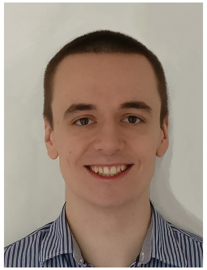
Coverability in VASS Revisited: Improving Rackoff's Bound to Obtain Conditional Optimality

Henry Sinclair-Banks

Joint work with Marvin Künnemann, Filip Mazowiecki, Lia Schütze, and Karol Węgrzycki in ICALP'23.



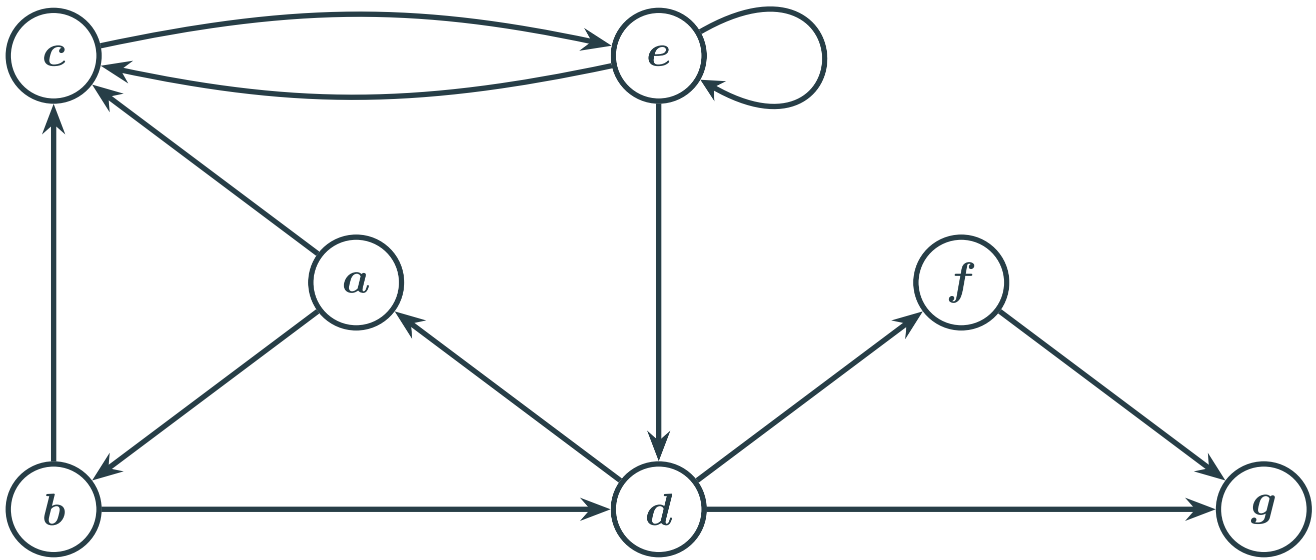




Highlights'23

26th July 2023

Kassel, Germany

2-Dimensional Vector Addition System with States



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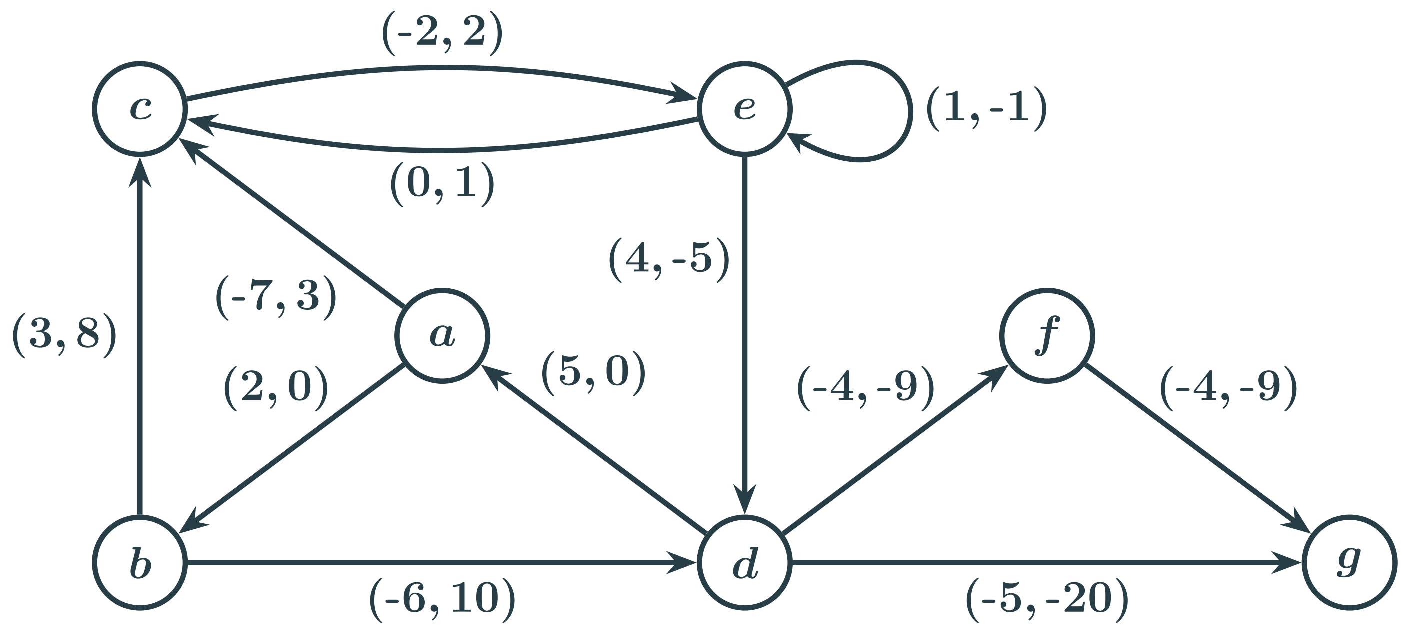
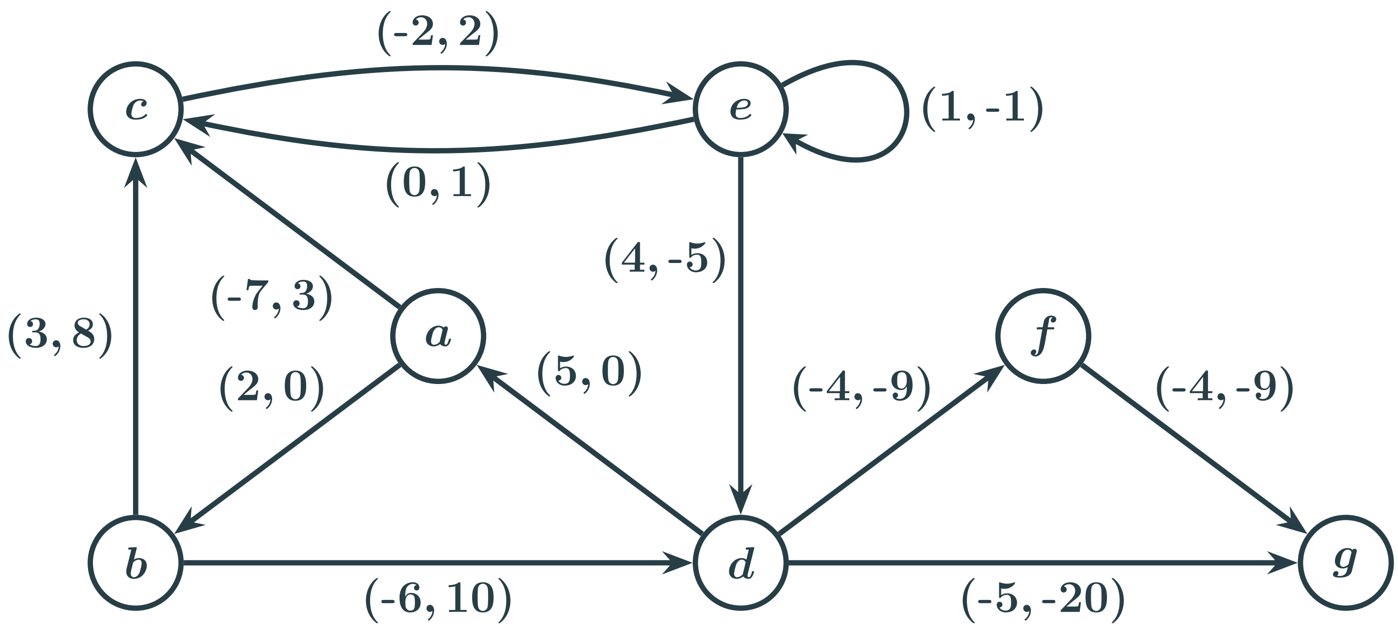
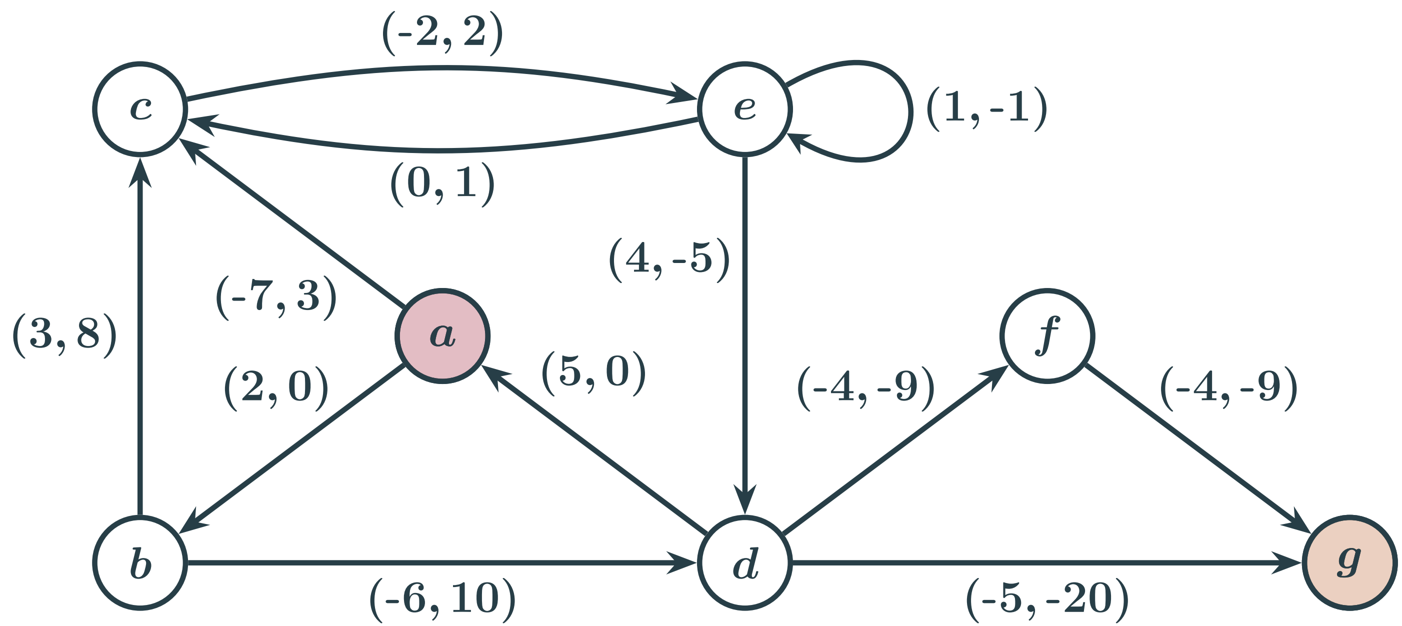


2-Dimensional VASS



Instance of Coverability in 2-Dimensional VASS



Question: from a can you reach g via a path that is *never negative on any component* ?

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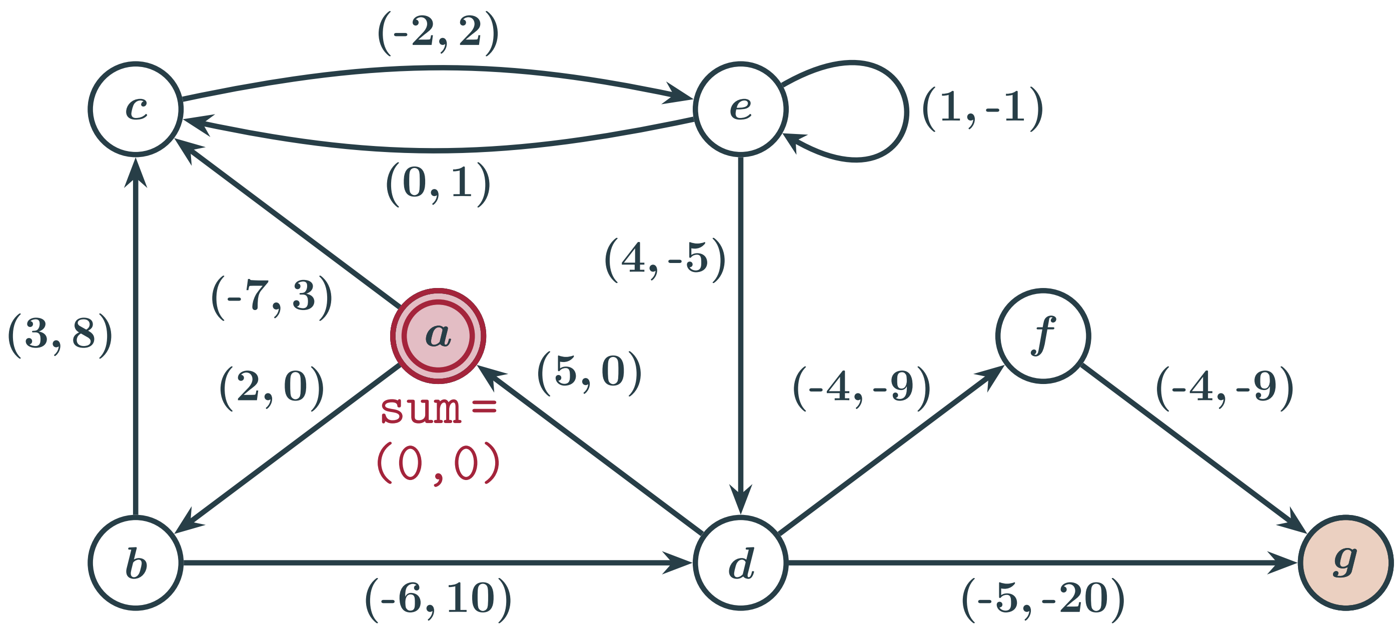

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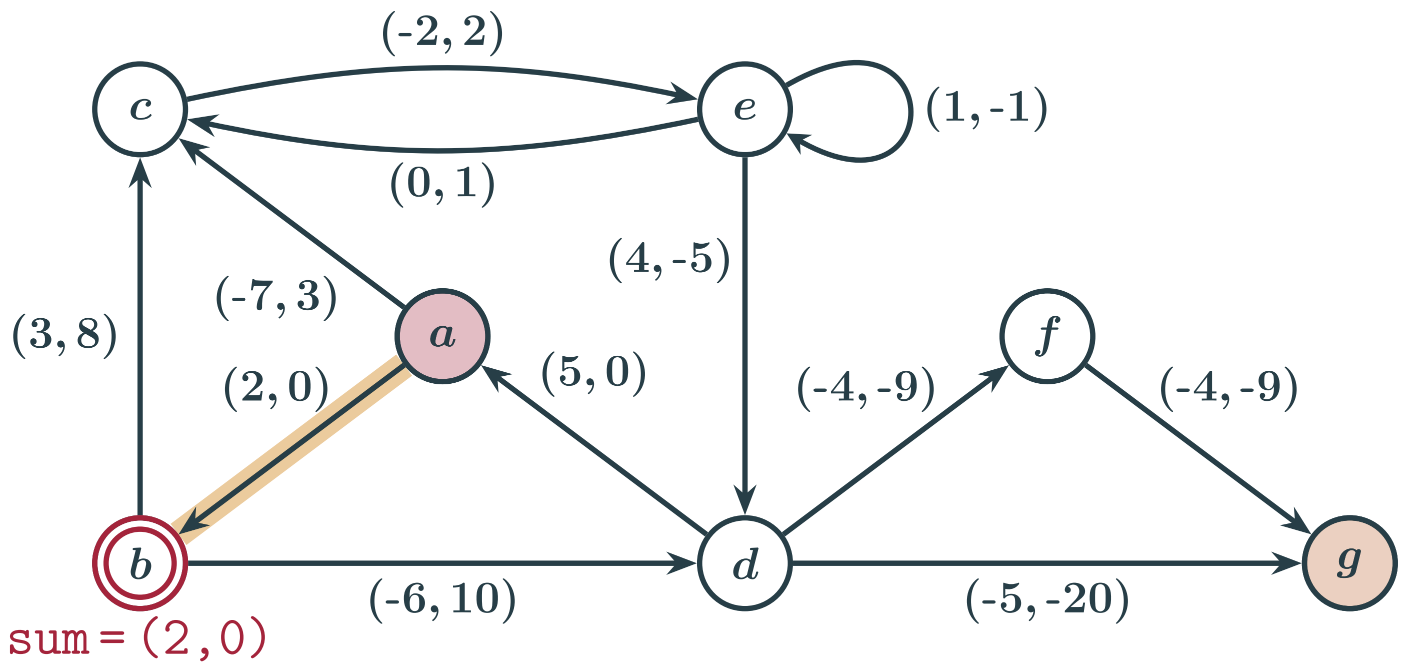

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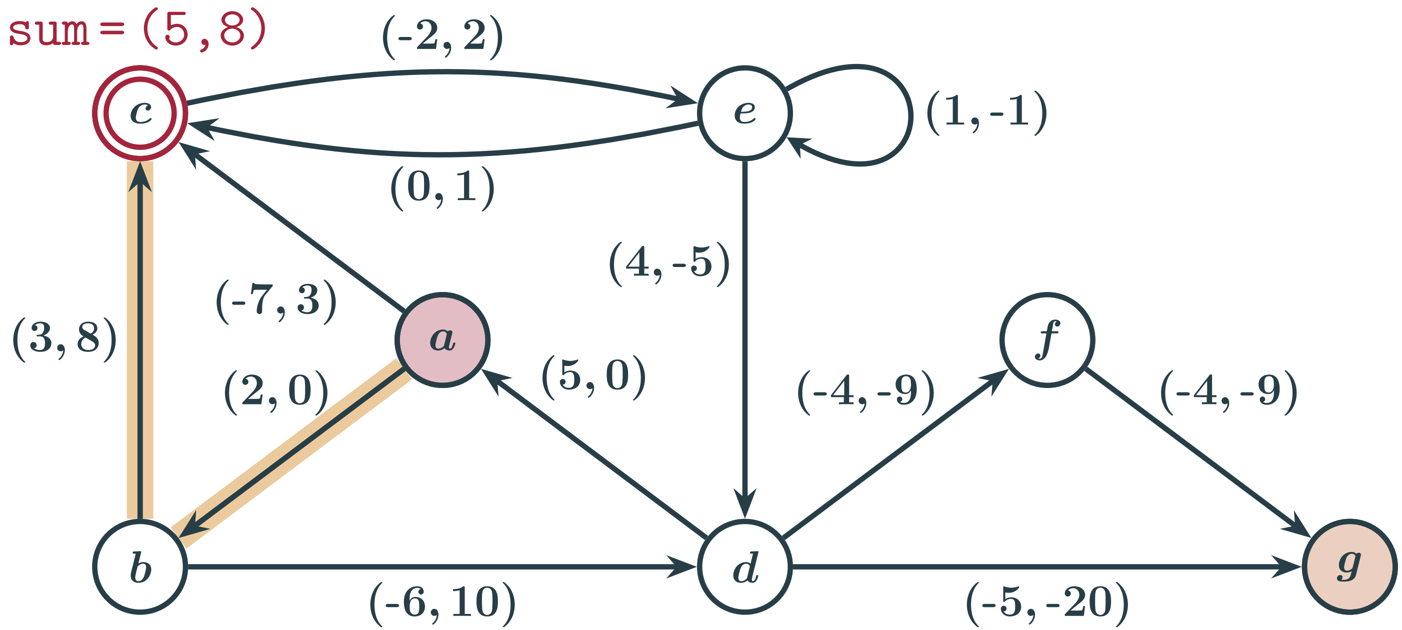

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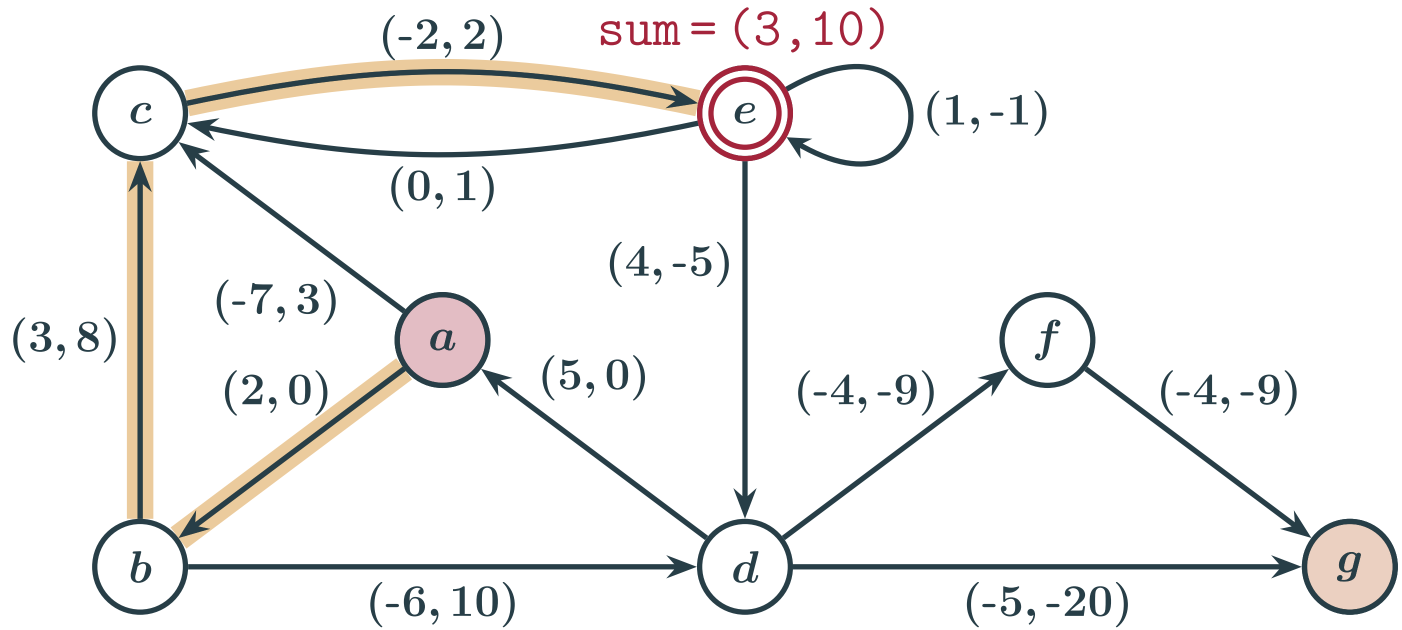

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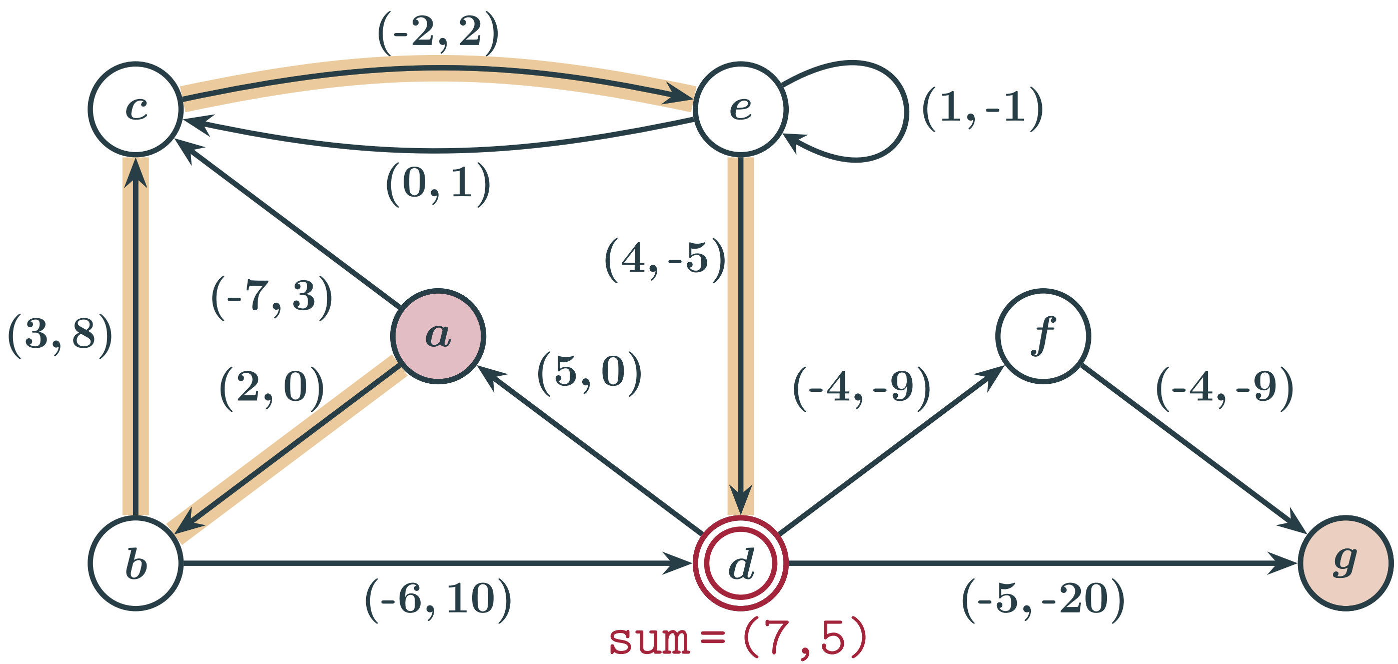

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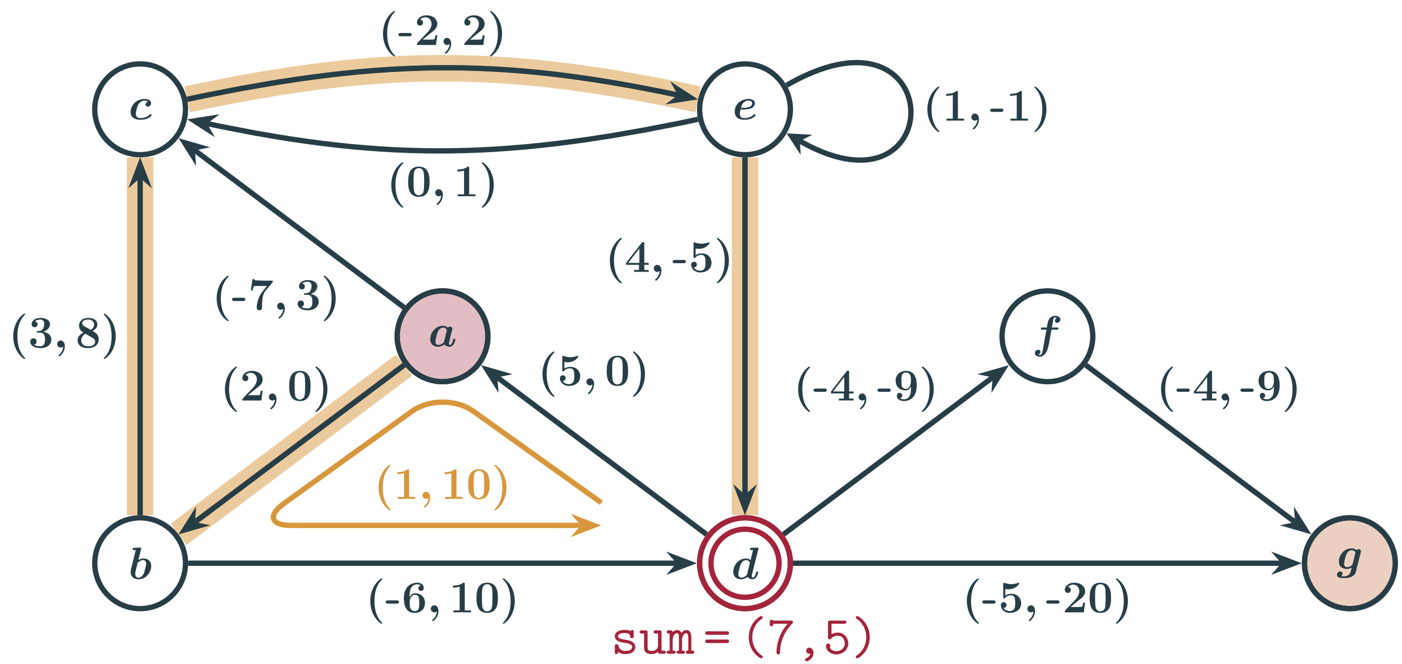

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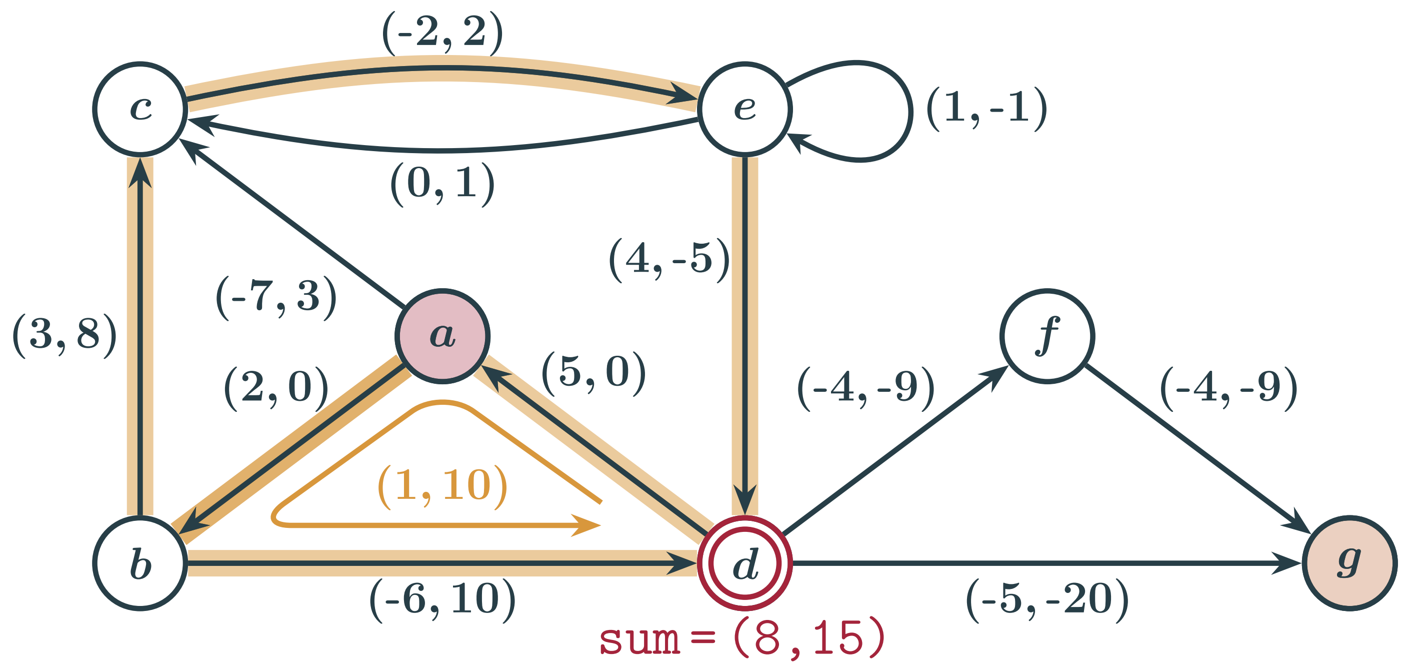

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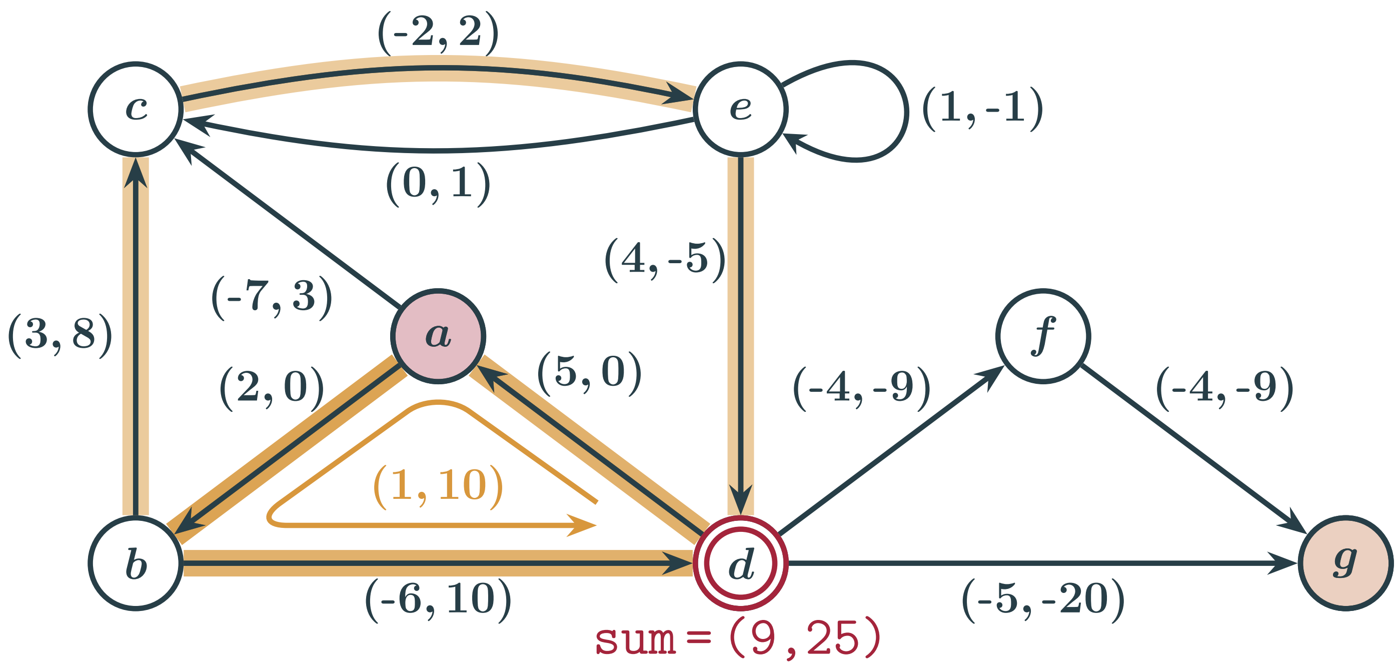

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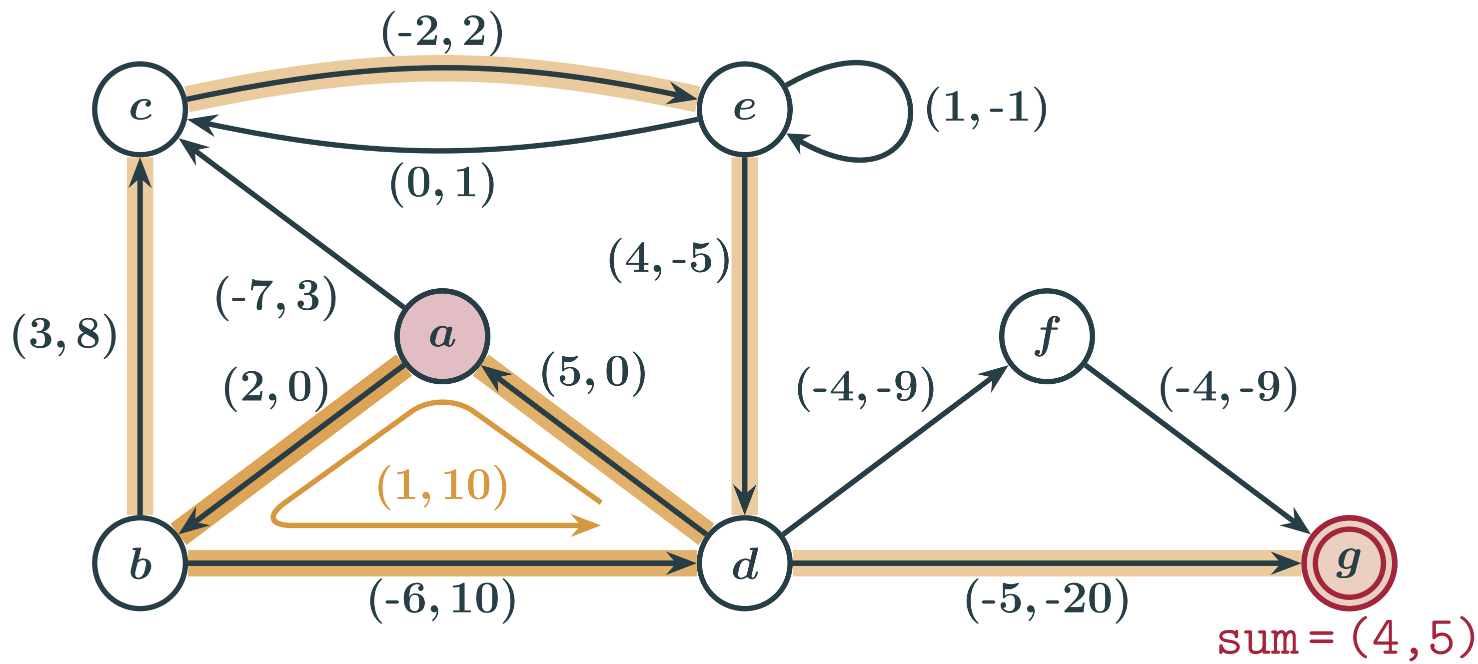

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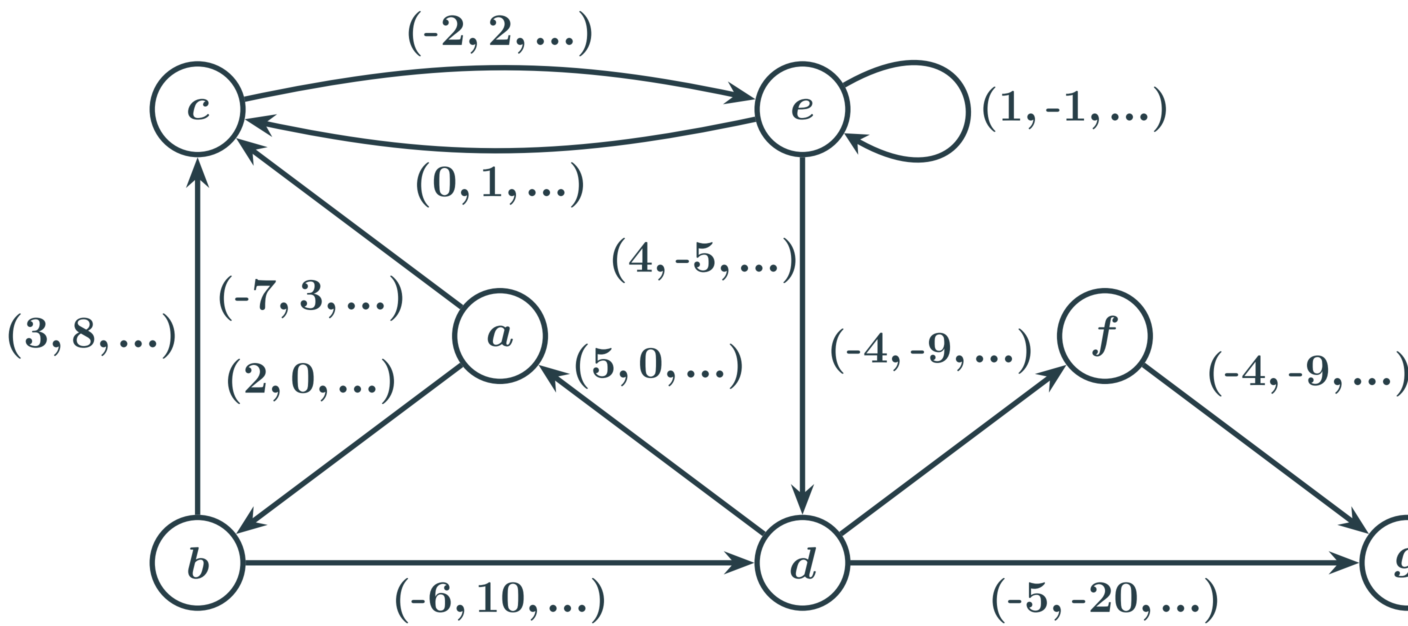
Coverability in VASS



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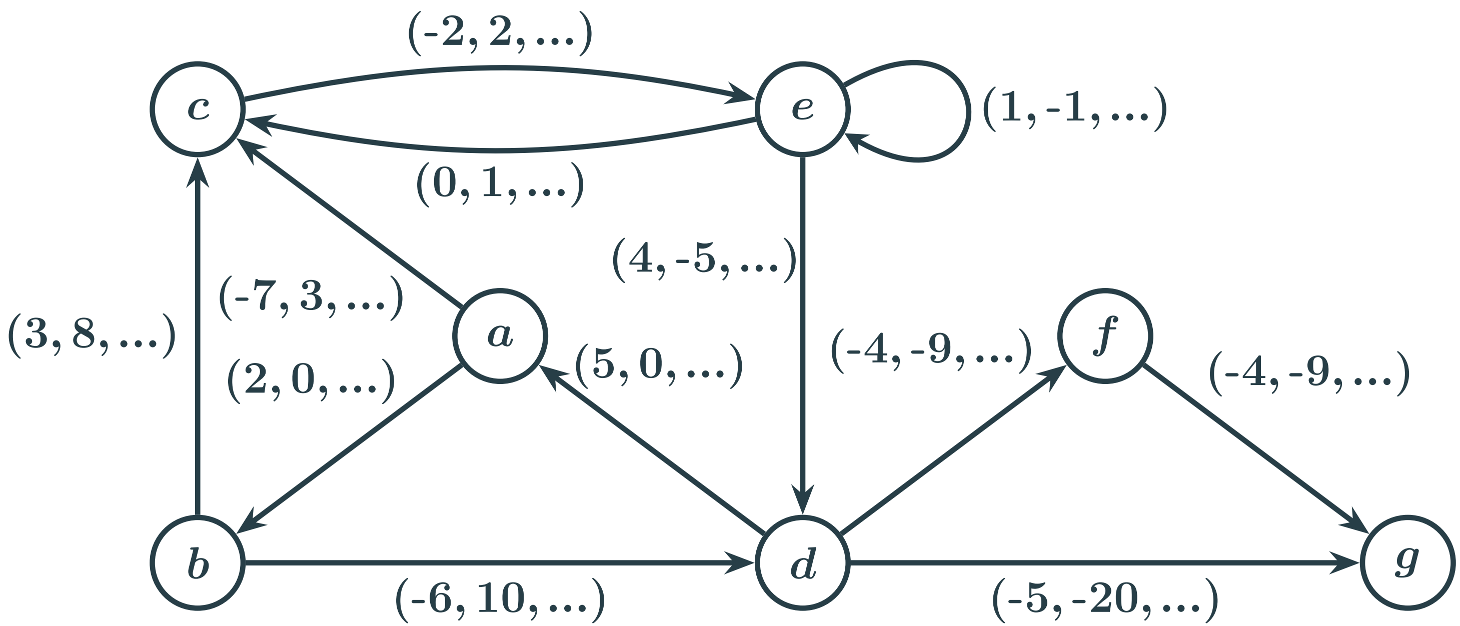

Coverability problem: from p can you reach q via a path that is *never negative on any component* ?

Motivation

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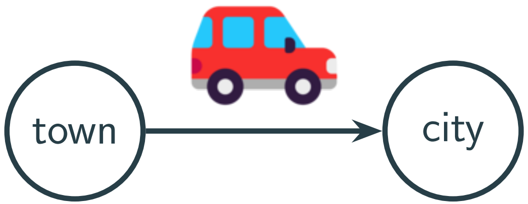
Resource Management



Road cost: (-1L fuel, $+2$kWh battery)

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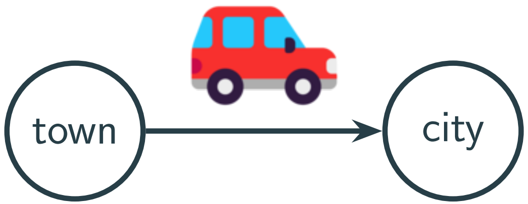

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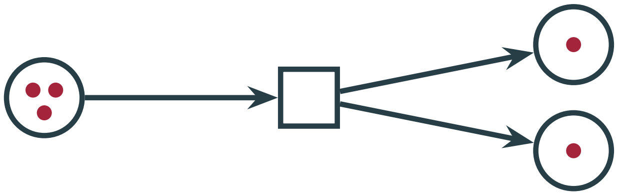
Model of Concurrency

VASS are equivalent to Petri nets



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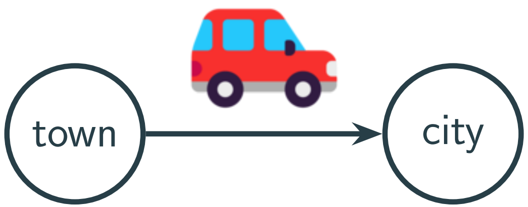

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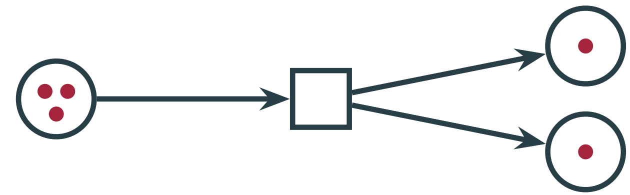

Testing Safety

Positive instance of coverability

$\Downarrow$

Some action sequence reaches a ‘bad’ state

$\Downarrow$

System is unsafe!

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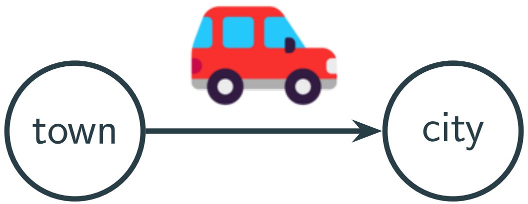

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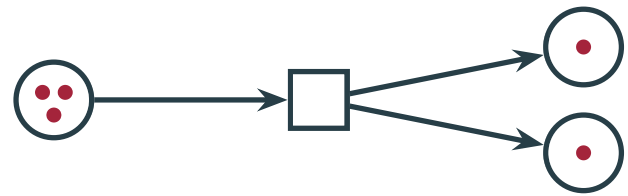

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Related Problems

Unboundedness

Reachability

Word problems for (commutative) semi-groups

History and Complexity

d is the dimension: number of components.

n is the size: number of states plus the absolute values of all updates.
(unary encoding)

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Richard Lipton

Theorem: Coverability in VASS requires $2^{\Omega(d)} \cdot \log(n)$-space. [Lipton '76]

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[Mayr and Meyer '82]

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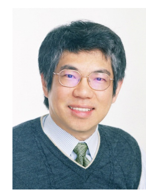
Hsu-Chun Yen

Refined via a multiparameter analysis.

[Rosier and Yen '85]

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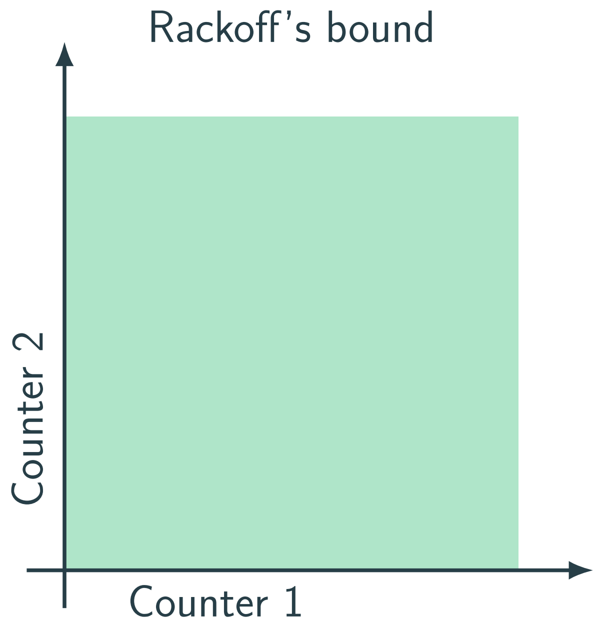

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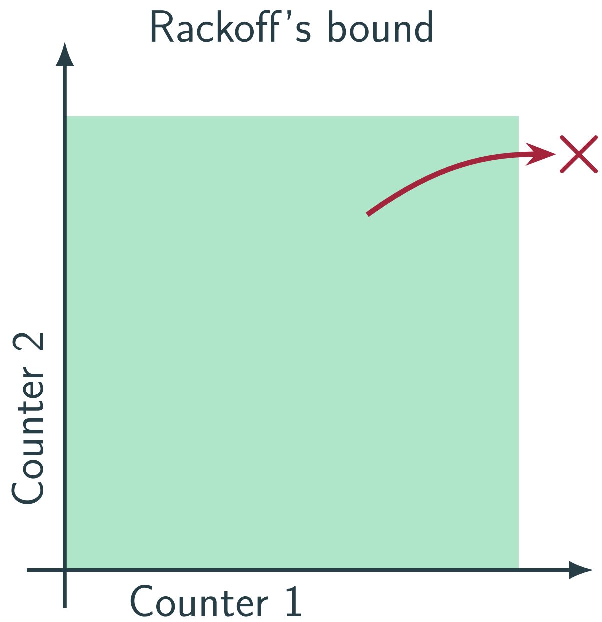

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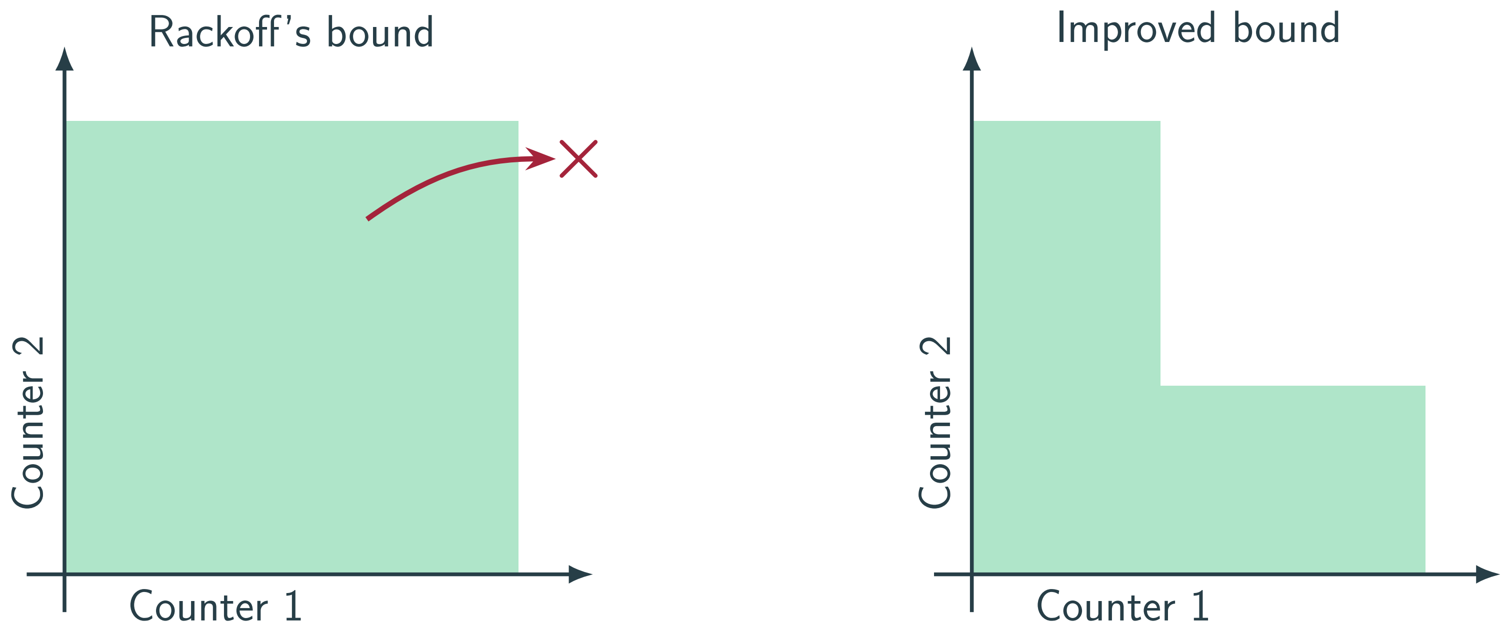

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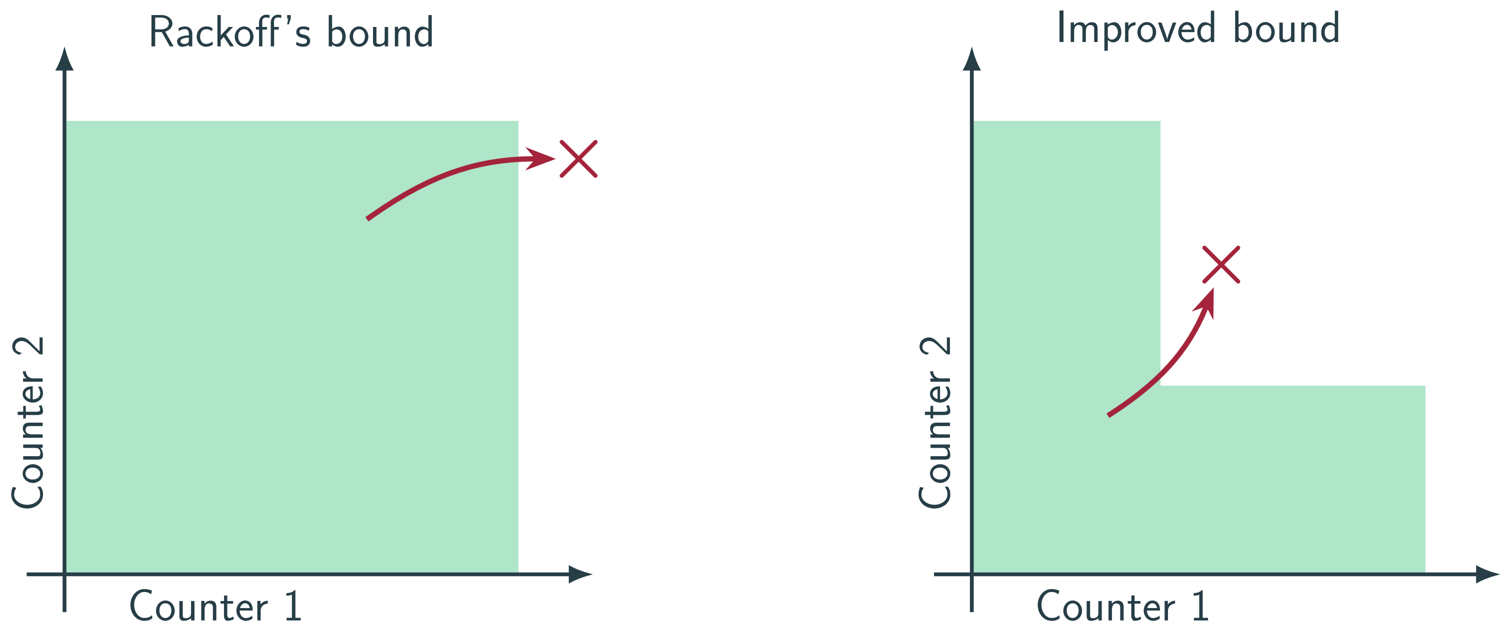

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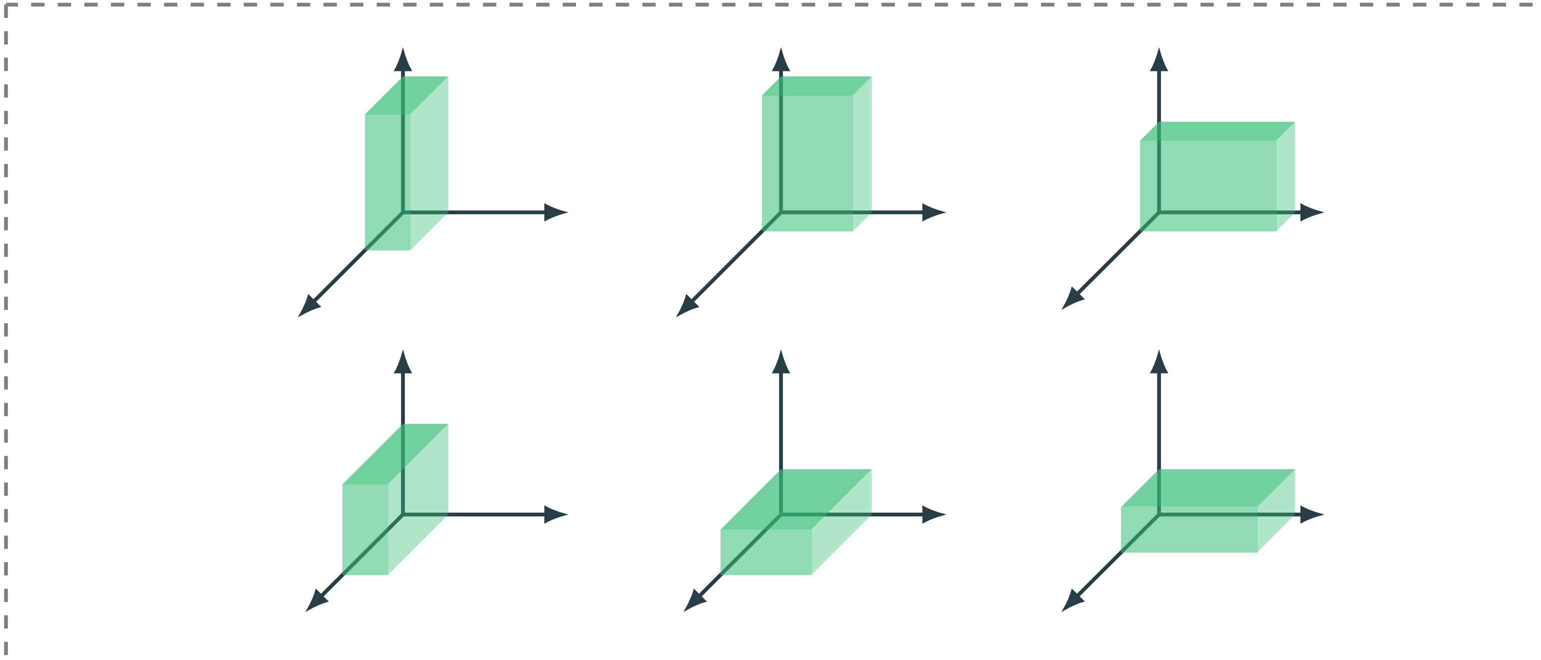

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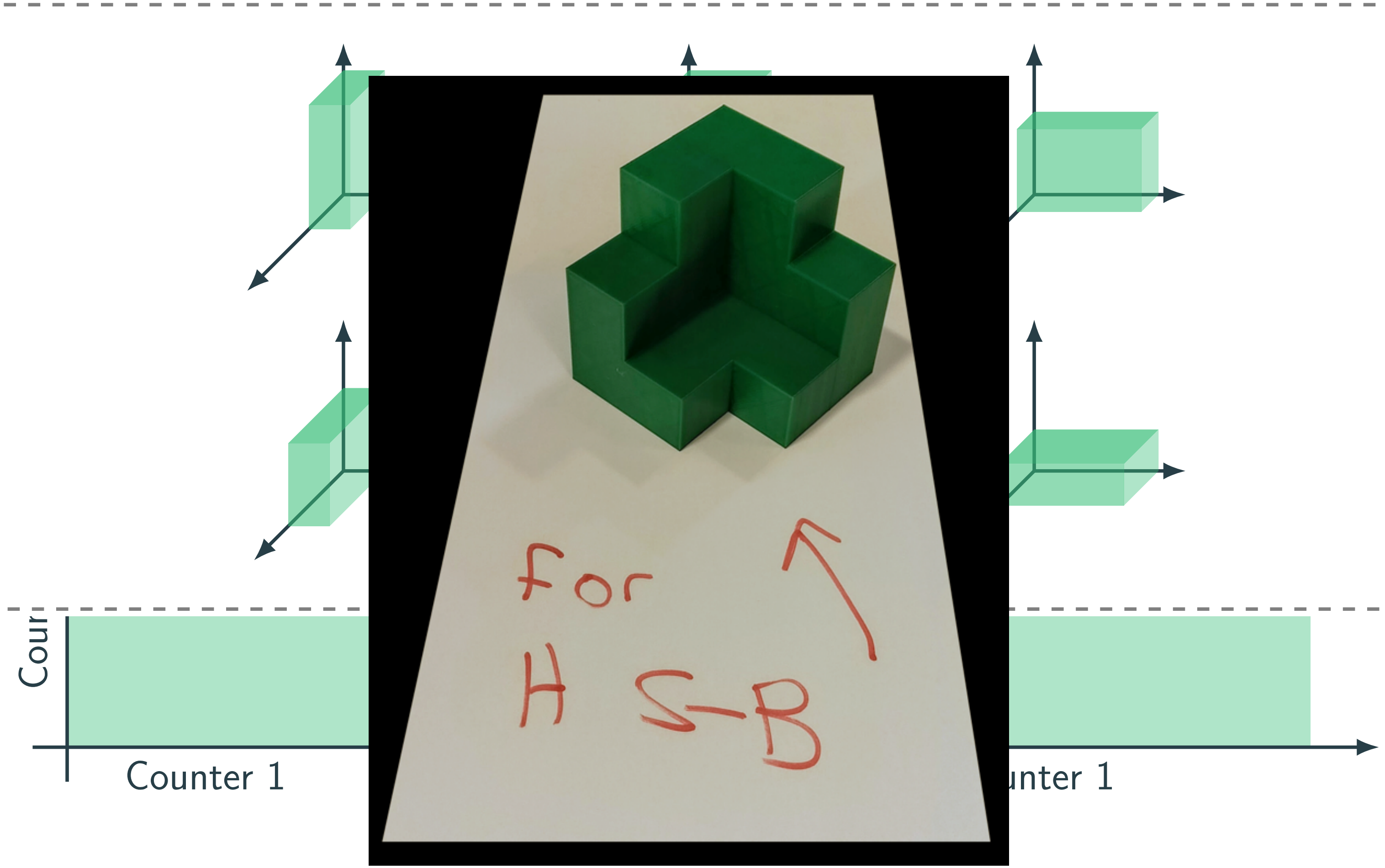

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Thank You!

Presented by Henry Sinclair-Banks, University of Warwick, UK

Highlights'23 in University of Kassel, Germany