Acyclic Petri and Workflow Nets with Resets

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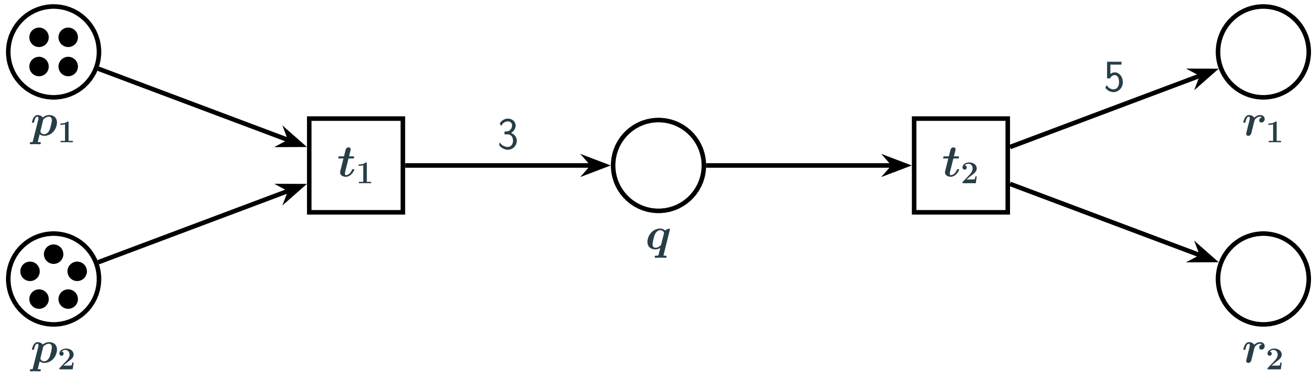
Petri Nets



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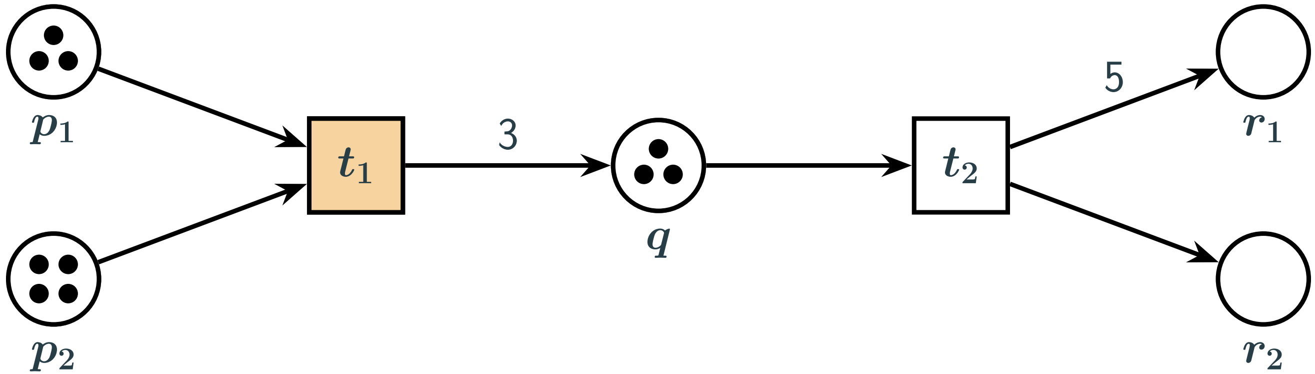

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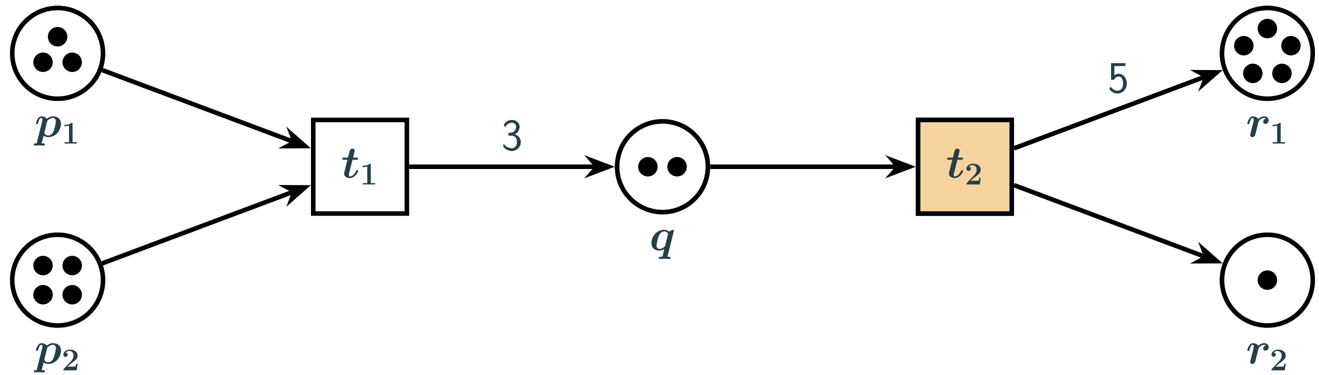

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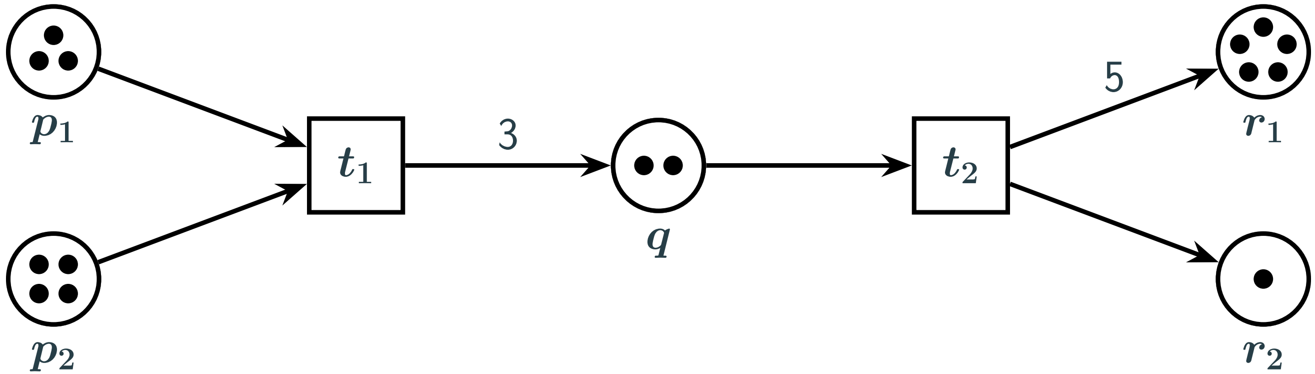


Marking: The number of tokens in each place.

p_1	p_2	q	r_1	r_2
(3,	4,	2,	5,	1)

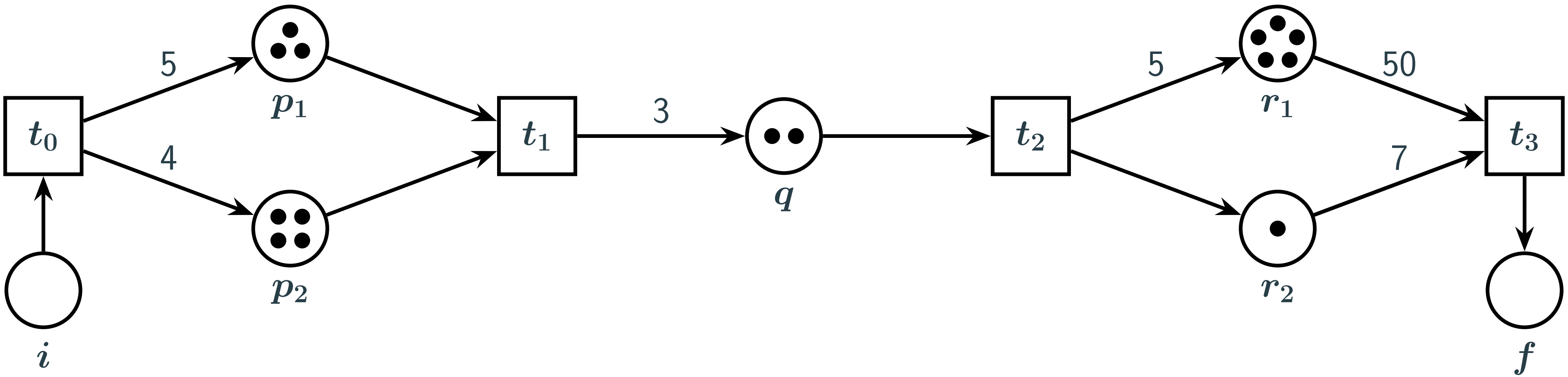
Run: A sequence of markings. $(4, 5, 0, 0, 0) \xrightarrow{t_1} (3, 4, 3, 0, 0) \xrightarrow{t_2} (3, 4, 2, 5, 1)$

Input to decision problems are: a Petri net $\mathcal{N}$, an initial marking $\mathbf{u}$, and a target marking $\mathbf{v}$.

Reachability: Does there exist a run from $\mathbf{u}$ to $\mathbf{v}$ in $\mathcal{N}$?

Coverability: Does there exist a run from $\mathbf{u}$ to $\mathbf{w}$ in $\mathcal{N}$ where $\mathbf{w} \geq \mathbf{v}$?

Workflow Nets



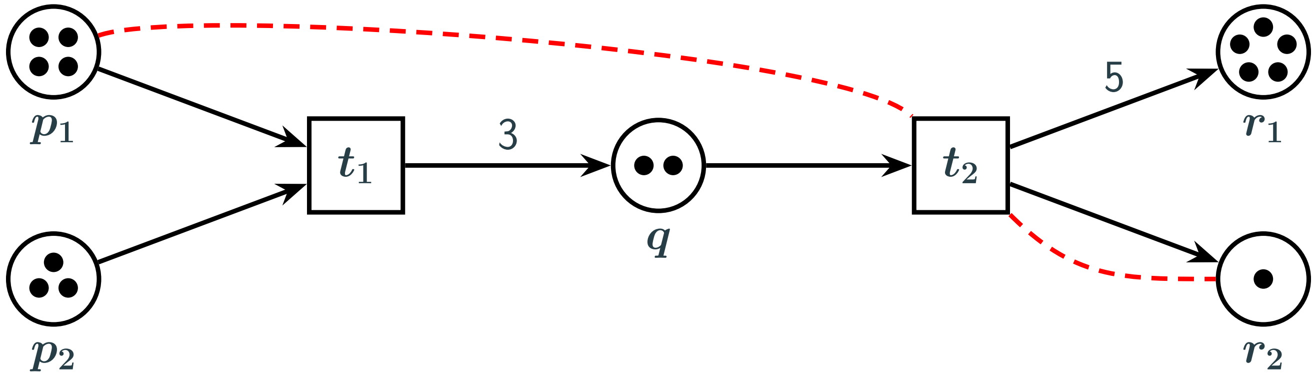
Features: A designated initial place i that cannot be produced to,

a designated final place f that cannot be consumed from, and

all places and transitions are on some path from i to f.

The complexities of reachability and coverability are the same for workflow nets as for Petri nets.

Petri Nets with Resets



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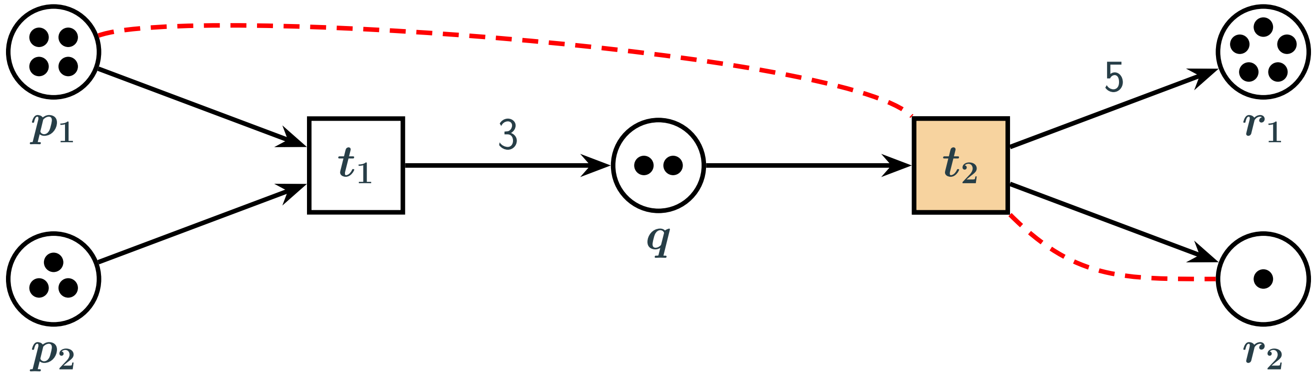
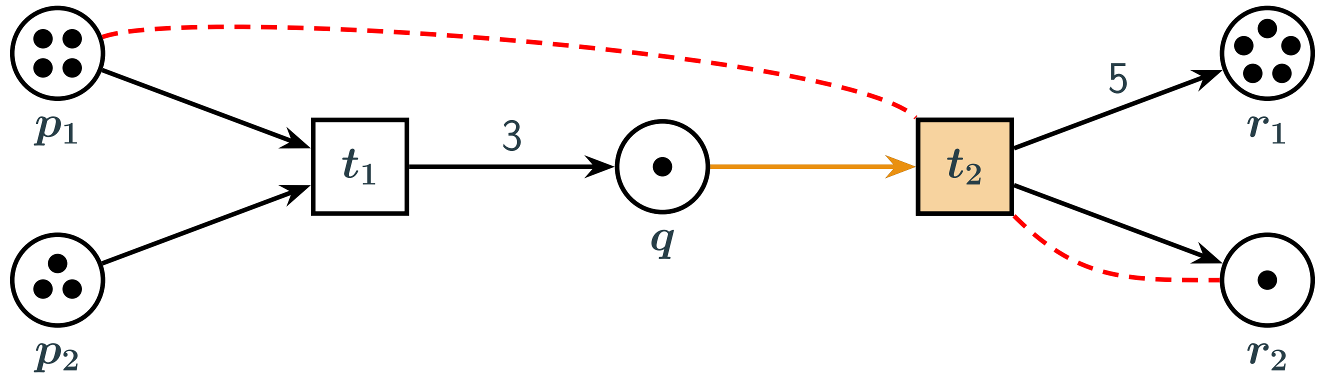


Semantics: First consume tokens, then reset places, then produce tokens.

Petri Nets with Resets



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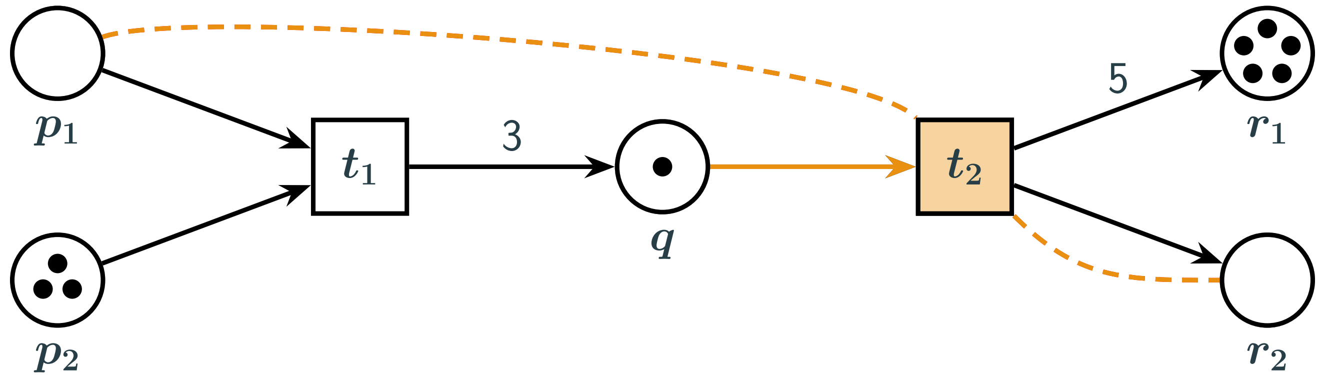

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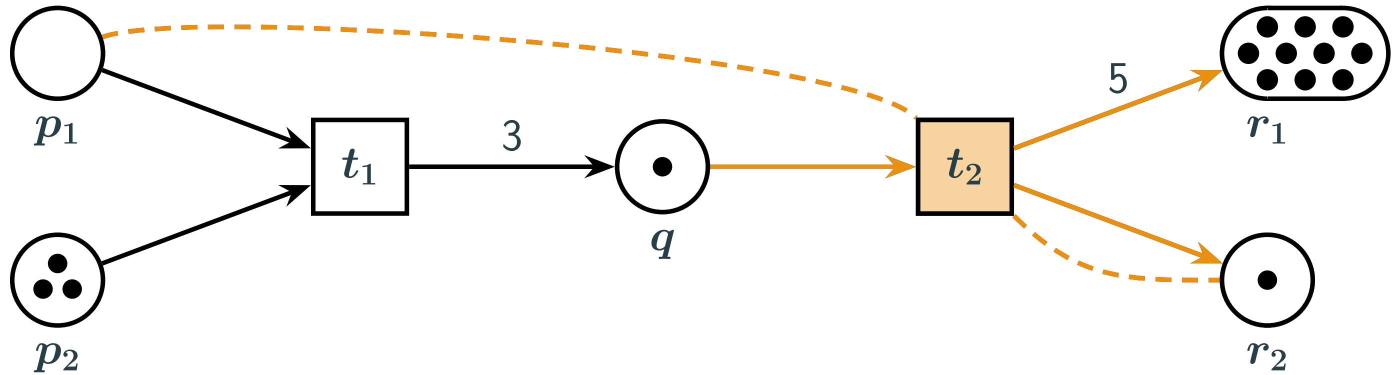

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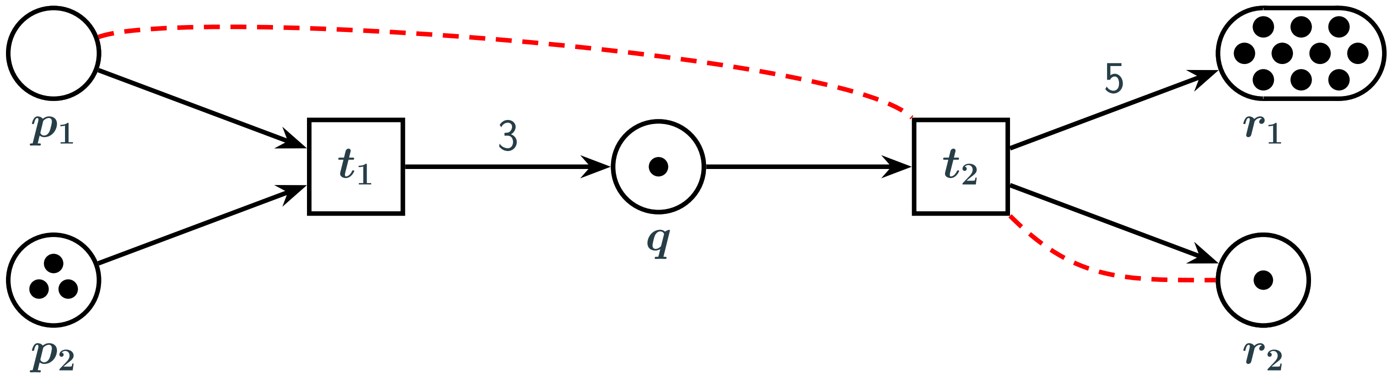


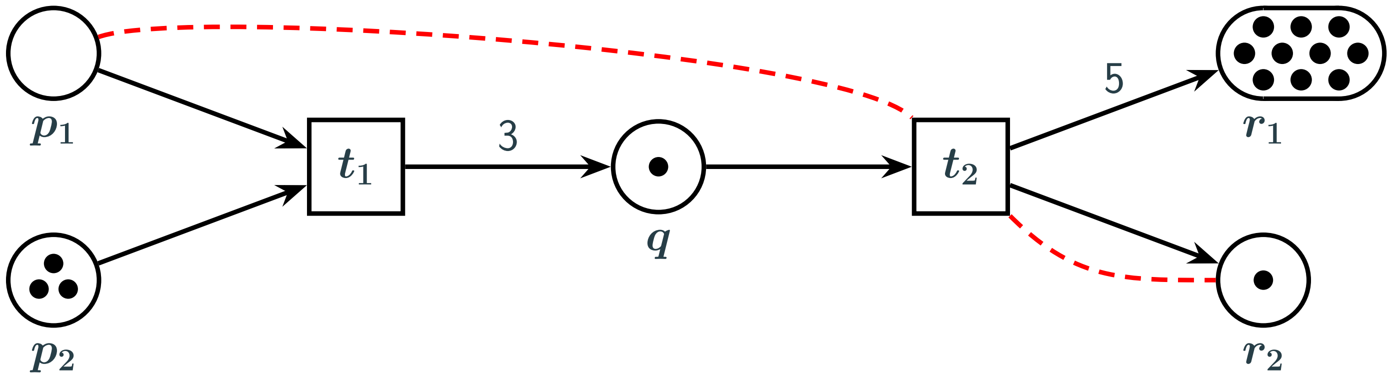
Semantics: First consume tokens, then reset places, then produce tokens.

Reachability in Petri nets with resets is undecidable. [Araki and Kasami '76]

Coverability in Petri nets with resets is Ackermann-complete. [Schnoebelen '10]

[Figueira, Figueira, Schmitz, and Schnoebelen '11]

Acyclic Petri Nets with Resets



Semantics: First consume tokens, then reset places, then produce tokens.

Acyclicity: No cycles in the graph of places with consumption arcs and production arcs.

Reset edges do not count!

We study both acyclic *Petri* nets with resets and acyclic *workflow* nets with resets.

Without resets, reachability and coverability in acyclic Petri and workflow nets are all NP-complete.

Results

	Coverability	Reachability
Acyclic workflow nets with resets	PSPACE-hard	in PSPACE
Acyclic Petri nets with resets	in PSPACE	

Theorem 1: Reachability in acyclic workflow nets with resets is in PSPACE.

Theorem 2: Coverability in acyclic Petri nets with resets is in PSPACE.

Theorem 3: Coverability in acyclic workflow nets with resets is PSPACE-hard.

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Acyclic workflow nets with resets	PSPACE-hard →	PSPACE-complete
Acyclic Petri nets with resets	↓ PSPACE-complete	

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Results

	Coverability	Reachability
Acyclic workflow nets with resets	PSPACE-complete	PSPACE-complete
Acyclic Petri nets with resets	PSPACE-complete	Undecidable

Theorem 1: Reachability in acyclic workflow nets with resets is in PSPACE.

Theorem 2: Coverability in acyclic Petri nets with resets is in PSPACE.

Theorem 3: Coverability in acyclic workflow nets with resets is PSPACE-hard.

Theorem 4: Reachability in acyclic Petri nets with resets is undecidable.

All results hold for both unary encoding and binary encoding.

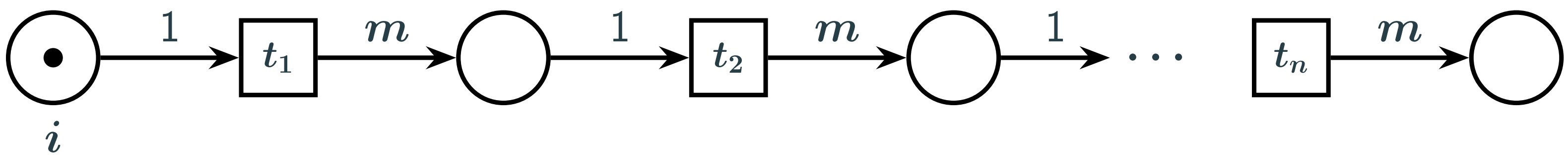
Ideas: PSPACE Upper Bound

Theorem 1: Reachability in acyclic workflow nets with resets is in PSPACE.

Proof idea: Suppose reachability holds, $(1, 0, ..., 0) \rightarrow \cdots \rightarrow (0, ..., 0, 1)$ in $\mathcal{N}$.

Ignore resets and consider number of times each transition can be fired.

Fact: The workflow features imply that all transitions consume at least one token.



So, t_1 can be only be fired once, then t_2 can be fired at most m times, ...

... t_n can be fired at most m^{n-1} times.

Thus, even without resets, a place cannot contain more than m^n many tokens.

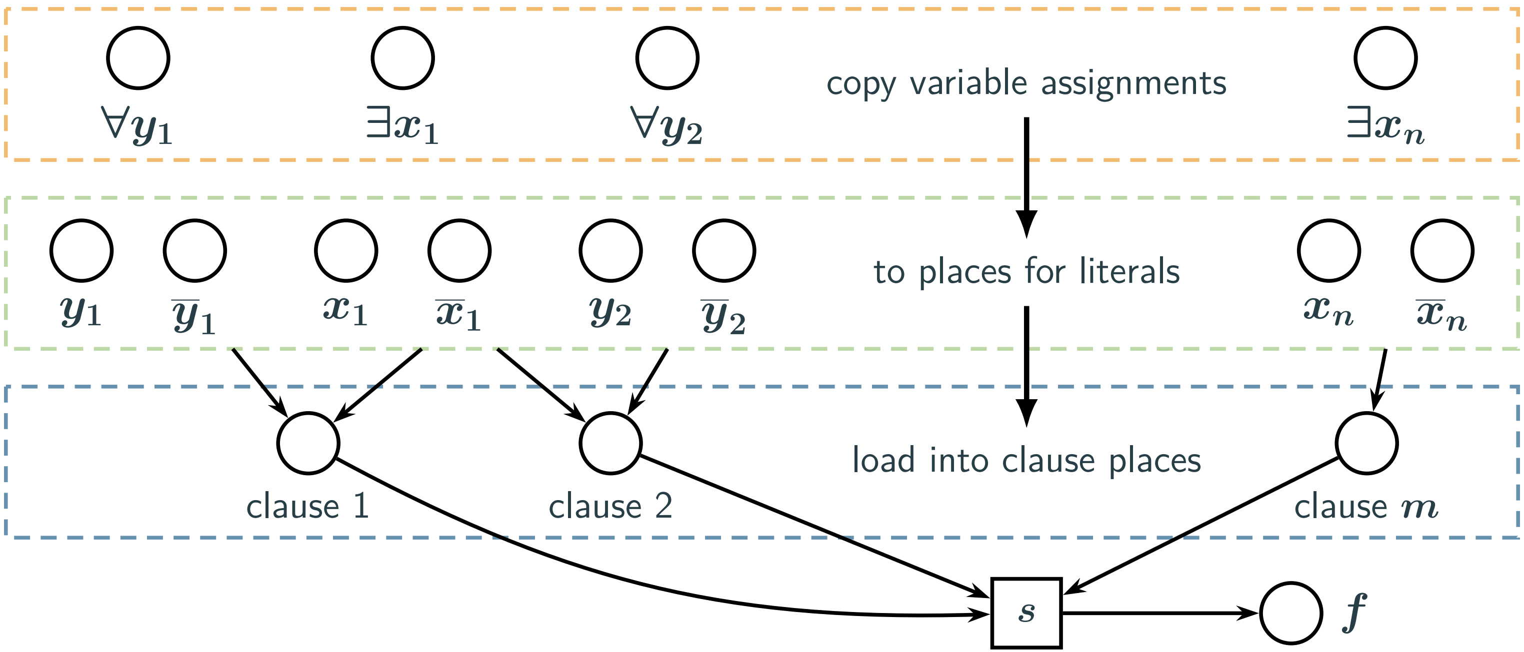
Finally, a marking can be written using $\log(m^n)$ many bits. Note: m, $n \leq \text{size}(\mathcal{N})$.

Ideas: PSPACE Lower Bound

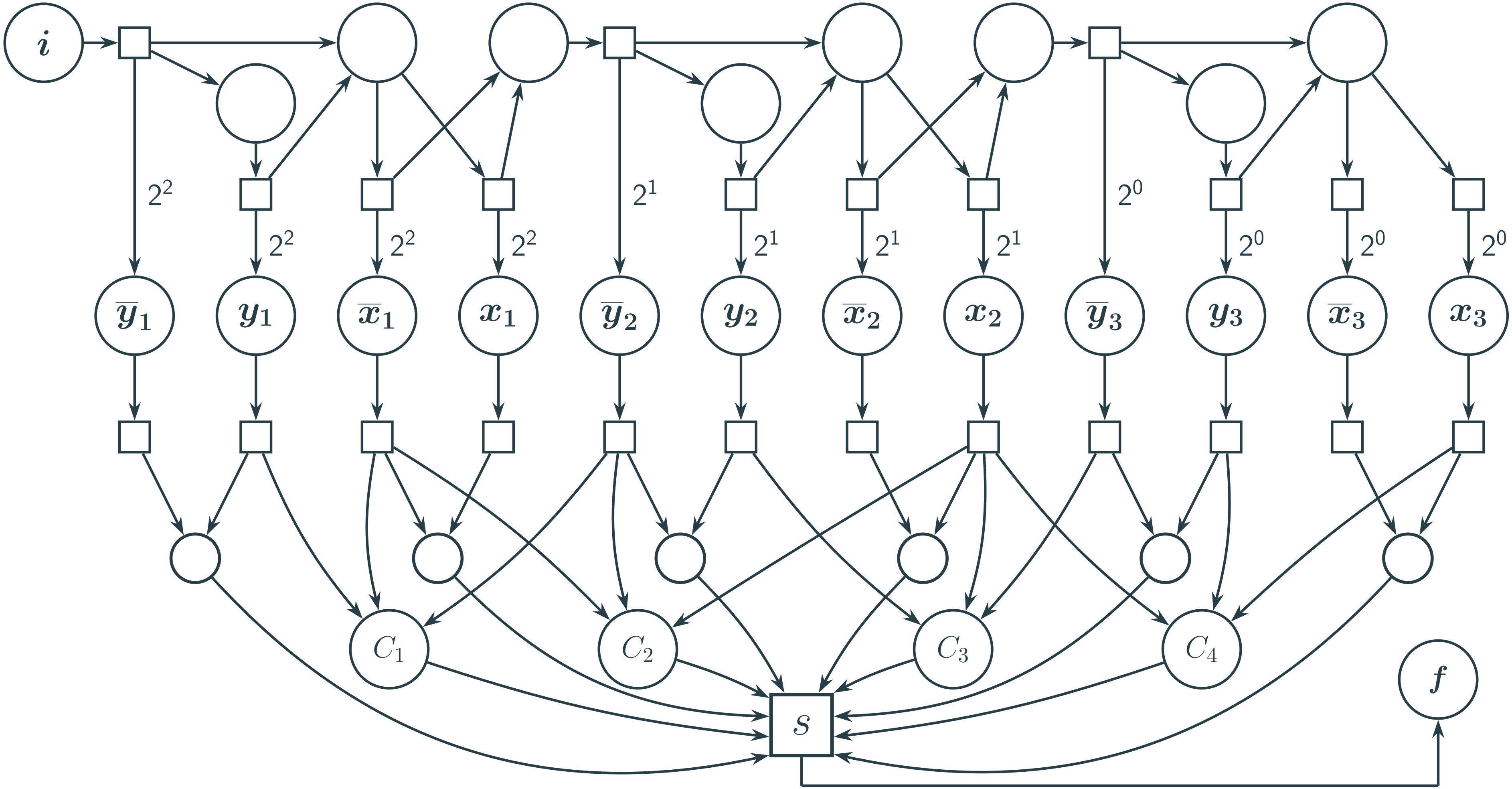
Theorem 3: Coverability in acyclic workflow nets with resets is PSPACE-hard.

Proof idea: Reduce from Quantified SATisfiability (QSAT) directly.

Given $\forall y_1 \exists x_1 \forall y_2 \exists x_2 \ldots \forall y_n \exists x_n \; \phi(y_1, x_1, y_2, x_2, \ldots, y_n, x_n)$.



Example: $(y_1 \vee \overline{x}_1 \vee \overline{y}_2) \wedge (\overline{x}_1 \vee \overline{y}_2 \vee x_2) \wedge (y_2 \vee x_2 \vee \overline{y}_3) \wedge (x_2 \vee y_3 \vee x_3)$



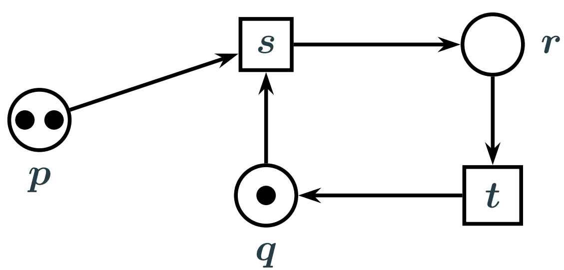
Ideas: Undecidability

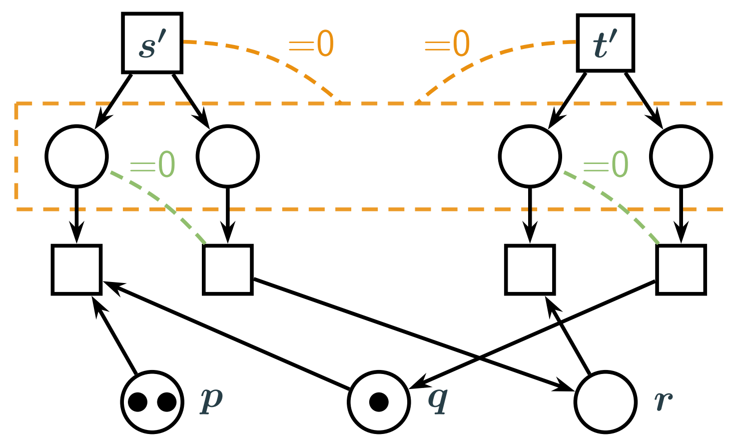
Theorem 4: Reachability in acyclic Petri nets with resets is undecidable.

Lemma: The reachability problem for acyclic Petri nets with zero tests is reducible in logarithmic space to the reachability problem for acyclic Petri nets with resets.

Lemma: The reachability problem in Petri nets with zero tests is reducible in logarithmic space to the reachability problem in *acyclic* Petri nets with zero tests.

Proof idea: Simulate (not necessarily acyclic) transitions using a "transition controller".





Acyclic Petri and Workflow Nets with Resets

	Coverability	Reachability
Acyclic workflow nets with resets	PSPACE-complete	PSPACE-complete
Acyclic Petri nets with resets	PSPACE-complete	Undecidable

Future work: Study (the decidability of) soundness in acyclic workflow nets with resets.

Future work: Is reachability or coverability in acyclic affine Petri or workflow nets decidable?

Thank You!

Presented by Henry Sinclair-Banks, University of Warwick, UK

FSTTCS'23 in International Institute of Information Technology, Hyderabad, India