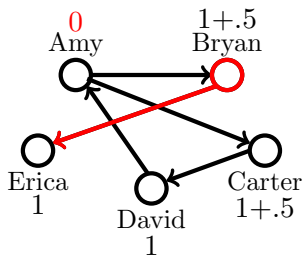
Page Rank and Eigenvector Centrality

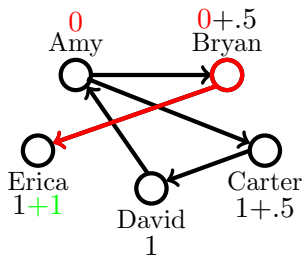


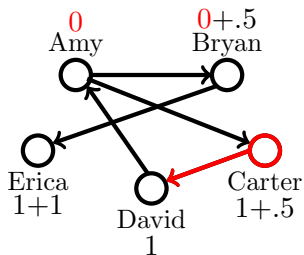


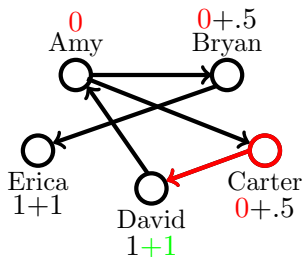


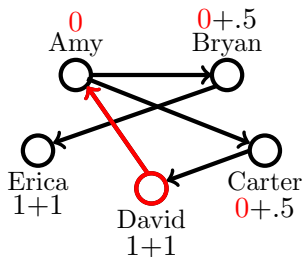


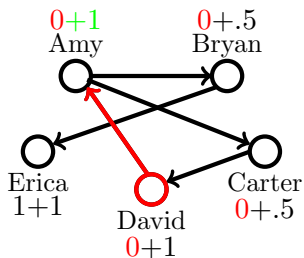


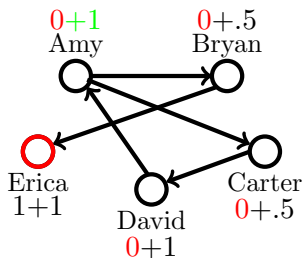


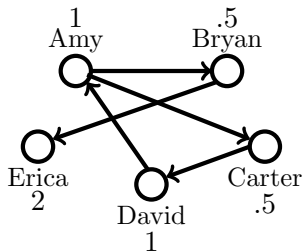












Other Centrality Measures

- Bonacich power centrality (1987)

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- Two-mode centrality measures (Everett and Borgatti 2005)

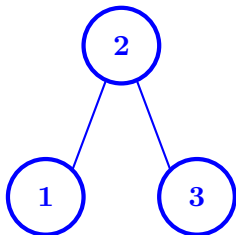
Graph Level Indices

- Number of ties, expressed as a percentage of the number of possible ties

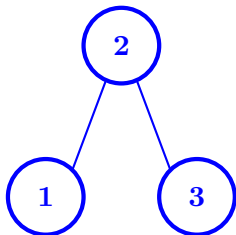
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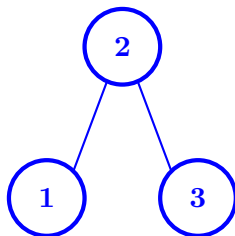


- Number of ties, expressed as a percentage of the number of possible ties
- For directed graphs: $\frac{E}{N(N-1)}$
- For undirected graphs: $\frac{\frac{E}{2}}{\frac{N(N-1)}{2}}$



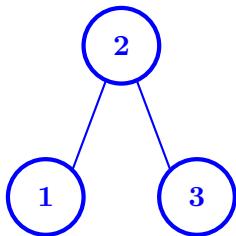
$$= \frac{\frac{2}{3(3-1)}}{\frac{2}{2}} = \frac{4}{6}$$

The mean degree, $\bar{d}$, of all nodes in the graph is



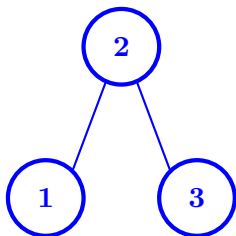
The mean degree, $\bar{d}$, of all nodes in the graph is

$$\bar{d}(n) = \frac{\sum_{i=1}^N d(n_i)}{N} = \frac{2E}{N}$$



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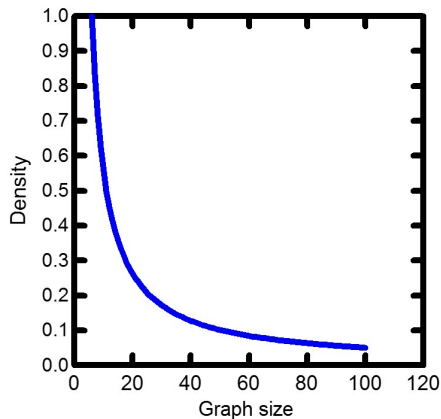
$$\begin{aligned}\bar{d}(n) &= \frac{\sum_{i=1}^N d(n_i)}{N} = \frac{2E}{N} \\ &= \frac{1+2+1}{3} = \frac{4}{3}\end{aligned}$$



Size, Density, and Mean Degree

If we hold the mean
degree constant, but
vary size, what
happens to density?

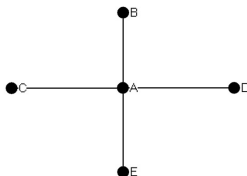
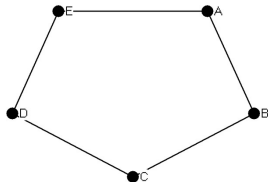
If we hold the mean degree constant, but vary size, what happens to density?



- Extent to which centrality is concentrated on a single vertex (Freeman 1978)

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- Calculated as the sum of the differences between each node's centrality score and the maximum score

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- Most centralized structure is usually a star network





Mutual (M)



Mutual (M)



Assymmetric (A)



Mutual (M)



Assymmetric (A)



Null (N)

- Dyadic: the proportion of dyads that are symmetric

$$\frac{M+N}{M+A+N}$$

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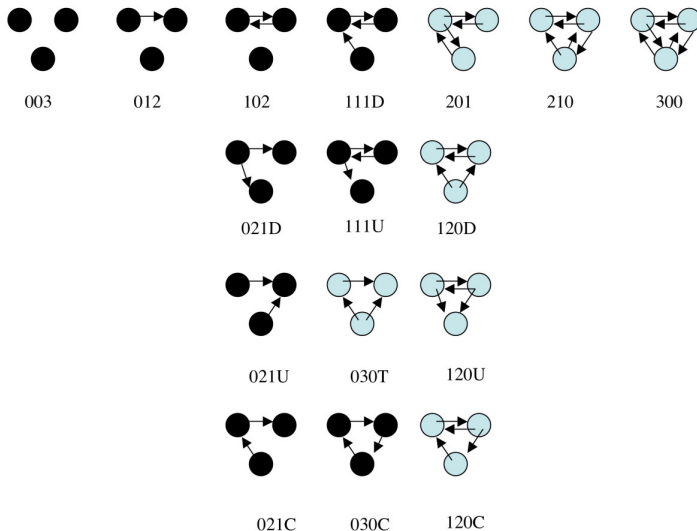
- Dyadic: the proportion of dyads that are symmetric

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- Dyadic non-null: the proportion of non-null dyads that

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- Edgewise: $\frac{2*M}{2*M+A}$



Holland and Leinhardt 1974

16 different triad types

One row per network



16 different triad types

One row per network

i, j cell is the number
of triad type j
in network i

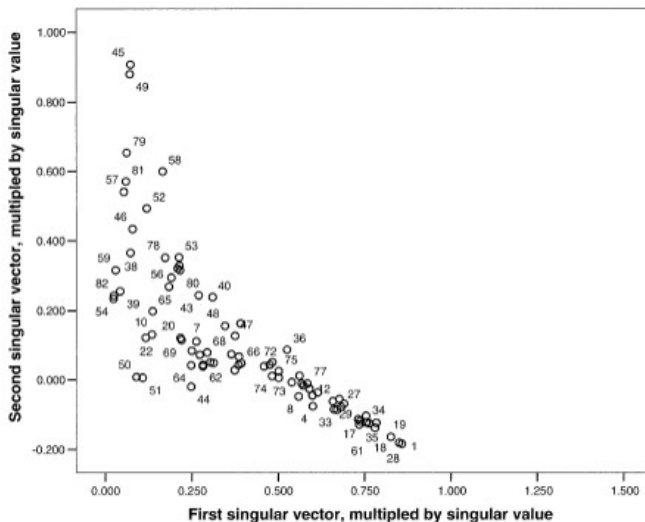


FIGURE 2. Singular value decomposition of triad census array, first two left singular vectors, multiplied by singular values, $N = 82$ networks.

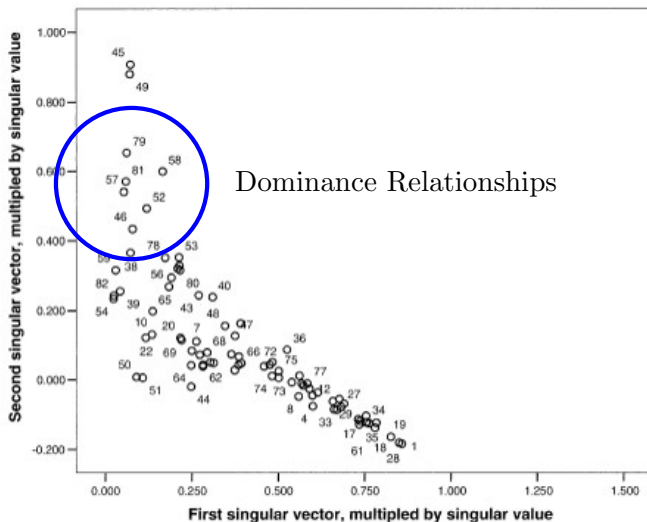


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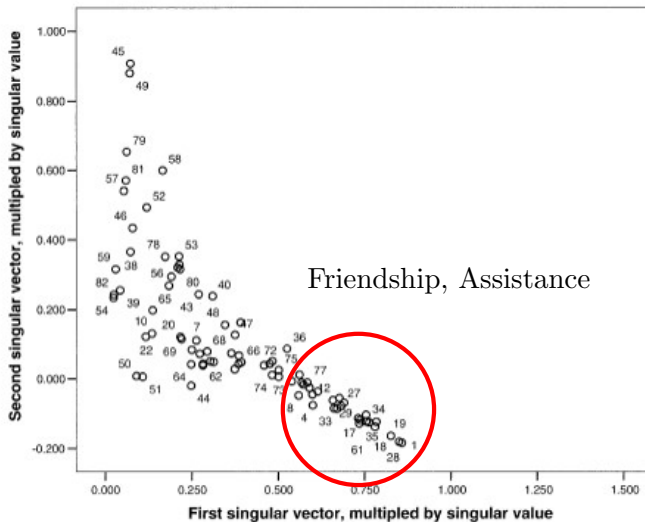
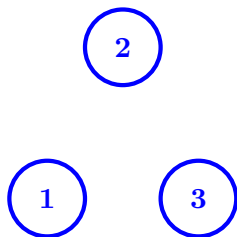
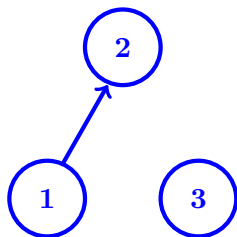
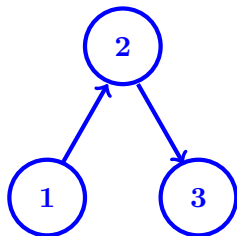
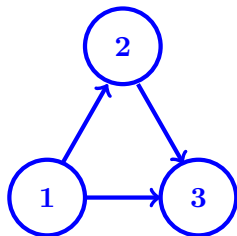


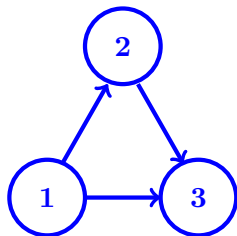
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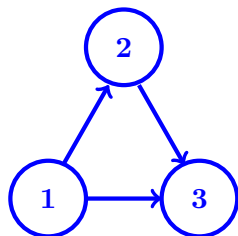




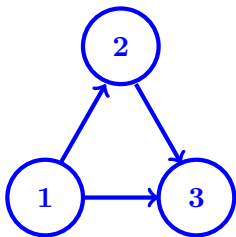




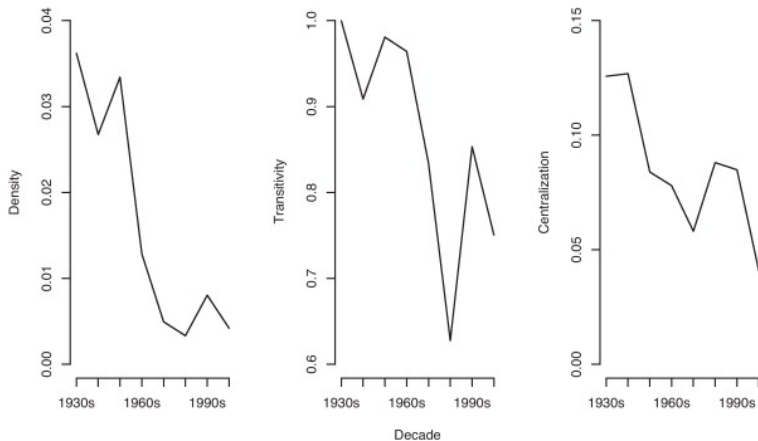
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- Related to Grannovetter's 'forbidden triad' (1983)

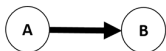


- Usually calculated as the fraction of completed two-paths
- Related to Grannovetter's 'forbidden triad' (1983)
- Can be directed or undirected



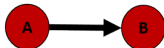
Janet Box-Steffensmeier and Dino Christenson
 “The evolution and formation of amicus curiae networks”
Social Networks 2012

a)



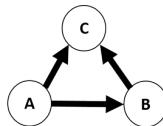
A sends information to B.

b)



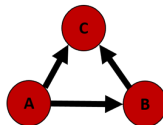
A and B agree so A's information echoes B's understanding.

c)



Transitive triad such that A sends information to B and C, and B also sends information to C. The smallest example of a 'chamber.'

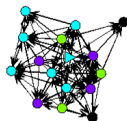
d)



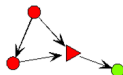
A transitive triad where each actor already holds the same position – an echo chamber.

Jasny et al 2015

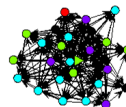
Representative Markey (D-MA)
16 actors, 90 ties, 82 chamber(s), 20 echo chamber(s)



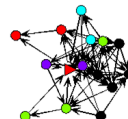
Representative Inhofe (R-OK)
4 actors, 4 ties, 1 chamber(s), 1 echo chamber(s)



Columbia University scientist
27 actors, 234 ties, 215 chamber(s), 39 echo chamber(s)



University of Alabama scientist
15 actors, 56 ties, 39 chamber(s), 4 echo chamber(s)



Jasny et al 2015



Jasny et al 2015

- Section 3-4.7

Forbidden Triad or Structural Hole?

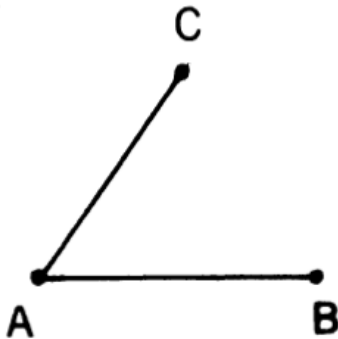


FIG. 1.—Forbidden triad

Forbidden Triad or Structural Hole?

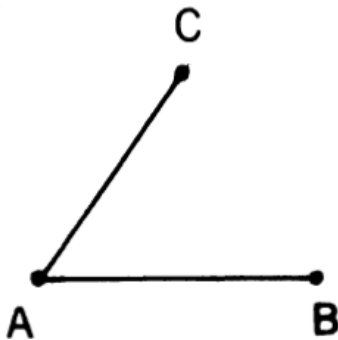
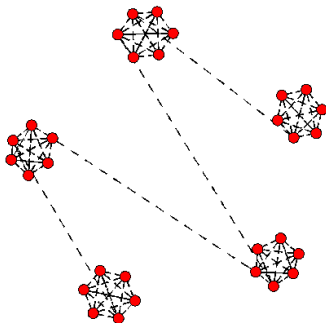


FIG. 1.—Forbidden triad

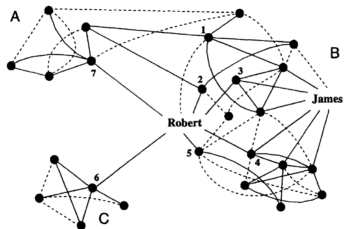
- Granovetter, Mark S. 1973.
“The Strength of Weak Ties”

Forbidden Triad or Structural Hole?



- Granovetter, Mark S. 1973.
“The Strength of Weak Ties”

Forbidden Triad or Structural Hole?



- Burt, Ronald S. 2004. “Structural Holes: The Social Structure of Competition”

Attributes!

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- Properties of nodes, edges, or even networks

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- Extension based on node attributes: Brokerage
- Extension based on edge attributes: Structural Balance

- Brokerage is a process “by which intermediary actors facilitate transactions between other actors lacking access to or trust in one another” (Marsden 1982)

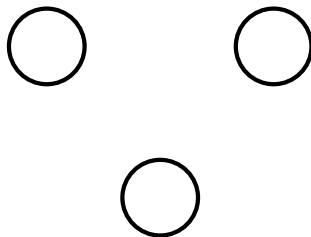
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- Brokers play a crucial role in knitting together diverse groups of people, organizations, parties
- Brokers can gain a lot – early access to information, prestige
- But can also be distrusted by everyone

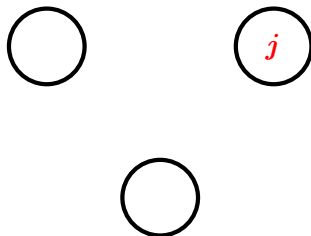
Brokerage: Formal Concept

In a network N with
edges E ,



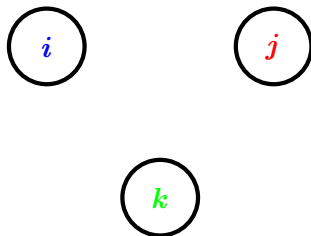
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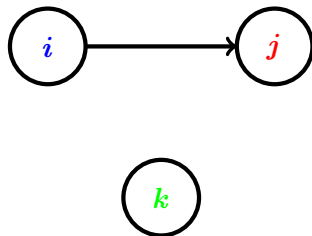
Brokerage: Formal Concept

In a network N with
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nodes i and k



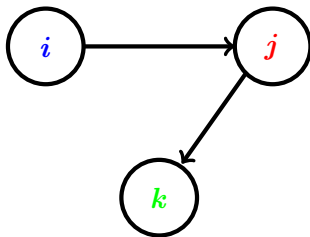
Brokerage: Formal Concept

In a network N with
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node j brokers
nodes i and k
if $e_{ij} \in E$



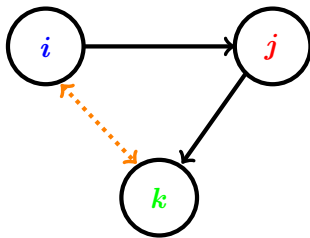
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Brokerage: Formal Concept

In a network N with
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Brokerage: Formal Concept

Gould and Fernandez (1989, 1994)

Brokerage: Formal Concept

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- formalized the concept

Brokerage: Formal Concept

Gould and Fernandez (1989, 1994)

- formalized the concept
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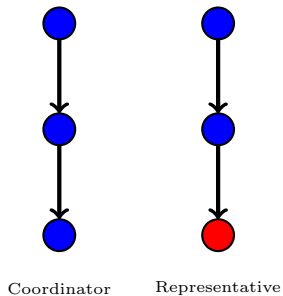
Brokerage: Formal Concept

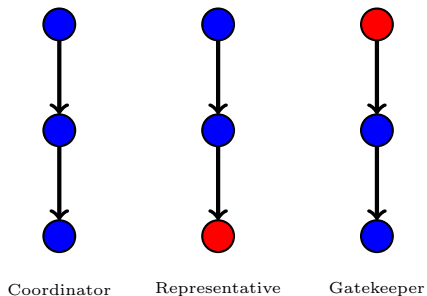
Gould and Fernandez (1989, 1994)

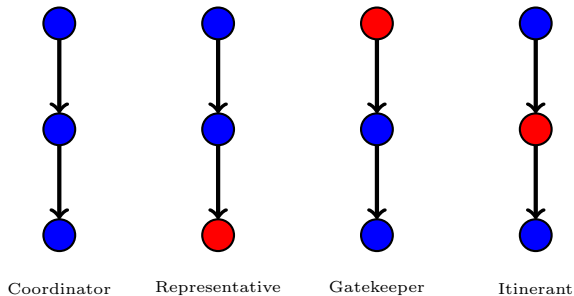
- formalized the concept
- added a vertex attribute component
- compared empirical brokerage counts to counts from random graphs conditioned on the number of edges

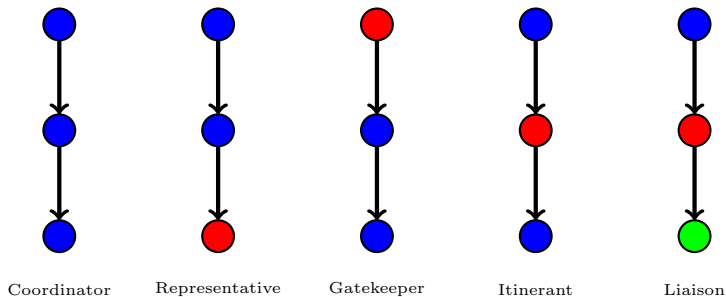


Coordinator









Gould and Fernandez' Findings

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- the benefits of brokerage are mediated both by the type of organization (the node sets) and the type of brokerage chain

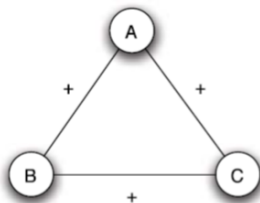
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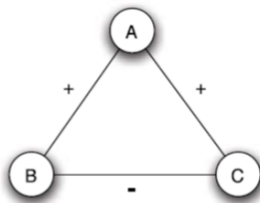
Gould and Fernandez' Findings

- the benefits of brokerage are mediated both by the type of organization (the node sets) and the type of brokerage chain
- non-governmental organizations were found to have more influence when they held any type of brokerage position
- governmental organizations gained influence only when they held “outsider” brokerage roles in itinerant and liaison chains

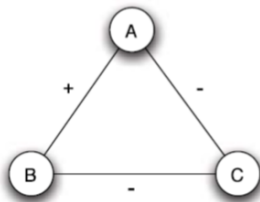
Structural Balance



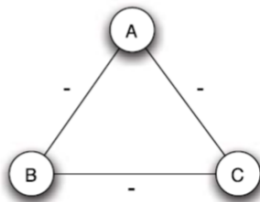
(a) *A, B, and C are mutual friends: balanced.*



(b) *A is friends with B and C, but they don't get along with each other: not balanced.*

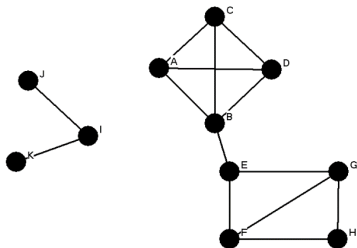


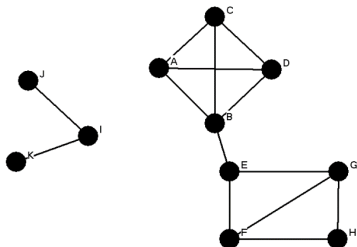
(c) *A and B are friends with C as a mutual enemy: balanced.*



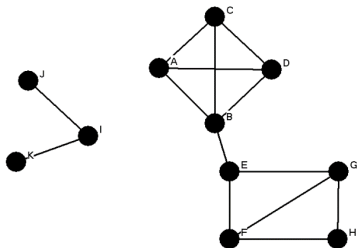
(d) *A, B, and C are mutual enemies: not balanced.*

Cartwright and Harary 1956

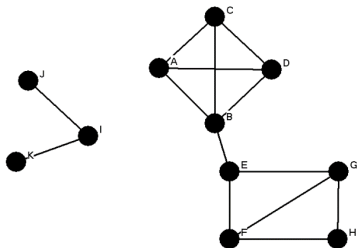


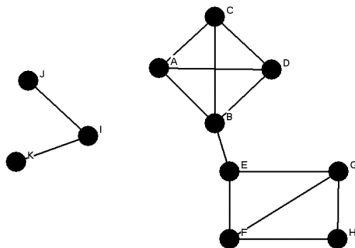


- Component: A maximal connected subgraph

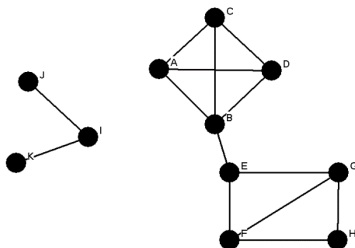


- Component: A maximal connected subgraph
- A subgraph is *maximal* with respect to some property if it has the property, but loses it with the addition of more nodes or edges

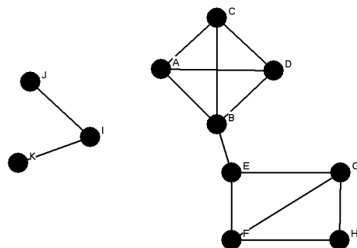




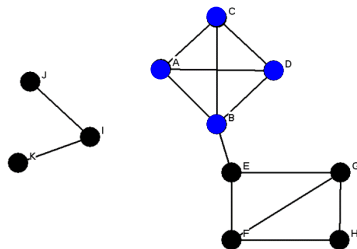
- A maximally complete subgraph of 3 or more



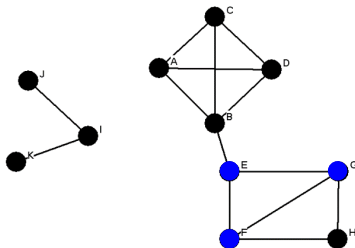
- A maximally complete subgraph of 3 or more
- All nodes are adjacent to all others in the subgraph



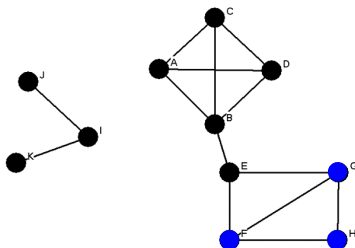
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Limitations of Cliques

- A very strict definition
- Often networks contain many small overlapping cliques
- Cliques have no internal structure

Subgroups based on reachability or diameter

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- n -clan: an n -clique in which the geodesic distance between all nodes in the subgraph is no greater than n **for paths within the subgraph**
- n -club: a maximal subgraph of diameter n

Subgroups based on nodal degree

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Subgroups based on nodal degree

- k -plex: a subgraph of g nodes in which each node is adjacent to no fewer than $g - k$ nodes in the subgraph
- k -core: a subgraph in which each node is adjacent to at least k other nodes in the subgroup

What to do with these subgroup definitions?

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- characterize the overall structure of the network

What to do with these subgroup definitions?

- characterize the overall structure of the network
- analyze the pattern of co-membership in cliques, etc.

RESEARCH ARTICLE

The Critical Periphery in the Growth of Social Protests

Pablo Barberá^{1*}, Ning Wang², Richard Bonneau^{3,4}, John T. Jost^{1,5,6}, Jonathan Nagler⁶, Joshua Tucker⁶, Sandra González-Bailón^{7*}

1 Center for Data Science, New York University, New York, New York, 10003, United States of America, **2** Mathematical Institute and Oxford Internet Institute, University of Oxford, Oxford, OX26GG, United Kingdom, **3** Center for Genomics and System Biology, New York University, New York, New York, 10003, United States of America, **4** Simons Center for Data Analysis, Simons Foundation, New York, New York, 10010, United States of America, **5** Department of Psychology, New York University, New York, New York, 10003, United States of America, **6** Department of Politics, New York University, New York, New York, 10012, United States of America, **7** Annenberg School for Communication, University of Pennsylvania, Philadelphia, Pennsylvania, 19104, United States of America

* pablo.barbera@nyu.edu (PB); sgonzalezbailon@asc.upenn.edu (SGB)

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al 2015

K-Core Example

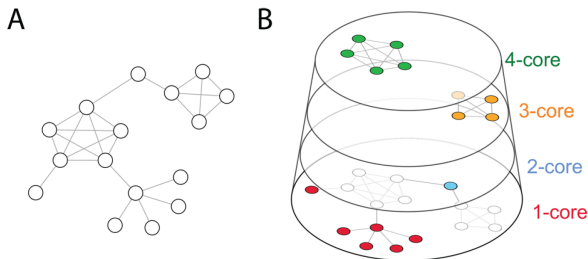


Fig 2. Schematic representation of the k -core decomposition for a random network with $N = 16$ vertices and $E = 24$ edges. This technique recursively prunes the network to remove nodes with the lowest degree. The coreness of a vertex is k if it belongs to the k -core but not to the $(k+1)$ -core.

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K-Core Example

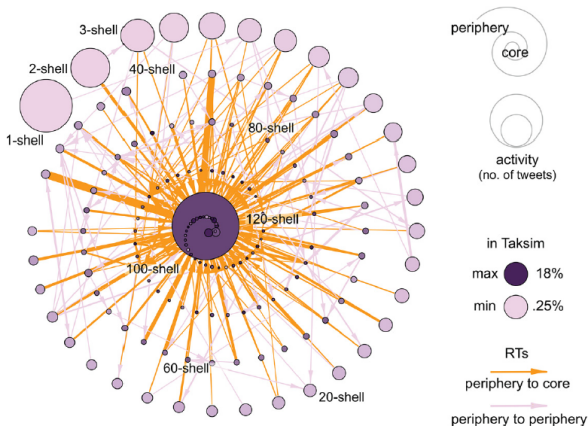
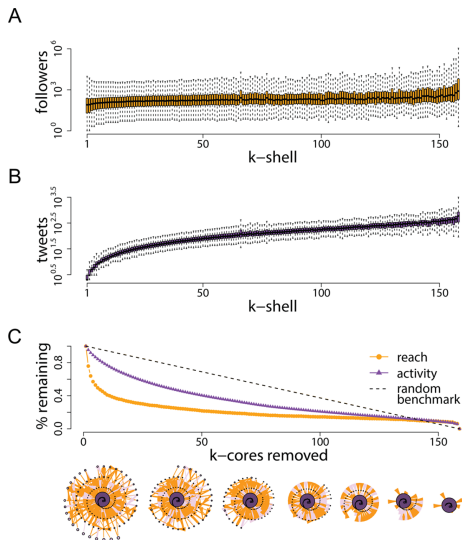


Fig 3. K-core decomposition of the network of retweets that emerged during the 2013 Taksim Gezi Park protests in Turkey (see [S1 Text](#)). Participants have been grouped in their corresponding k-shells, here represented by nodes. Lower k-shells contain participants at the periphery of the network; higher k-shells contain core participants. Node size is proportional to aggregated activity, measured as total number of protest messages (not just retweets). Arcs indicate retweeting activity, and their width is proportional to normalized strength (arcs with lower strength have been filtered to improve the visualization of the network). The darkness of nodes is proportional to the percentage of participants who reported being in the Taksim Gezi Park (the geographical epicenter of the protests), as indicated by the geographic information attached to their tweets. Most of these participants are at the core of the network where most RTs are also sourced from, thus allowing information to flow from the core to the periphery.

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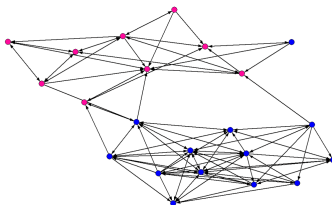
K-Core Example



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- Attribute Based

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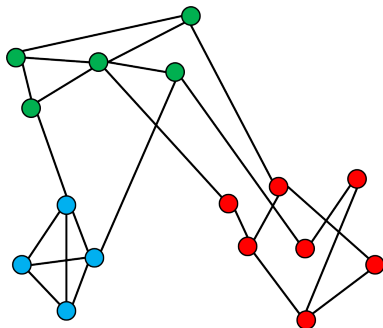


- Attribute Based

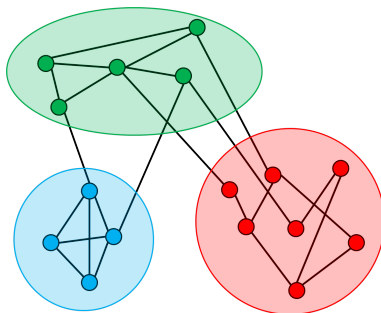
		Boys												Girls											
		1	2	4	5	5	7	8	9	9	0	1	2	6	3	4	3	6	7	8	0	1	2		
		1	2	1	1	5	1	1	1	9	1	1	1	1	1	4	3	6	7	8	2	2	2		
Boys	1 1				1	1	1						1			1	1								
	2 2				1	1	1		1	1	1	1													
	14 14	1	1		1	1	1	1	1	1	1	1	1												
	15 15	1	1		1	1	1	1	1	1	1	1													
	5 5	1	1	1				1	1				1					1							
	17 17	1	1		1	1	1	1	1	1	1	1													
	18 18	1			1	1						1													
	19 19				1	1				1			1												
	9 9				1	1							1												
	10 10																								
	11 11		1	1			1													1	1				
	12 12		1	1			1	1		1	1												1		
	16 16			1	1				1																
Girls	13 13															1				1	1	1			
	4 4		1													1					1	1			
	3 3																		1	1					
	6 6										1														
	7 7																		1						
	8 8																			1	1	1			
	20 20															1	1	1	1	1	1	1			
	21 21															1	1		1						
	22 22															1		1	1	1					

- Attribute Based
- Relation Based

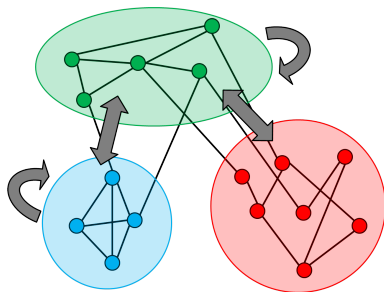
- Attribute Based
- Relation Based



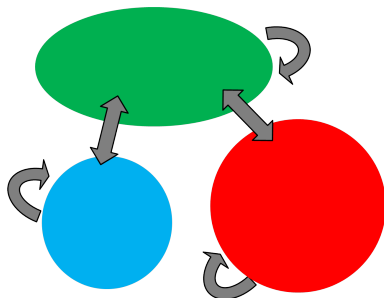
- Attribute Based
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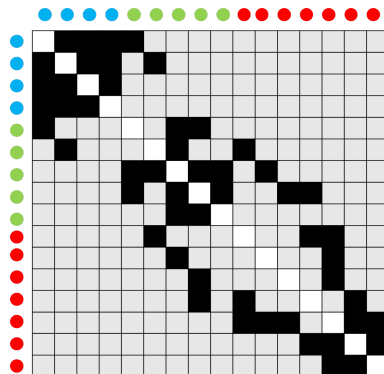
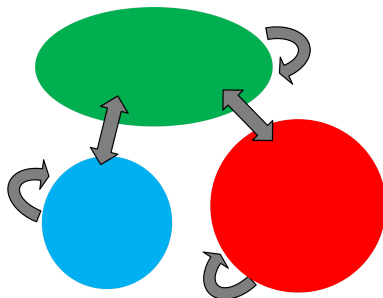
- Attribute Based
- Relation Based

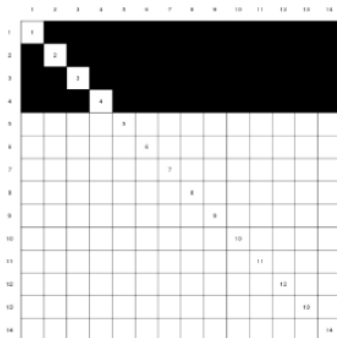


- Attribute Based
- Relation Based



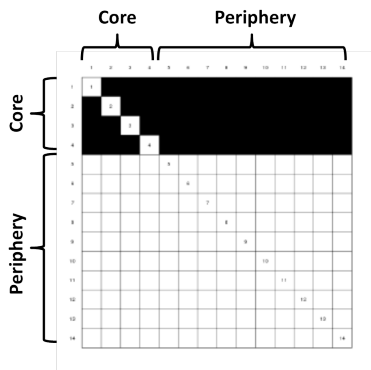
- Attribute Based
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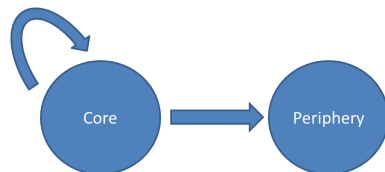
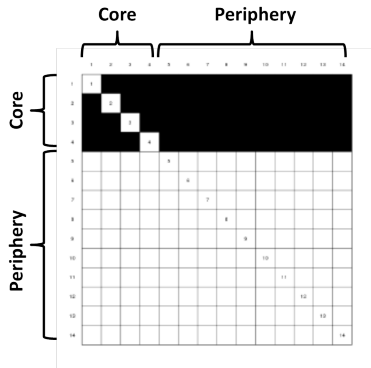


Core-Periphery Models

Core-Periphery Models

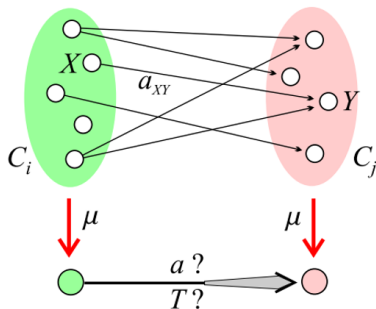


Core-Periphery Models



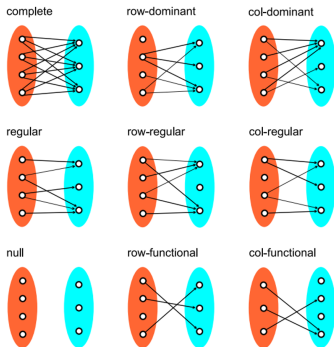
Generalized Blockmodeling

A *blockmodel* consists of structures obtained by identifying all units from the same cluster of the clustering C . For an exact definition of a blockmodel we have to be precise also about which blocks produce an arc in the *reduced graph* and which do not, and of what *type*. Some types of connections are presented in the figure on the next slide. The reduced graph can be represented by relational matrix, called also *image matrix*.



Doreian, Batagelj, and Ferligoj *Generalized Blockmodeling*, 2005

Even more blockmodeling



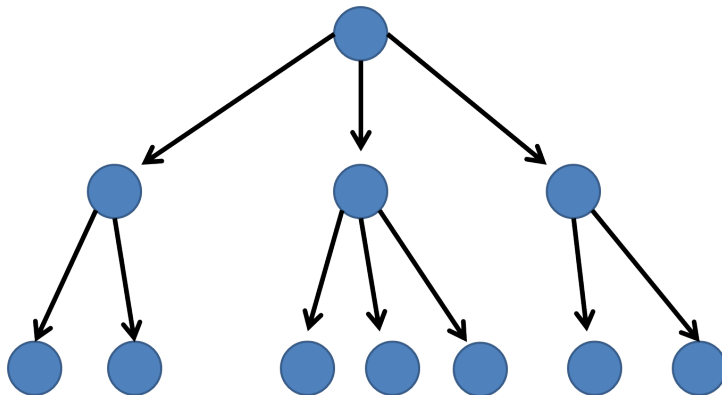
null	nul	all 0 *	
complete	com	all 1 *	
regular	reg	1-covered rows and columns	
row-regular	rre	each row is 1-covered	
col-regular	cre	each column is 1-covered	
row-dominant	rdo	$\exists$ all 1 row *	
col-dominant	cdo	$\exists$ all 1 column *	
row-functional	rfn	$\exists!$ one 1 in each row	
col-functional	cfn	$\exists!$ one 1 in each column	
non-null	one	$\exists$ at least one 1	

* except this may be diagonal

Doreian, Batagelj, and Ferligoj *Generalized Blockmodeling*, 2005

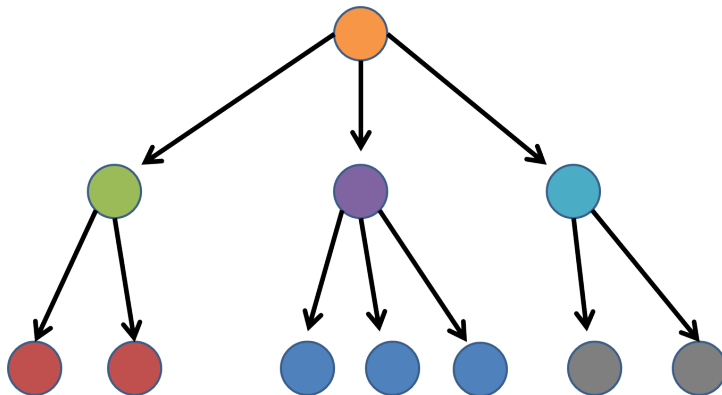
Structural Equivalence

Structural Equivalence



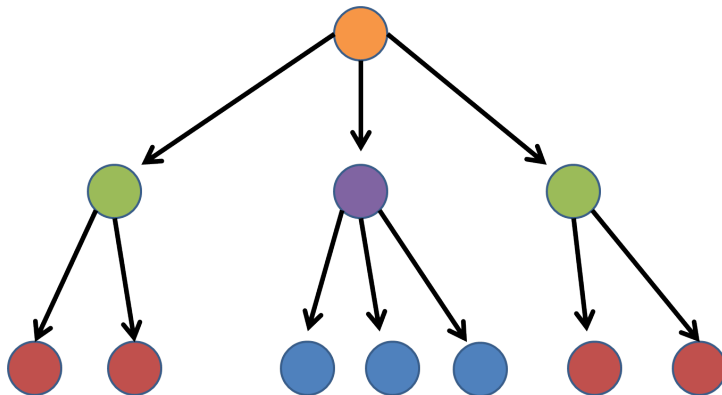
Sailer 1978

Structural Equivalence



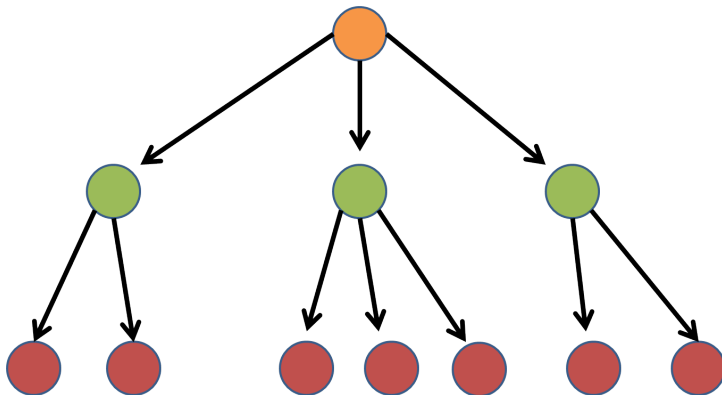
Sailer 1978

Automorphic Equivalence



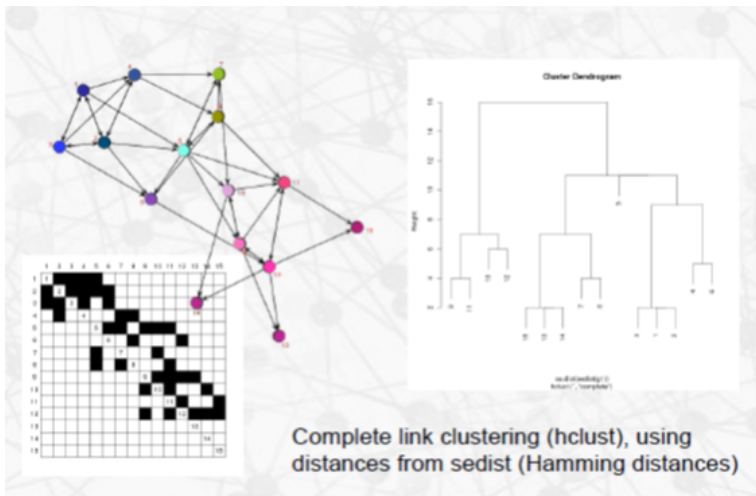
Sailer 1978

Regular Equivalence

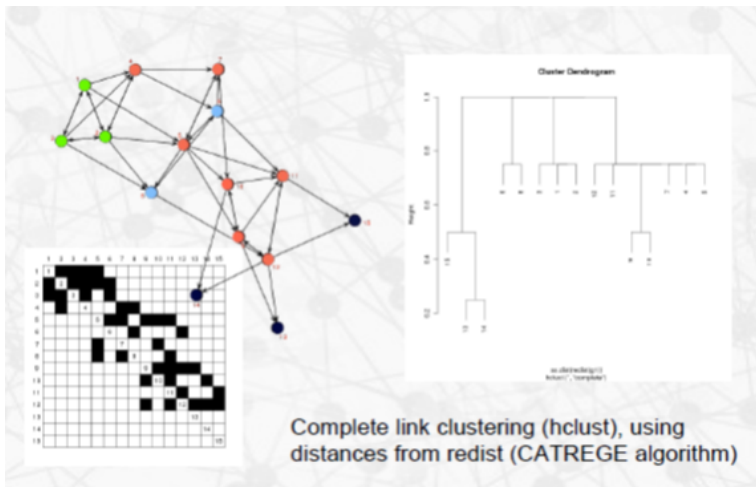


Sailer 1978

Structural Equivalence Example



Regular Equivalence Example



Everett and Borgatti 1994

Community Structure

- Modularity index measures the ‘goodness’ of an assignment of nodes to subgroups by comparing the number of ties within groups as compared to the number of expected ties

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