QS101: Introduction to Quantitative Methods in Social Science Week 11: Statistical Inference

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The Five Steps of a Significance Test

Significance Test for a Mean

Type I and Type II Errors

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Outline	The Five Steps of a Significance Test	Significance Test for a Mean	Type I and Type I
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The Five Steps of a Significance Test

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Hypotheses

- In empirical social science research, we try to find out, whether the data agree with certain predictions
- These predictions result from theories we want to test
- The predictions are called hypotheses

"In statistics, a hypothesis is a statement about a population. It is usually a prediction that a parameter describing some characteristic of a variable takes a particular numerical value or falls in a certain range of values." Agresti and Finlay, 2014, p. 143)

Examples

- For workers in service jobs, women and men have the same chance to be employed.
- Half of the UK population is happy with the Conservative government.

Significance Tests

- A significance test uses data to summarise the evidence about a hypothesis.
- It compares point estimates of parameters to the values predicted by the hypothesis.
- It has five parts:
 - 1. Assumptions
 - 2. Hypotheses
 - 3. Test statistic
 - 4. p-value
 - 5. Conclusion

1. Assumptions

- Type of data
- Randomisation
- Popuation Distribution
- Sample Size

2. Hypotheses

- Each significance test has TWO hypotheses about the value of a parameter
 - ▶ Null hypothesis (*H*₀): is a statement that the parameter takes a particular value, that usually indicates no effect.
 - Alternative hypothesis (*H_a*): states that the parameter falls into some alternative range of values, representing an effect of some type

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Examples again

- For workers in service jobs, women and men have the same chance to be employed.
- Half of the UK population is happy with the Conservative government.

3. Test Statistic

"The parameter to which the hypotheses refer has a point estimate. The test statistic summarizes how far that estimate falls from the parameter value in H_0 . Often this is expressed by the number of standard errors between the estimate and the H_0 value." (Agresti and Finlay, 2014, p. 145)

4. *p*-value

- ► We need to create a probability statement of the evidence against H₀.
- ► For this, we use the test statistic, under the assumption that H₀ is true.
- ► The purpose is to find out how unusual the observed test statistic value is compared to what *H*₀ predicts

Example 1 again

- For workers in service jobs, women and men have the same chance to be employed.
- Here we test against H_0 : $\pi = 0.5$, where π is the probability that a potential employee is male.

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$$H_0: \pi = 0.5$$

- H_a: π > 0.5
- ▶ If we observe 9/10 employees are male, (see graph)

Definition: *p*-value

The p-value is the probability that the test statistic equals the observed value or a value even more extreme in the direction predicted by H_a . It is calculated presuming that H_0 is true. The p-value is denoted by p. (Agresti and Finlay, 2014, p. 145)

The smaller the *p*-value, the stronger the evidence against H_0 .

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5. Conclusion

- *p*-value summarises the evidence against H_0
- ▶ If the *p*-value is sufficiently small, we reject H_0 , and accept H_a
- Most studies require $p \le 0.05$

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Significance Tests for a Mean

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1. Assumptions

- Randomisation
- Normal Distribution

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2. Hypotheses

▶ $H_0: \mu = \mu_0$, where μ_0 is a particular value for the population mean

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2. Hypotheses

- ▶ $H_0: \mu = \mu_0$, where μ_0 is a particular value for the population mean
- $H_a: \mu \neq \mu_0$, such as $H_a: \mu \neq 0$

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2. Hypotheses

- ▶ $H_0: \mu = \mu_0$, where μ_0 is a particular value for the population mean
- $H_a: \mu \neq \mu_0$, such as $H_a: \mu \neq 0$
- This is called a two-sided test

3. Test Statistic

- The sample mean \bar{y} estimates the population mean μ .
- We assume under H_0 that $\mu = \mu_0$ (see graph on the board)
- Center of the sampling distribution of \bar{y} is the value μ_0
- ► a value of ȳ that falls far out in the tail of the distribution would be unusual, and provide strong evidence against H₀

t-test statistic

- The evidence about H₀ is summarised by the number of standard errors that ȳ falls from the null hypothesis value μ₀
- The true standard error is $\sigma_{\bar{y}} = \frac{\sigma}{\sqrt{n}}$
- In reality, we do not know what σ (the standard deviation of the population) is
- ▶ We can estimate it, however, by $se = \frac{s}{\sqrt{n}}$, where s is the sample standard deviation
- The resulting test-statistic is the t-score

$$t = \frac{\bar{y} - \mu_0}{se}$$
, where $se = \frac{s}{\sqrt{n}}$

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t-test statistic (contd.)

- ▶ In principle, this is the same as the z-value from week 9
- BUT: We use s to estimate σ, and therefore introduce additional error
- This test uses the t-distribution (read chapter 5 in Finlay and Agresti, 2014)

4. *p*-value

- ► The *p*-value is the probability that the test statistic equals the observed value or a value even more extreme in the direction predicted by *H_a*.
- We have a two-sided test here
- ► It is therefore the probability that ȳ falls at least as far from µ₀ in either direction as the observed value of ȳ
- Assume t = 0.68, this gives us a p-value of 0.5 (0.25 on either side of the t-distribution)

5. Conclusion

• The smaller p, the stronger the evidence against H_0 .

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Type I and Type II Errors

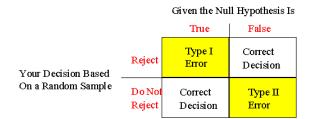
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Why not go for p = 0?

- Not possible (Therefore you CANNOT PROVE anything)
- We can merely make a decision between committing either of two errors
- These errors are called the Type I and Type II Errors

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The Relationship between Type I and Type II Errors



Two Types of Errors in Decision Making

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Court Trial

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- Court Trial
- ► H₀: Defendant is innocent

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- Court Trial
- ► *H*₀: Defendant is innocent
- ► *H*_a: Defendant is guilty

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Why does it matter?

- Court Trial
- ► *H*₀: Defendant is innocent
- H_a: Defendant is guilty
- Type I error: We send an innocent person to jail

- Court Trial
- ► *H*₀: Defendant is innocent
- H_a: Defendant is guilty
- Type I error: We send an innocent person to jail
- ► Type II error: We let a guilty person run free