QS101: Introduction to Quantitative Methods in Social Science

Week 11: Statistical Inference

Dr. Florian Reiche
Teaching Fellow in Quantitative Methods
Course Director BA Politics and Sociology
Deputy Director of Student Experience and Progression

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The Five Steps of a Significance Test

Hypotheses

- In empirical social science research, we try to find out, whether the data agree with certain predictions
- These predictions result from theories we want to test
- The predictions are called hypotheses

"In statistics, a hypothesis is a statement about a population. It is usually a prediction that a parameter describing some characteristic of a variable takes a particular numerical value or falls in a certain range of values." Agresti and Finlay, 2014, p. 143)

Examples

- For workers in service jobs, women and men have the same chance to be employed.
- Half of the UK population is happy with the Conservative government.

Significance Tests

- A significance test uses data to summarise the evidence about a hypothesis.
- It compares point estimates of parameters to the values predicted by the hypothesis.
- It has five parts:
    1. Assumptions
    2. Hypotheses
    3. Test statistic
    4. *p*-value
    5. Conclusion

1. Assumptions

- Type of data
- Randomisation
- Popuation Distribution
- Sample Size

2. Hypotheses

- Each significance test has TWO hypotheses about the value of a parameter
  - Null hypothesis (H_0): is a statement that the parameter takes a particular value, that usually indicates no effect.
  - Alternative hypothesis (H_a): states that the parameter falls into some alternative range of values, representing an effect of some type

Examples again

- For workers in service jobs, women and men have the same chance to be employed.
- Half of the UK population is happy with the Conservative government.

3. Test Statistic

"The parameter to which the hypotheses refer has a point estimate. The test statistic summarizes how far that estimate falls from the parameter value in H_0. Often this is expressed by the number of standard errors between the estimate and the H_0 value." (Agresti and Finlay, 2014, p. 145)

4. p-value

- We need to create a probability statement of the evidence against H_0.
- For this, we use the test statistic, under the assumption that H_0 is true.
- The purpose is to find out how unusual the observed test statistic value is compared to what H_0 predicts

Example 1 again

- For workers in service jobs, women and men have the same chance to be employed.
- Here we test against H_0: $\pi = 0.5$, where π is the probability that a potential employee is male.
- H_0: $\pi = 0.5$
- H_a: $\pi > 0.5$
- If we observe 9/10 employees are male, (see graph)

Definition: p-value

> *The p-value is the probability that the test statistic equals the observed value or a value even more extreme in the direction predicted by H_a. It is calculated presuming that H_0 is true. The p-value is denoted by p. (Agresti and Finlay, 2014, p. 145)*

The smaller the p-value, the stronger the evidence against H_0.

5. Conclusion

- p-value summarises the evidence against H_0
- If the p-value is sufficiently small, we reject H_0, and accept H_a
- Most studies require $p \leq 0.05$

Significance Tests for a Mean

1. Assumptions

- Randomisation
- Normal Distribution

2. Hypotheses

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2. Hypotheses

- $H_0 : \mu = \mu_0$, where μ_0 is a particular value for the population mean
- $H_a : \mu \neq \mu_0$, such as $H_a : \mu \neq 0$
- This is called a *two-sided* test

3. Test Statistic

- The sample mean $\bar{y}$ estimates the population mean μ.
- We assume under H_0 that $\mu = \mu_0$ (see graph on the board)
- Center of the sampling distribution of $\bar{y}$ is the value μ_0
- a value of $\bar{y}$ that falls far out in the tail of the distribution would be unusual, and provide strong evidence against H_0

t-test statistic

- The evidence about H_0 is summarised by the number of standard errors that $\bar{y}$ falls from the null hypothesis value μ_0
- The true standard error is $\sigma_{\bar{y}} = \frac{\sigma}{\sqrt{n}}$
- In reality,we do not know what σ (the standard deviation of the population) is
- We can estimate it, however, by $se = \frac{s}{\sqrt{n}}$, where s is the sample standard deviation
- The resulting test-statistic is the t-score

$$t = \frac{\bar{y} - \mu_0}{se}, \text{ where } se = \frac{s}{\sqrt{n}}$$

t-test statistic (contd.)

- In principle, this is the same as the z-value from week 9
- BUT: We use s to *estimate* σ, and therefore introduce additional error
- This test uses the t-distribution (read chapter 5 in Finlay and Agresti, 2014)

4. p-value

- The p-value is the probability that the test statistic equals the observed value or a value even more extreme in the direction predicted by H_a.
- We have a two-sided test here
- It is therefore the probability that $\bar{y}$ falls at least as far from μ_0 in either direction as the observed value of $\bar{y}$
- Assume $t = 0.68$, this gives us a p-value of 0.5 (0.25 on either side of the t-distribution)

5. Conclusion

- The smaller p, the stronger the evidence against H_0.

Type I and Type II Errors

Why not go for $p = 0$?

- Not possible (Therefore you CANNOT PROVE anything)
- We can merely make a decision between committing either of two errors
- These errors are called the Type I and Type II Errors

The Relationship between Type I and Type II Errors

Your Decision Based On a Random Sample	Given the Null Hypothesis Is: True	Given the Null Hypothesis Is: False
Reject	Type I Error	Correct Decision
Do Not Reject	Correct Decision	Type II Error

Two Types of Errors in Decision Making

Why does it matter?

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- Court Trial
- H_0: Defendant is innocent
- H_a: Defendant is guilty
- Type I error: We send an innocent person to jail
- Type II error: We let a guilty person run free