

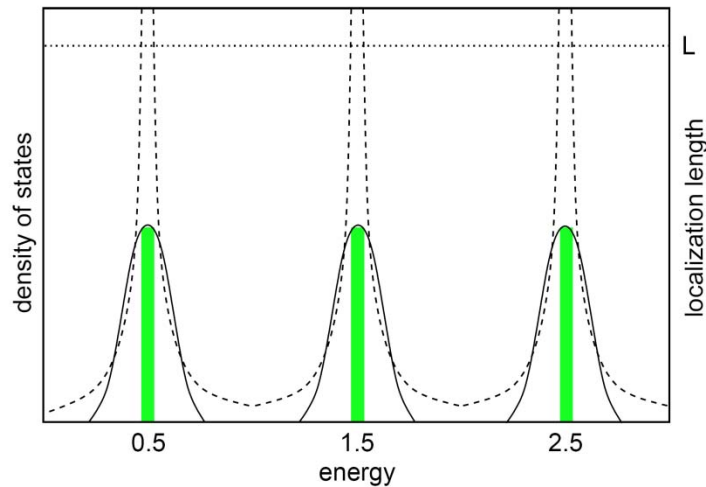
Critical exponent of the quantum Hall transition

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Quantum Hall Effect

B. Kramer et al. / Physics Reports 417 (2005) 211–342



$$\xi \approx \xi_0 |E - E_0|^{-\nu}$$

- Landau Levels (LL)
 - Broadened by disorder
 - All states localised except for a single state at the centre of the LL
 - Localisation length diverges at the centre of the LL
 - Divergence described by a critical exponent

Kramer, B., T. Ohtsuki, et al. (2005). "Random network models and quantum phase transitions in two dimensions." *Physics Reports* **417(5-6): 211-342.**

Li et al

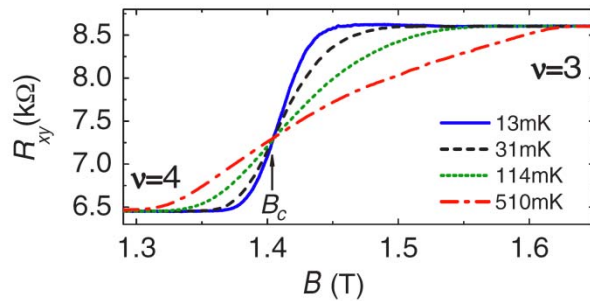


FIG. 1 (color online). Hall resistance around the 4-3 transition at different temperatures. A critical field of $B_c = 1.4$ T is observed.

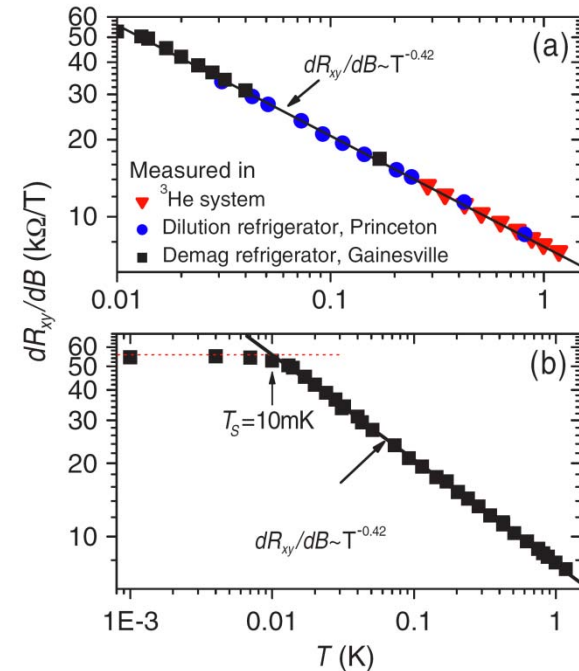


FIG. 2 (color online). (a) Perfect temperature scaling $(dR_{xy}/dB)|_{B_c} \propto T^{-0.42}$ of the 4-3 transition over two decades of temperature between 1.2 K and 12 mK. (b) Saturation of $(dR_{xy}/dB)|_{B_c}$ at low temperatures. The saturation temperature $T_s = 10$ mK is obtained from the cross point between extrapolations of the higher temperature data (black line) and the lower temperature saturated data (horizontal dotted line).

Li et al

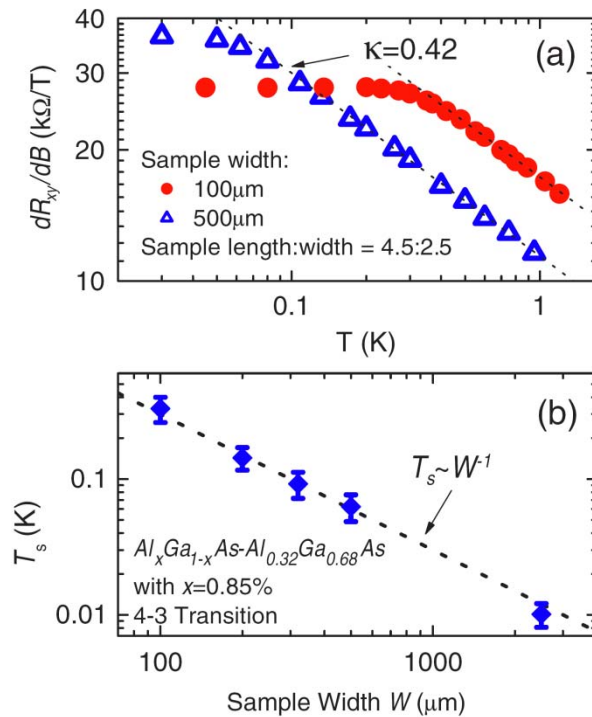


FIG. 3 (color online). (a) $(dR_{xy}/dB)|_{B_c}$ vs T of the 4-3 transition for two samples of different size. The length/width of samples is kept to be 4.5:2.5. (b) The sample size dependence of the saturation temperature T_s of $(dR_{xy}/dB)|_{B_c}$. The value of T_s is inversely proportional to the sample width W within the error.

$$\frac{1}{z\nu} = 0.42 \pm 0.01$$

$$z = 1$$

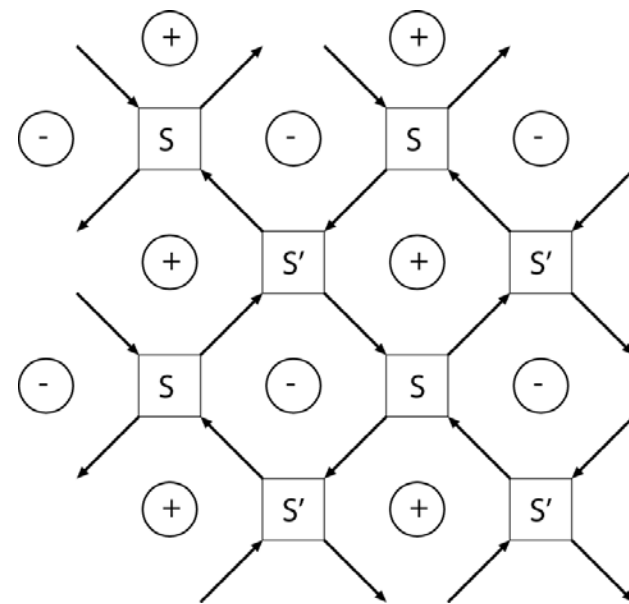
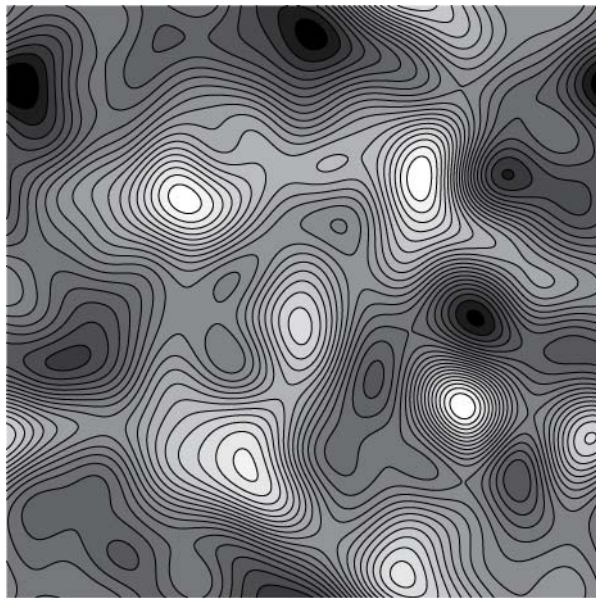
$$\nu = 2.38$$

The consensus

Chalker & Coddington	2.5 ± 0.5
Huckestein & Kramer	2.34 ± 0.04
Cain et al	2.37 ± 0.02
(Milnikov & Sokolov)	(7/3)
Lee & Wang	2.33 ± 0.03

“It is now generally believed that $\nu = 2.4$ ”, Li et al.

Network Model of 2DEG in B field



Chalker-Coddington Model

- Nodes described by 2×2 scattering matrices
- Random phases on the links
- Energy x in units of Landau band width
- LL center $x=0$

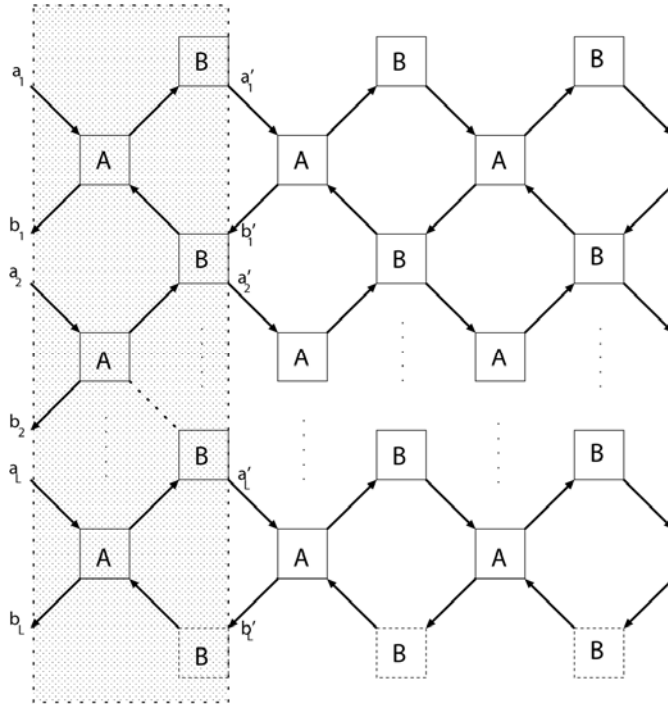
$$S = \begin{pmatrix} e^{i\varphi_1} & 0 \\ 0 & e^{i\varphi_2} \end{pmatrix} \begin{pmatrix} -r & t \\ t & r \end{pmatrix} \begin{pmatrix} e^{i\varphi_3} & 0 \\ 0 & e^{i\varphi_4} \end{pmatrix}$$

$$S' = \begin{pmatrix} e^{i\varphi'_1} & 0 \\ 0 & e^{i\varphi'_2} \end{pmatrix} \begin{pmatrix} -t & r \\ r & t \end{pmatrix} \begin{pmatrix} e^{i\varphi'_3} & 0 \\ 0 & e^{i\varphi'_4} \end{pmatrix}$$

$$t = \frac{1}{\sqrt{e^{+2x} + 1}} \quad r = \frac{1}{\sqrt{e^{-2x} + 1}}$$

Transfer Matrix of CC Model

- Divide system into layers



$$T_l = BV_lAU_l$$

$$T = \prod_{l=1}^L T_l$$

Lyapunov Exponents

- As a consequence of current conservation the eigenvalues of the matrix

$$\Omega = \ln T^\dagger T$$

- occur in pairs of opposite sign

$$\{+v_1, \dots, +v_N, -v_N, \dots, -v_1\} \quad v_1 > v_2 > \dots > v_N > 0$$

- The Lyapunov exponents (LEs) are the limiting values

$$\gamma_i = \lim_{L \rightarrow \infty} \frac{v_i}{2L}$$

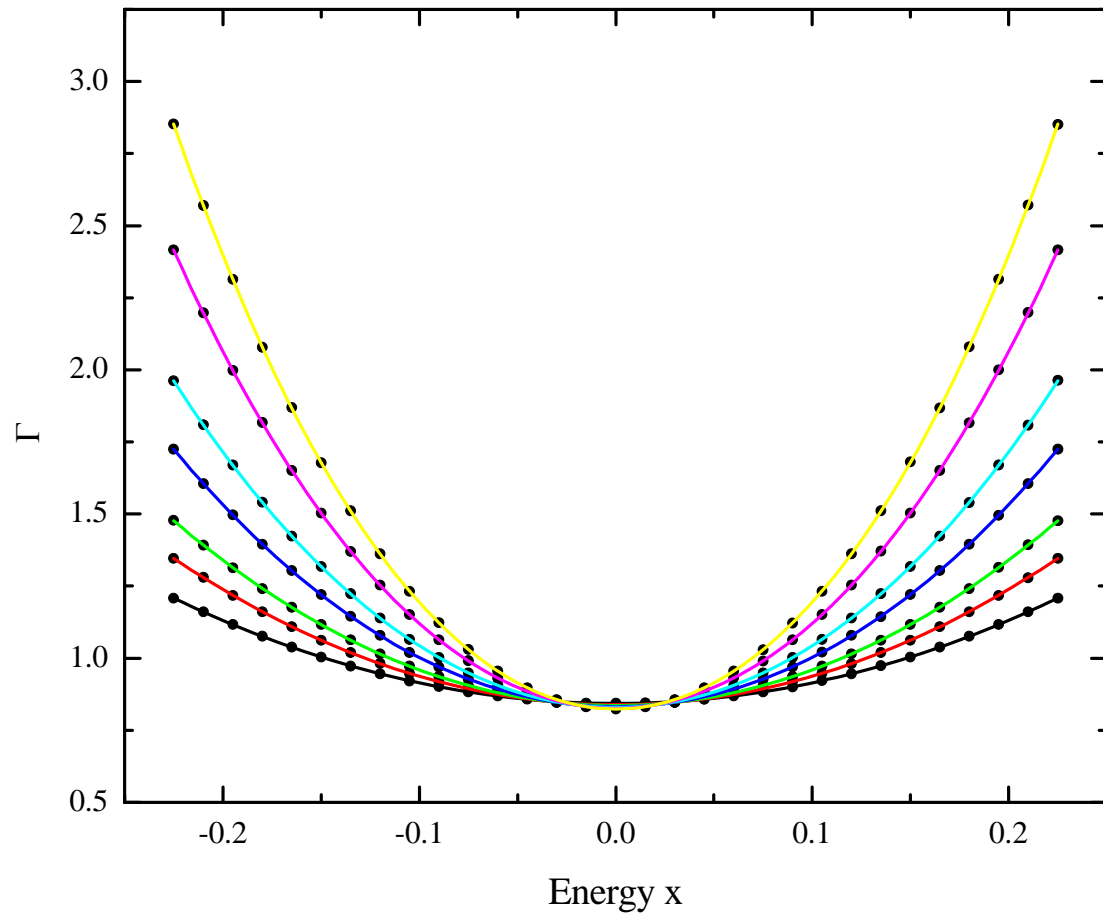
Smallest positive exponent

- It is usual (but not necessary) to focus on the smallest positive exponent

$$\gamma \equiv \gamma(x, N) \quad \Gamma \equiv \gamma N$$

- The output of the simulation is estimates of the LE as a function of
 - energy x
 - cross section Nwith specified precision σ

Γ vs x



$N=16, 24, 32, 48, 64, 96, 128$ Precision 0.03%

Finite Size Scaling

- Localised Phase

$$\Gamma \approx \frac{N}{\xi} \quad N \rightarrow \infty$$

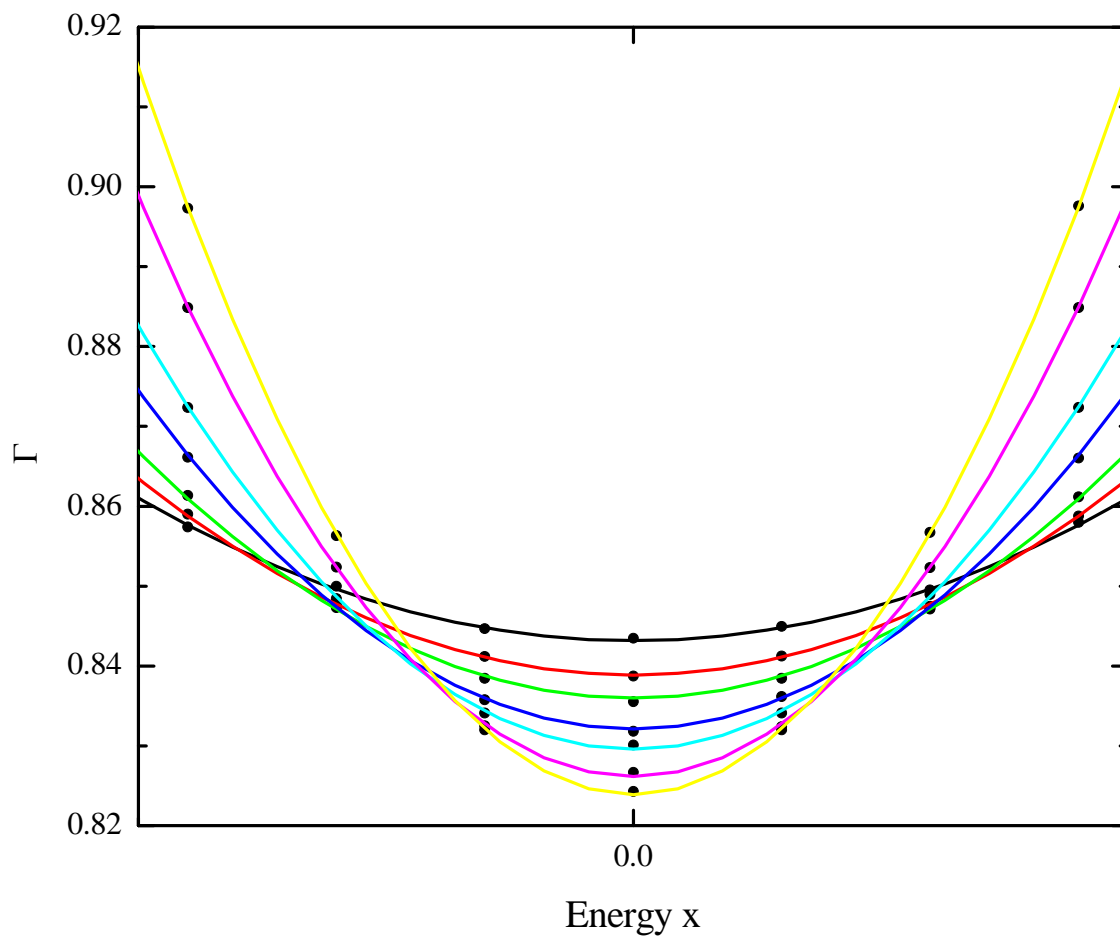
- Scale invariance at critical point

$$N \approx \text{constant} \quad N \rightarrow \infty$$

- FSS law

$$\Gamma = F_0 \left(N^\alpha (x - x_c) \right) \quad \alpha = \frac{1}{\nu} \quad (x_c = 0)$$

Near the Critical Point



Corrections to Scaling

- Irrelevant scaling variable
- Non-linearity of the scaling variables

$$\begin{aligned}\Gamma &= F\left(N^\alpha u_0(x), N^y u_1(x)\right) \\ &\approx F_0\left(N^\alpha u_0(x)\right) + N^y u_1(x) F_1\left(N^\alpha u_0(x)\right)\end{aligned}$$

K. Slevin and T. Ohtsuki (1999). "Corrections to Scaling at the Anderson Transition."
Physical Review Letters **82(2): 382-385.**

FSS Results

N	16, 24, 32, 48, 64, 96, 128
number of data	217
number of parameters	9
chi-squared	199.8
goodness of fit	0.6
α	0.3857 [0.3849, 0.3866]
Γ_c	0.78 [0.767, 0.788]
y	-0.17 [-0.21, -0.14]

$$\nu = 2.593 \quad [2.587, 2.598]$$

End of the consensus...

Slevin & Ohtsuki	2.593 [2.587, 2.598]
Chalker & Coddington	2.5 ± 0.5
Huckestein & Kramer	2.34 ± 0.04
Cain et al	2.37 ± 0.02
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Conformal Symmetry

- Scaling relations for 2D systems with ATs
 - strips \Leftrightarrow 2D
 - Position α_0 of the maximum of the $f(\alpha)$ spectrum for a 2D system with Λ_c

$$\Gamma_c = \pi(\alpha_0 - 2)$$

- Confirmed for 2D SU(2) model
 - $\alpha_0 = 2.173 \pm 0.001$ (Obuse et al 2007)
 - $\alpha_0 = 2.1727 \pm 0.0001$
 - (applying scaling relation to Asada et al 2004)

Conformal symmetry and QHE?

- Is this relation obeyed at the QHE transition?

	α_0
Obuse et al	2.2617 ± 0.0006
Evers et al	2.2596 ± 0.0004
Slevin et al	2.248 [2.244, 2.251]

- Obuse, H., A. R. Subramaniam, A. Furusaki, I. A. Gruzberg, A. W. W. Ludwig. Physical Review Letters, 2008. **101** 116802
- Evers, F., A. Mildenberger, and A.D. Mirlin. Physical Review Letters, 2008. **101** 116803

Summary

- Previous work seems to have underestimated the exponent

$$\nu = 2.593 \quad [2.587, 2.598]$$

- Agreement with experiment seems to have been fortuitous
- Further work needed to confirm conformal invariance

Slevin, K. and T. Ohtsuki, *Critical exponent for the quantum Hall transition*. Physical Review B (Condensed Matter and Materials Physics), 2009. **80**(4): p. 041304-4.