Non-classicality, quantum resources, and quantum advantage

Samson Abramsky

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Warwick Computer Science Colloquium, November 2019

Quantum advantage and non-classicality

Quantum advantage in information-processing tasks:

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Quantum advantage in information-processing tasks:

- When do we get it?

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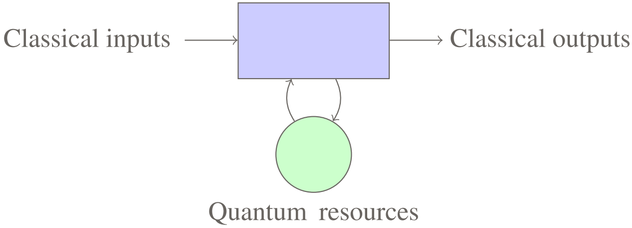
An emerging paradigm, combining quantum information and quantum foundations:

- The possibility of quantum advantage is intimately related to the non-classicality of quantum mechanics. And this non-classicality manifests itself in *logical* terms.
- This non-classical picture of the world lives "at the borders of paradox", as indicated by foundational results such as the EPR paradox, the Kochen-Specker paradox, the Hardy paradox, etc.

Methodology



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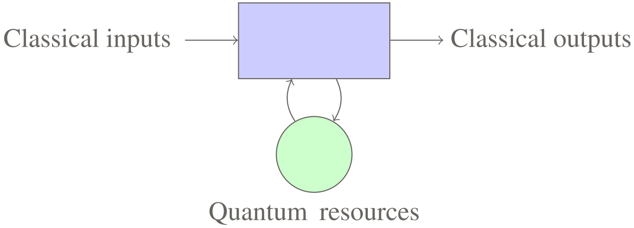

- Non-classicality is present *in the data*.

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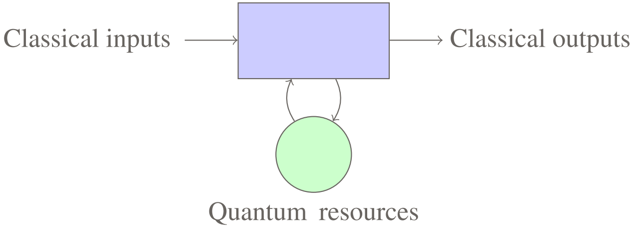

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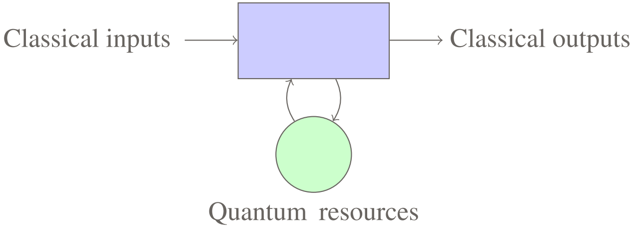

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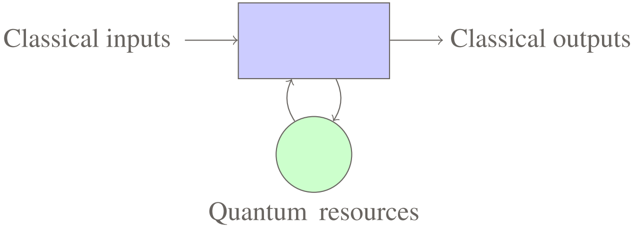

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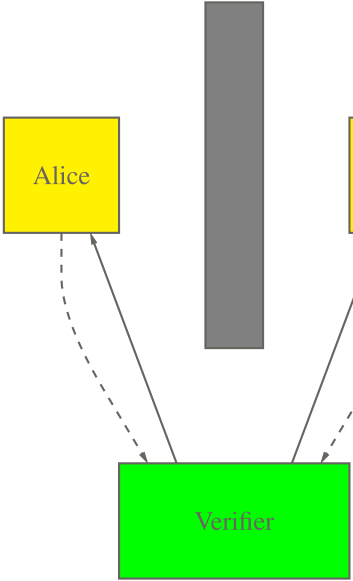
- Physics-independent – cf. device independence.
- Non-classicality relates to logic and probability - we want to understand its mathematical structure.
- Concretely: we want to *characterize classes of tasks* where quantum advantage can be gained, and to *quantify the degree of quantum advantage* which can be obtained.

Alice-Bob games



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A table of conditional probabilities $p(a,b|x,y)$ defines a *probabilistic strategy* for this game. The *success probability* for this strategy is:

$$1/4[p(a=b|x=0,y=0) + p(a=b|x=0,y=1) + p(a=b|x=1,y=0)$$

$$+ p(a \neq b|x=1,y=1)]$$

A Strategy for the Alice-Bob game

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Example: The Bell Model

A	B	$(0,0)$	$(1,0)$	$(0,1)$	$(1,1)$
0	0	$1/2$	0	0	$1/2$
0	1	$3/8$	$1/8$	$1/8$	$3/8$
1	0	$3/8$	$1/8$	$1/8$	$3/8$
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If the Verifier sends Alice a_1 *and Bob* b_2*, then with probability* $1/8$*, Alice outputs a* 0 *and Bob outputs a* 1.

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The Bell table exceeds this bound. Since it is *quantum realizable* using an entangled pair of qubits, it shows that quantum resources yield a *quantum advantage* in an information-processing task.

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Using elementary probability theory, we can calculate:

$$p_N \leq \mathsf{Prob}\left(\bigvee_{i=1}^{N-1} \neg\phi_i\right) \leq \sum_{i=1}^{N-1} \mathsf{Prob}(\neg\phi_i) = \sum_{i=1}^{N-1} (1 - p_i) = (N-1) - \sum_{i=1}^{N-1} p_i.$$

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Hence we obtain the inequality

$$\sum_{i=1}^{N} p_i \le N-1.$$

Logical analysis of the Bell table

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	$(0,0)$	$(1,0)$	$(0,1)$	$(1,1)$
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If we read 0 as true and 1 as false, the highlighted entries in each row of the table are represented by the following propositions:

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\begin{array}{lcccccccc}
\varphi_1 & = & (a_1 \wedge b_1) & \vee & (\neg a_1 \wedge \neg b_1) & = & a_1 & \leftrightarrow & b_1 \\
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All Bell inequalities arise this way.
Abramsky, Hardy, *Logical Bell inequalities*, Physical Review A 2012.

Science Fiction? – The News from Delft

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First Loophole-free Bell test, 2015

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NATURE | LETTER

日本語要約

Loophole-free Bell inequality violation using electron spins separated by 1.3 kilometres

B. Hensen, H. Bernien, A. E. Dréau, A. Reiserer, N. Kalb, M. S. Blok, J. Ruitenberg, R. F. L. Vermeulen, R. N. Schouten, C. Abellán, W. Amaya, V. Pruneri, M. W. Mitchell, M. Markham, D. J. Twitchen, D. Elkouss, S. Wehner, T. H. Taminiau & R. Hanson

Nature **526**, 682–686 (29 October 2015) doi:10.1038/nature15759
Received 19 August 2015 Accepted 28 September 2015 Published online 21 October 2015

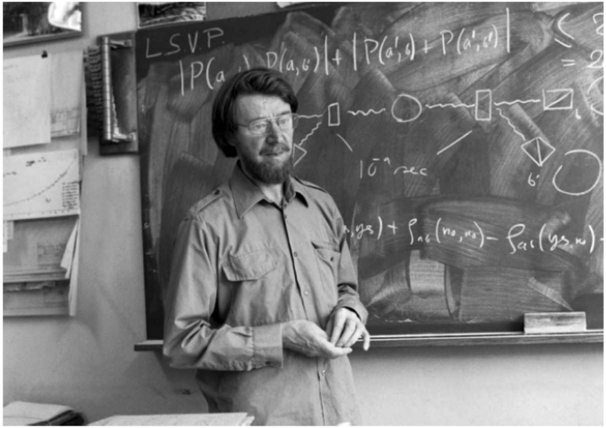
More than 50 years ago[1], John Bell proved that no theory of nature that obeys locality and realism[2] can reproduce all the predictions of quantum theory: in any local-realist theory, the correlations between outcomes of measurements on distant particles satisfy an inequality that can be violated if the particles are entangled. Numerous Bell inequality tests have been reported[3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13]; however, all experiments reported so far required additional assumptions to obtain a contradiction with local realism, resulting in 'loopholes'[13, 14, 15, 16]. Here we report a Bell experiment that is free of any such additional assumption and thus directly tests the principles underlying Bell's inequality. We use an event-ready scheme[17, 18, 19] that enables the generation of robust entanglement between distant electron spins (estimated state fidelity of 0.92 ± 0.03). Efficient spin read-out avoids the fair-sampling assumption (detection loophole[14, 15]), while the use of fast random-basis selection and spin read-out combined with a spatial separation of 1.3 kilometres ensure the required locality conditions[13]. We performed 245 trials that tested the CHSH–Bell inequality[20] $S \leq 2$ and found $S = 2.42 \pm 0.20$ (where S quantifies the correlation between measurement outcomes). A null-hypothesis test yields a probability of at most $P = 0.039$ that a local-realist model for space-like separated sites could produce data with a violation at least as large as we observe, even when allowing for memory[16, 21] in the devices. Our data hence imply statistically significant rejection of the local-realist null hypothesis. This conclusion may be further consolidated in future experiments; for instance, reaching a value of $P = 0.001$ would require approximately 700 trials for an observed $S = 2.4$. With improvements, our experiment could be used for testing less-conventional theories, and for implementing device-independent quantum-secure communication[22] and randomness certification[23, 24].

Quantum ‘spookiness’ passes toughest test yet

Experiment plugs loopholes in previous demonstrations of 'action at a distance', against Einstein's objections — and could make data encryption safer.

Zeeya Merali

27 August 2015



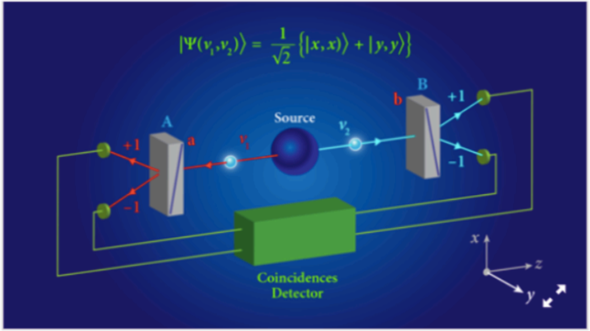
CERN

Viewpoint: Closing the Door on Einstein and Bohr's Quantum Debate

Alain Aspect, Laboratoire Charles Fabry, Institut d'Optique Graduate School, CNRS, Université Paris-Saclay, Palaiseau, France

December 16, 2015 • *Physics* 8, 123

By closing two loopholes at once, three experimental tests of Bell's inequalities remove the last doubts that we should renounce local realism. They also open the door to new quantum information technologies.



APS/Alan Stonebraker

Figure 1: An apparatus for performing a Bell test. A source emits a pair of entangled photons ν_1 and ν_2. Their polarizations are analyzed by polarizers A and B (grey blocks), which are aligned, respectively

Timeline

1932	von Neumann's Mathematical Foundations of Quantum Mechanics
1935	EPR Paradox, the Einstein-Bohr debate
1964	Bell's Theorem
1982	First experimental test of EPR and Bell inequalities (Aspect, Grangier, Roger, Dalibard)
1984	Bennett-Brassard quantum key distribution protocol
1985	Deutch Quantum Computing paper
1993	Quantum teleportation (Bennett, Brassard, Crépeau, Jozsa, Peres, Wooters)
1994	Shor's algorithm
2015	First loophole-free Bell tests (Delft, NIST, Vienna)

Formalising empirical data*

*SA, Brandenburger, *New Journal of Physics*, 2011.

in\out	$(0,0)$	$(0,1)$	$(1,0)$	$(1,1)$
(a,b)	—	—	—	—
(a,b')	—	—	—	—
(a',b)	—	—	—	—
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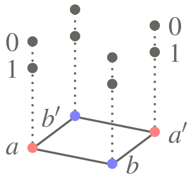
A **measurement scenario** $\mathbf{X} = \langle X, \Sigma, O \rangle$:

- X – a finite set of measurements
- Σ – a simplicial complex on X
  faces are called the *measurement contexts*
- $O = (O_x)_{x \in X}$ – for each $x \in X$ a finite non-empty set of possible outcomes O_x



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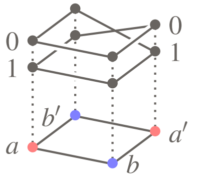
An **empirical model** $e = \{e_\sigma\}_{e \in \Sigma}$ on $\mathbf{X}$:

- Each e_σ is a prob. distribution over joint
  outcomes $\prod_{x \in \sigma} O_x$ for σ
- *generalised no-signalling* holds:
  $\forall \sigma, \tau \in \Sigma, \sigma \subseteq \tau$.

$$e_\tau|_\sigma = e_\sigma$$

(i.e. marginals are well-defined)



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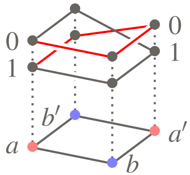
(a',b)	$1/2$	0	0	$1/2$
(a',b')	0	$1/2$	$1/2$	0

An **empirical model** $e = \{e_\sigma\}_{e \in \Sigma}$ on $\mathbf{X}$:

- Each e_σ is a prob. distribution over joint outcomes $\prod_{x \in \sigma} O_x$ for σ
- *generalised no-signalling* holds:
  $\forall \sigma, \tau \in \Sigma, \sigma \subseteq \tau.$

$$e_\tau|_\sigma = e_\sigma$$

(i.e. marginals are well-defined)



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The import of Bell's theorem and similar results is that there are empirical models arising from quantum mechanics which are contextual.

Categorical formulation

Given a scenario $\mathbf{X} = \langle X, \Sigma, O \rangle$, we can define a presheaf $\mathscr{D} \circ \mathscr{E} : \Sigma^{\mathsf{op}} \to \mathbf{Set}$, where:

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There is also topology at work here. We can use *Čech cohomology* of our (pre)sheaf to define invariants to capture contextuality.

- Abramsky, Barbosa, Mansfield, *The cohomology of non-locality and contextuality*, QPL 2011.
- Abramsky, Barbosa, Kishida, Lal, Mansfield, *Contextuality, Cohomology and Paradox*, CSL 2015.

Bundle Pictures

Logical Contextuality

- Ignore precise probabilities
- Events are possible or not
- E.g. the Hardy model:

	00	01	10	11
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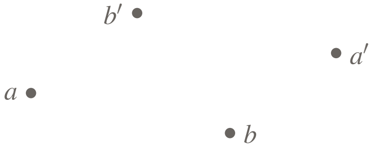
$a'b'$	$\checkmark$	$\checkmark$	$\checkmark$	$\times$



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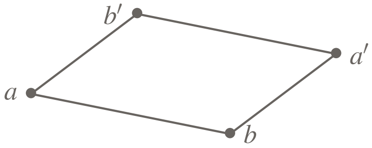

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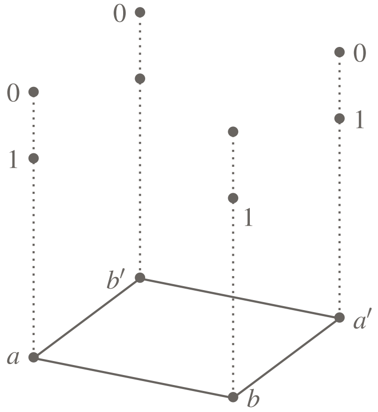

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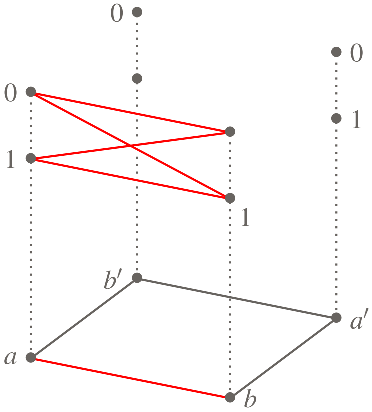

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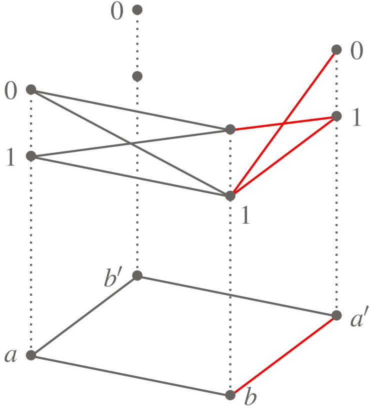

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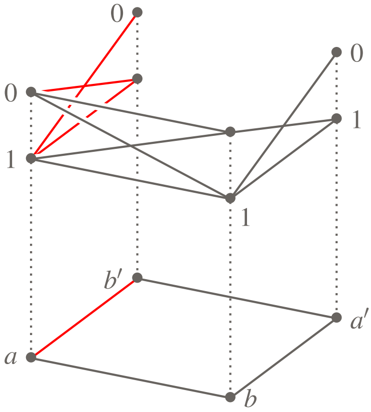

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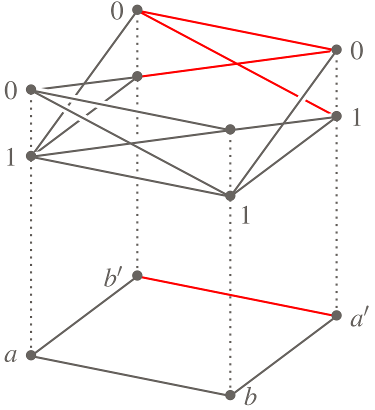

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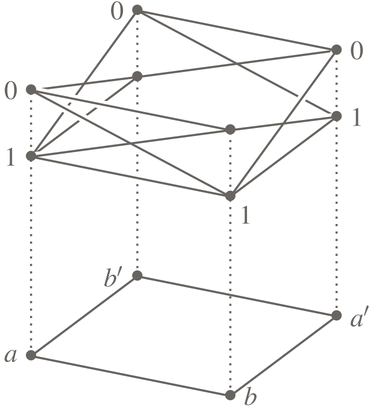

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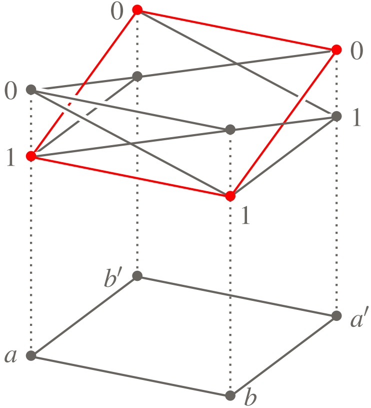

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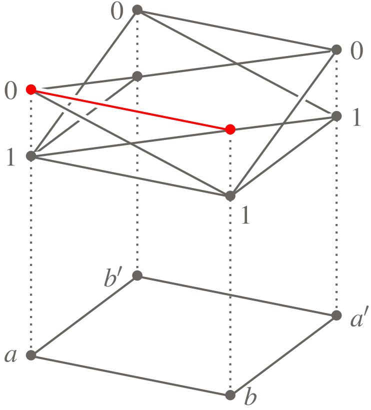

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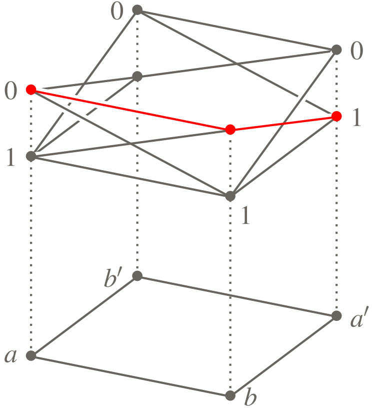

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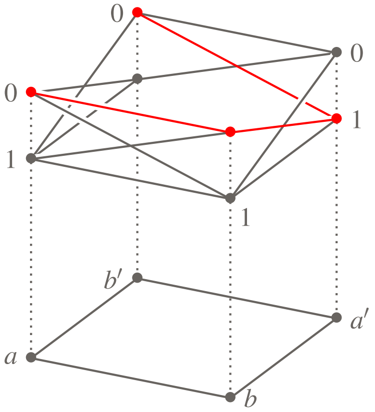

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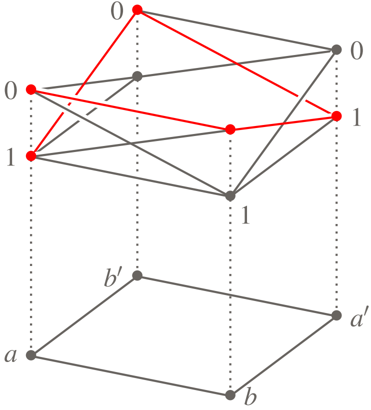

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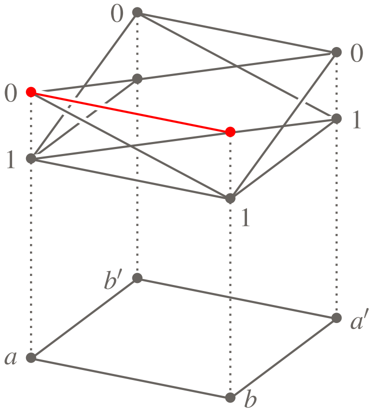

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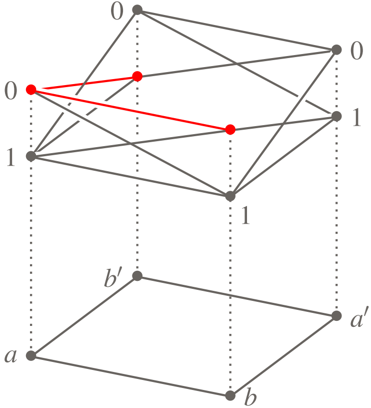

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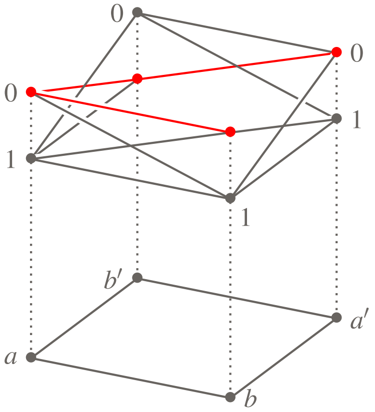

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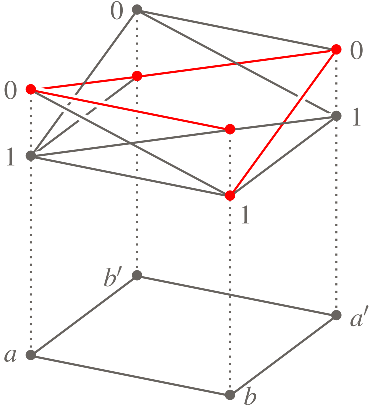

Bundle Pictures

Strong Contextuality

- E.g. the PR box:

	00	01	10	11
ab	✓	×	×	✓
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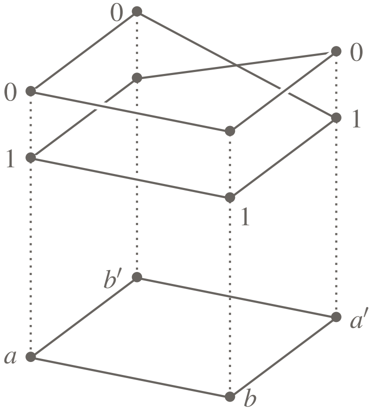
$a'b$	✓	×	×	✓
$a'b'$	×	✓	✓	×



The contextual fraction

Quantifying contextuality: we ask for a convex decomposition

$$e = \lambda e^{NC} + (1-\lambda)e' \tag{1}$$

where e^{NC} is a non-contextual model and e' is another empirical model.

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1. Computable by a linear program.
2. The normalised violation by e of any Bell inequality is at most $\mathsf{CF}(e)$;
3. this bound is attained, *i.e.* there exists a Bell inequality whose normalised violation by e is $\mathsf{CF}(e)$;
4. moreover, for any decomposition of the form $e = \mathsf{NCF}(e)e^{NC} + \mathsf{CF}(e)e^{SC}$, this Bell inequality is tight at the non-contextual model e^{NC} and maximally violated at the strongly contextual model e^{SC}.

Contextuality and quantum advantage

- Measurement-based quantum computation (MBQC)
  - Raussendorf, *Physical Review A*, 2018.
  - SA, Barbosa, Mansfield, *Physical Review Letters*, 2018.

$$\overbrace{1-\bar{p}_S}^{\text{error}} \;\geq\; \underbrace{[1-\mathsf{CF}(e)]}_{\text{classicality}}\;\overbrace{\nu(f)}^{\text{hardness}}$$

quantifiable relationship!

- The same quantitative relationship arises for
  - cooperative games (ABM)
  - communication complexity (Linde Wester D.Phil thesis)
- Shallow circuits
  - Bravyi, Gossett, Kœnig, *Science*, 2018.

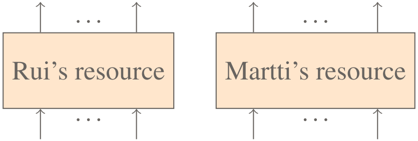
Contextuality analysis using empirical models, logical Bell inequalities, contextual fraction:

- Aasnæss, *Forthcoming*, 2019.

Contextuality as a resource

Comparing contextual behaviours

- When can we say that one resource is more powerful than another?
- Can one resource simulate the usefulness of another?



Example

Barrett, Pironio, *PRL*, 2005.

- PR boxes simulate all 2-outcome bipartite boxes
- A tripartite quantum box that cannot be simulated from PR boxes

Structure of resources

Two views

1. **Resource theories:** An algebraic theory of *free operations* which do not use any of the resource in question, *i.e.* under which contextuality is non-increasing (Physics approach).

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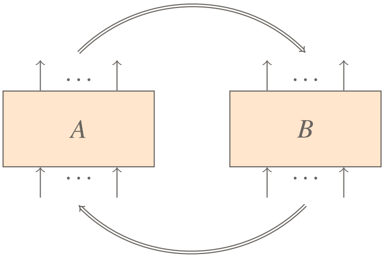
   A *category* of resources and simulations (CS approach*).
   *Cf. (in)computability, degrees of unsolvability, complexity classes

   - Karvonen, *QPL*, 2018.
     SA, Barbosa, Karvonen, Mansfield, *LiCS*, 2019.

Basic simulations



To simulate B using A:

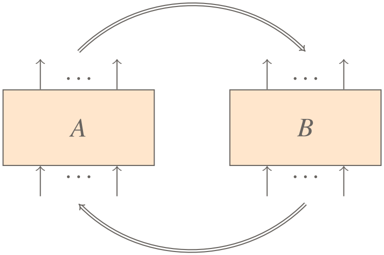
- map inputs of B (measurements) to inputs of A
- run A
- map outputs of A (measurement outcomes) back to outputs of B

Formally

A morphism of scenarios $(\pi, h) : \langle X, \Sigma, O \rangle \to \langle Y, \Theta, P \rangle$ is given by:

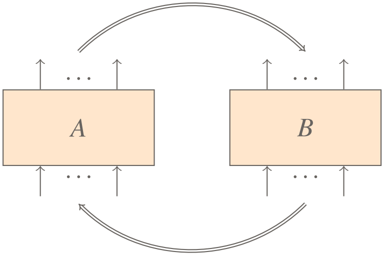
- A simplicial map $\pi : \Theta \to \Sigma$.
- For each $y \in Y$, a map $h_y : O_{\pi(y)} \to P_y$.

General simulations



Idea: allow *adaptive* use of A

General simulations



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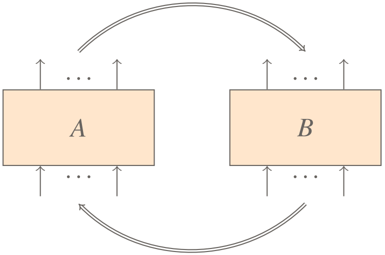

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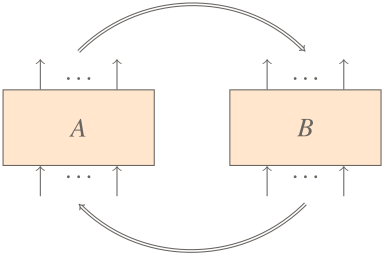

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Thus they incorporate adaptive classical processing, of the kind used e.g. in Measurement-Based Quantum Computing.

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in $\mathbf{Emp}_{\mathsf{MP}}$, the coKleisli category of MP, *i.e.* a map

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We denote the existence of a general simulation by $d \rightsquigarrow e$.

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More generally, we can ask for conditions on scenarios (X,Σ,O) and (Y,Δ,P) such that every empirical model over (Y,Δ,P) can be simulated by some empirical model over (X,Σ,O).

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- Graded versions of simulability: e.g. by adaptivity width or depth, available classical randomness, numbers of copies of resource, approximate simulations, ...

People

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Adam Brandenburger, Lucien Hardy, Shane Mansfield, Rui Soares Barbosa, Ray Lal, Mehrnoosh Sadrzadeh, Phokion Kolaitis, Georg Gottlob, Carmen Constantin, Kohei Kishida. Giovanni Caru, Linde Wester, Nadish de Silva, Martti Karvonen