Critical exponent of the quantum Hall transition

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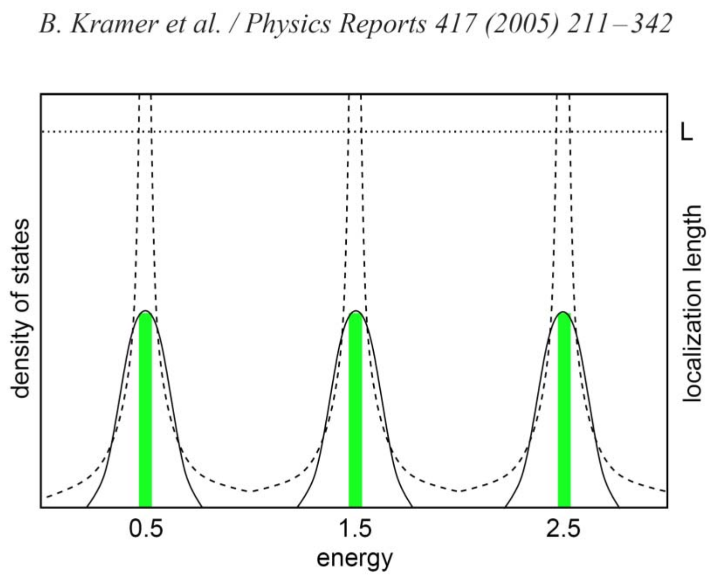
Quantum Hall Effect



$$\xi \approx \xi_0 \left| E - E_0 \right|^{-\nu}$$

- Landau Levels (LL)
  - Broadened by disorder
  - All states localised except for a single state at the centre of the LL
  - Localisation length diverges at the centre of the LL
  - Divergence described by a critical exponent

Kramer, B., T. Ohtsuki, et al. (2005). "Random network models and quantum phase transitions in two dimensions." Physics Reports **417(5-6): 211-342.**

Li et al

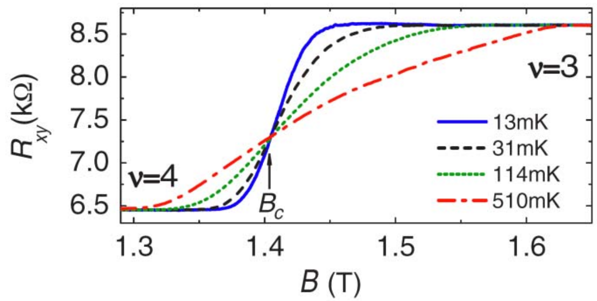


FIG. 1 (color online). Hall resistance around the 4-3 transition at different temperatures. A critical field of $B_c = 1.4$ T is observed.

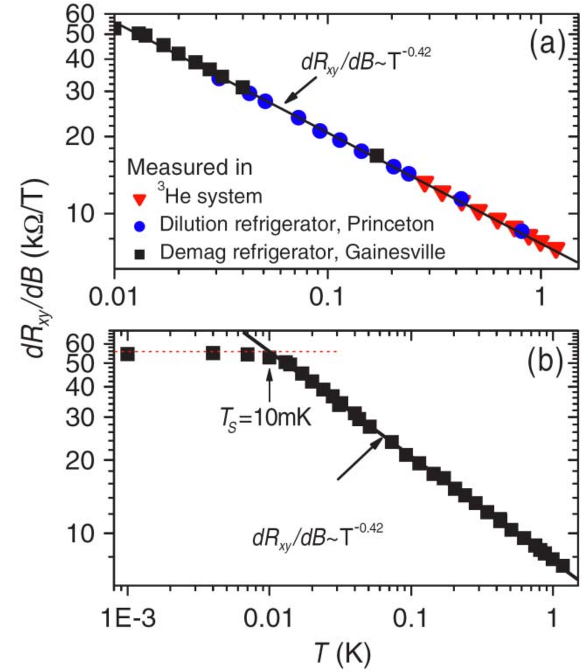


FIG. 2 (color online). (a) Perfect temperature scaling $(dR_{xy}/dB)|_{B_c} \propto T^{-0.42}$ of the 4-3 transition over two decades of temperature between 1.2 K and 12 mK. (b) Saturation of $(dR_{xy}/dB)|_{B_c}$ at low temperatures. The saturation temperature $T_s = 10$ mK is obtained from the cross point between extrapolations of the higher temperature data (black line) and the lower temperature saturated data (horizontal dotted line).

Li, W., C. L. Vicente, et al. (2009). Physical Review Letters **102(21): 216801-4.**

Li et al

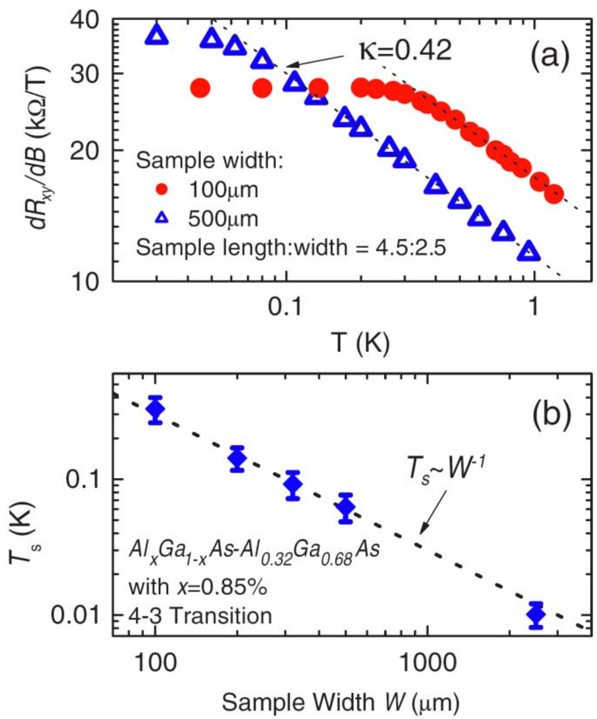


FIG. 3 (color online). (a) $(dR_{xy}/dB)|_{B_c}$ vs T of the 4-3 transition for two samples of different size. The length/width of samples is kept to be 4.5:2.5. (b) The sample size dependence of the saturation temperature T_s of $(dR_{xy}/dB)|_{B_c}$. The value of T_s is inversely proportional to the sample width W within the error.

$$\frac{1}{z\nu} = 0.42 \pm 0.01$$

$$z = 1$$

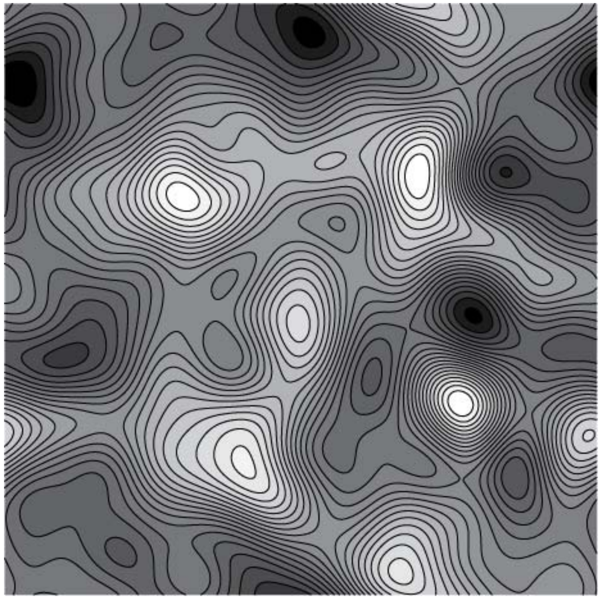
$$\nu = 2.38$$

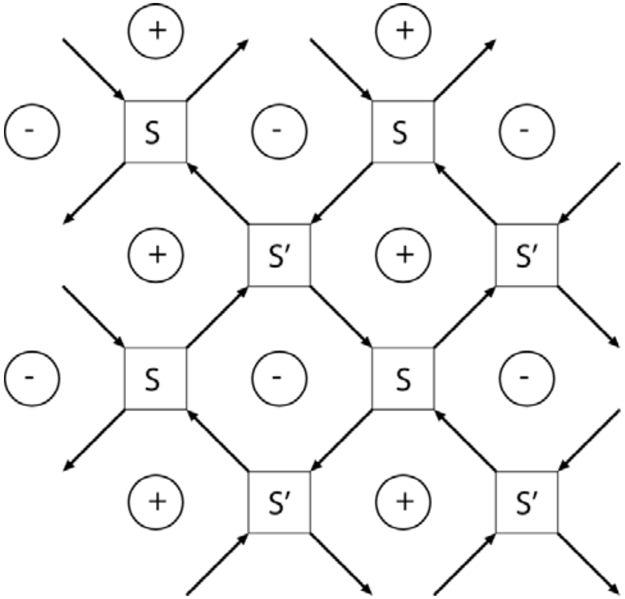
The consensus

Chalker & Coddington	2.5 ± 0.5
Huckestein & Kramer	2.34 ± 0.04
Cain et al	2.37 ± 0.02
(Milnikov & Sokolov)	(7/3)
Lee & Wang	2.33 ± 0.03

"It is now generally believed that $\nu = 2.4$", Li et al.

Network Model of 2DEG in B field





Chalker-Coddington Model

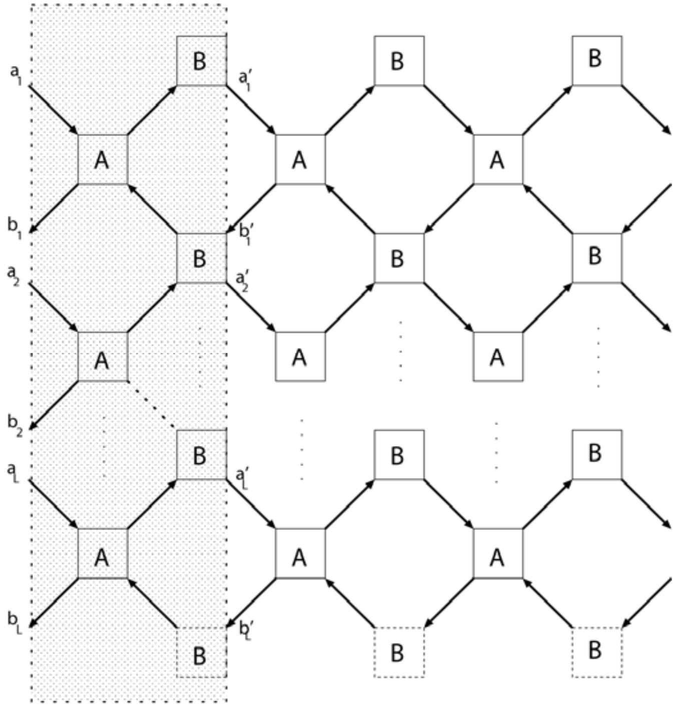
- Nodes described by 2 × 2 scattering matrices
- Random phases on the links
- Energy x in units of Landau band width
- LL center x=0

$$S = \begin{pmatrix} e^{i\varphi_1} & 0 \\ 0 & e^{i\varphi_2} \end{pmatrix} \begin{pmatrix} -r & t \\ t & r \end{pmatrix} \begin{pmatrix} e^{i\varphi_3} & 0 \\ 0 & e^{i\varphi_4} \end{pmatrix}$$

$$S' = \begin{pmatrix} e^{i\varphi_1'} & 0 \\ 0 & e^{i\varphi_2'} \end{pmatrix} \begin{pmatrix} -t & r \\ r & t \end{pmatrix} \begin{pmatrix} e^{i\varphi_3'} & 0 \\ 0 & e^{i\varphi_4'} \end{pmatrix}$$

$$t = \frac{1}{\sqrt{e^{+2x}+1}} \qquad r = \frac{1}{\sqrt{e^{-2x}+1}}$$

Transfer Matrix of CC Model



- Divide system into layers

$$T_l = BV_l AU_l$$

$$T = \prod_{l=1}^{L} T_l$$

Lyapunov Exponents

- As a consequence of current conservation the eigenvalues of the matrix

$$\Omega = \ln T^{\dagger} T$$

- occur in pairs of opposite sign

$$\{+\nu_1, \cdots, +\nu_N, -\nu_N, \cdots, -\nu_1\} \quad \nu_1 > \nu_2 > \cdots > \nu_N > 0$$

- The Lyapunov exponents (LEs) are the limiting values

$$\gamma_i = \lim_{L \to \infty} \frac{\nu_i}{2L}$$

Smallest positive exponent

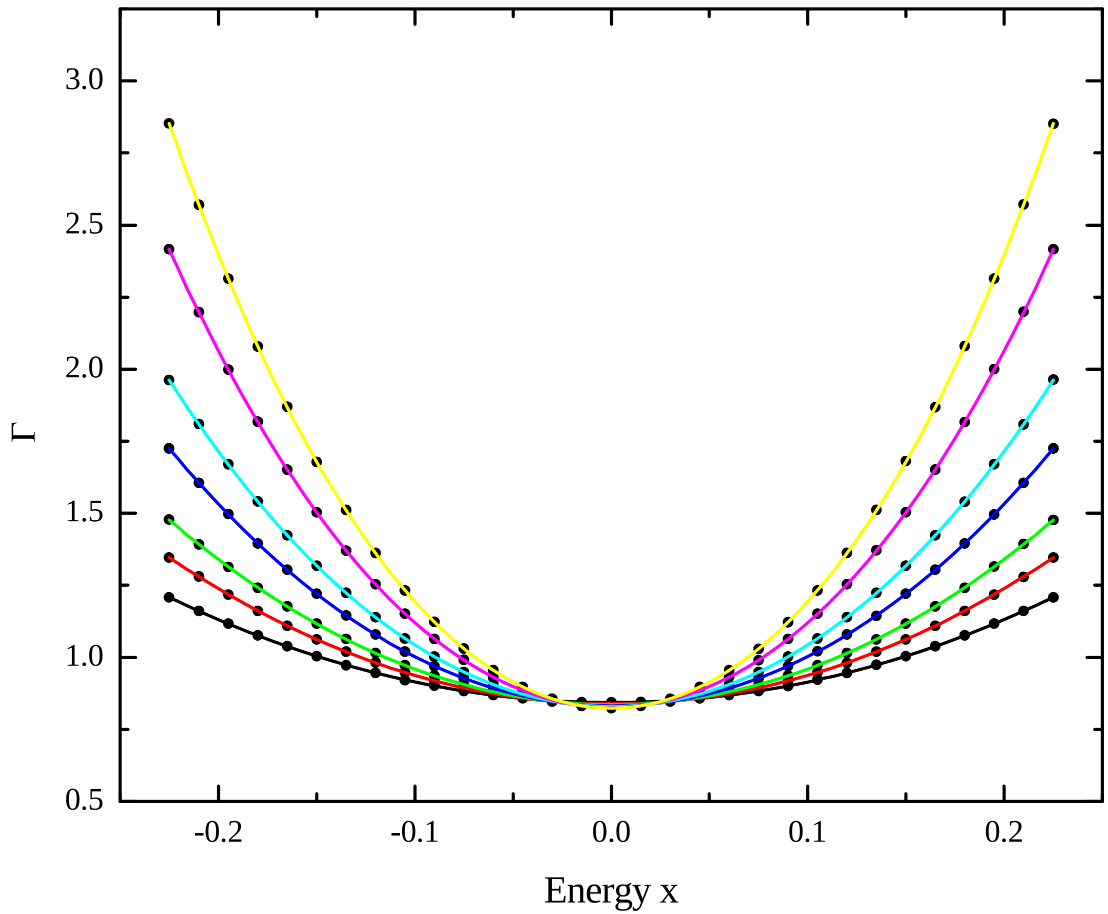
- It is usual (but not necessary) to focus on the smallest positive exponent

$$\gamma \equiv \gamma\left(x, N\right) \quad \Gamma \equiv \gamma N$$

- The output of the simulation is estimates of the LE as a function of
  - energy x
  - cross section N

  with specified precision σ

Γ vs x



N=16, 24, 32, 48, 64, 96, 128 Precision 0.03%

Finite Size Scaling

- Localised Phase

$$\Gamma \approx \frac{N}{\xi} \quad N \to \infty$$

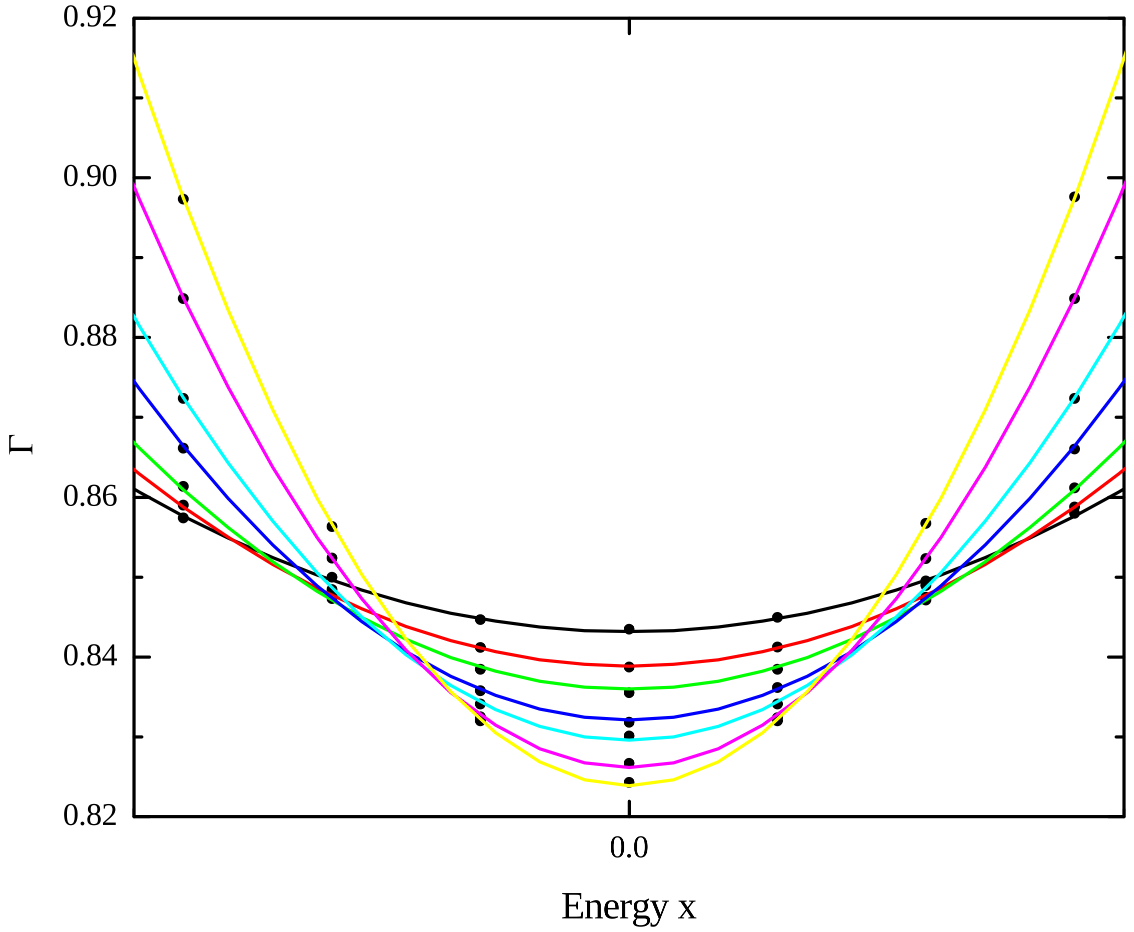
- Scale invariance at critical point

$$N \approx \text{constant} \quad N \to \infty$$

- FSS law

$$\Gamma = F_0\left(N^{\alpha}\left(x - x_c\right)\right) \quad \alpha = \frac{1}{\nu} \quad \left(x_c = 0\right)$$

Near the Critical Point



Corrections to Scaling

- Irrelevant scaling variable
- Non-linearity of the scaling variables

$$\begin{aligned}\Gamma &= F\left(N^{\alpha}u_0(x), N^{y}u_1(x)\right)\\ &\approx F_0\left(N^{\alpha}u_0(x)\right) + N^{y}u_1(x)F_1\left(N^{\alpha}u_0(x)\right)\end{aligned}$$

K. Slevin and T. Ohtsuki (1999). "Corrections to Scaling at the Anderson Transition." Physical Review Letters **82(2): 382-385.**

FSS Results

N	16, 24, 32, 48, 64, 96, 128
number of data	217
number of parameters	9
chi-squared	199.8
goodness of fit	0.6
α	0.3857 [0.3849,0.3866]
Γc	0.78 [0.767, 0.788]
y	-0.17 [-0.21, -0.14]

$$\nu = 2.593 \quad \left[2.587, 2.598\right]$$

End of the consensus…

Slevin & Ohtsuki	2.593 [2.587, 2.598]
Chalker & Coddington	2.5 ± 0.5
Huckestein & Kramer	2.34 ± 0.04
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Conformal Symmetry

- Scaling relations for 2D systems with ATs
  - strips $\Leftrightarrow$ 2D
  - Position α_0 of the maximum of the $f(\alpha)$ spectrum for a 2D system with Λ_c

$$\Gamma_c = \pi\left(\alpha_0 - 2\right)$$

- Confirmed for 2D SU(2) model
  - α_0=2.173±0.001 (Obuse et al 2007)
  - α_0=2.1727±0.0001
    - (applying scaling relation to Asada et al 2004)

Conformal symmetry and QHE?

- Is this relation obeyed at the QHE transition?

	α_0
Obuse et al	2.2617 ± 0.0006
Evers et al	2.2596 ± 0.0004
Slevin et al	2.248 [2.244,2.251]

- Obuse, H., A. R. Subramaniam, A. Furusaki, I. A. Gruzberg, A. W. W. Ludwig. Physical Review Letters, 2008. **101** 116802
- Evers, F., A. Mildenberger, and A.D. Mirlin. Physical Review Letters, 2008. **101** 116803

Summary

- Previous work seems to have underestimated the exponent

$$\nu = 2.593 \quad [2.587, 2.598]$$

- Agreement with experiment seems to have been fortuitous
- Further work needed to confirm conformal invariance

Slevin, K. and T. Ohtsuki, *Critical exponent for the quantum Hall transition*. Physical Review B (Condensed Matter and Materials Physics), 2009. **80**(4): p. 041304-4.