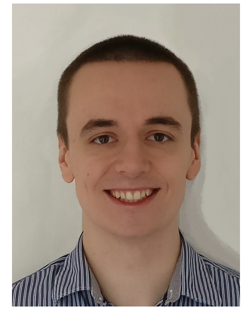
Coverability in VASS Revisited: Improving Rackoff's Bound to Obtain Conditional Optimality

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ICALP'23: Track B Best Paper

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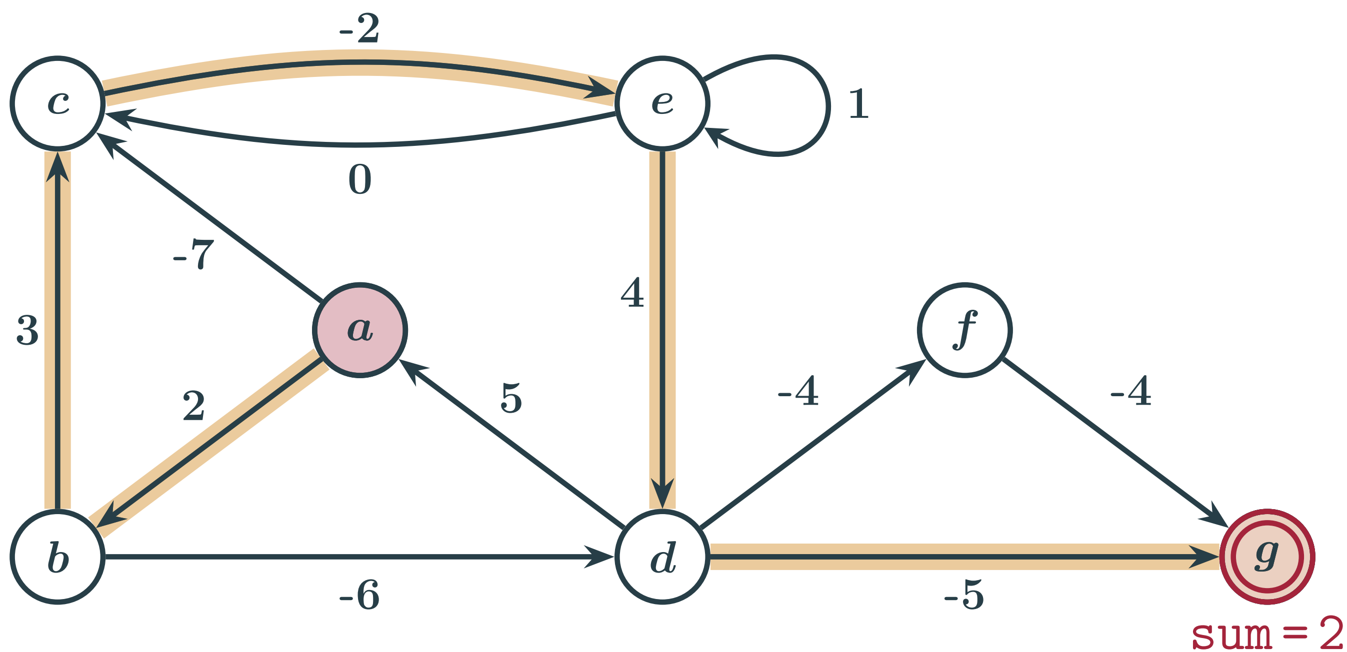
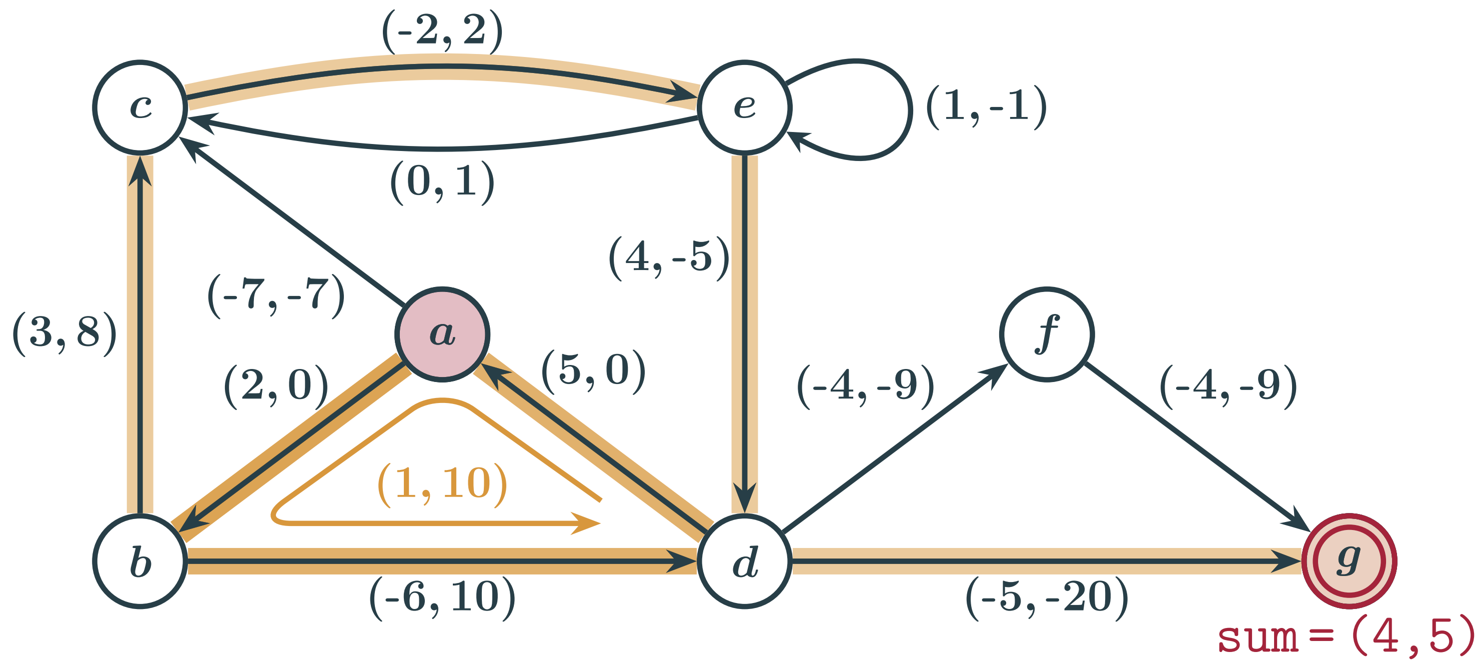
Heinz Nixdorf MuseumsForum, Paderborn, Germany

Never Negative Paths in Weighted Graphs



Question: from a can you reach g via a path that is *never negative*? **YES!**

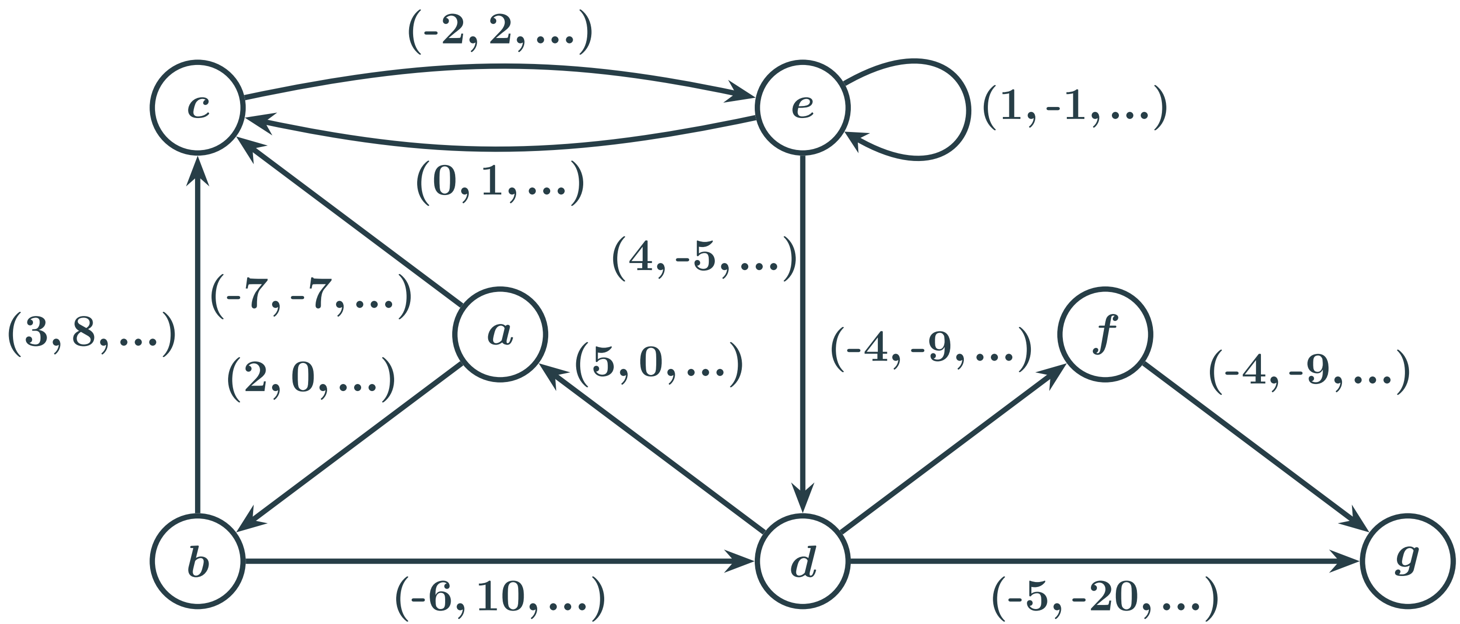
Never Negative Paths in Multi-Weighted Graphs



YES!

Question: from a can you reach g via a path that is *never negative on any component* ?

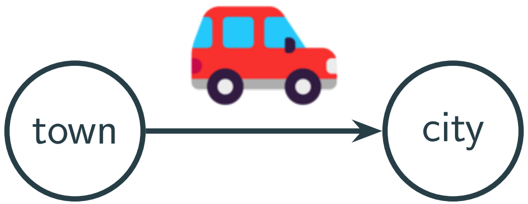
Coverability in Vector Addition Systems with States



Coverability problem: from p can you reach q via a path that is never negative on any component ?

Motivation

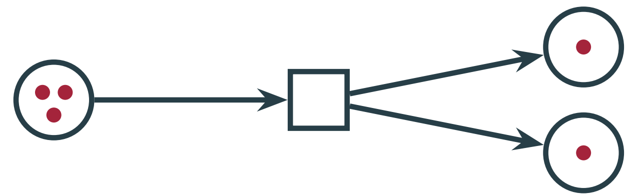
Resource Management



Road cost: $(-1\text{L fuel}, +2\text{kWh battery})$

Model of Concurrency

VASS are equivalent to Petri nets



Testing Safety

Positive instance of coverability

$\Downarrow$

Some action sequence reaches a 'bad' state

$\Downarrow$

System is unsafe!

Related Problems

Unboundedness

Reachability

Word problems for (commutative) semi-groups

History and Complexity

Richard Lipton

Theorem: Coverability in VASS is EXPSPACE-hard. [Lipton '76]

Charles Rackoff

Theorem: Coverability in VASS is in EXPSPACE. [Rackoff '78]

d is the dimension: number of components.

n is the size: number of states plus the absolute values of all updates (unary encoding).

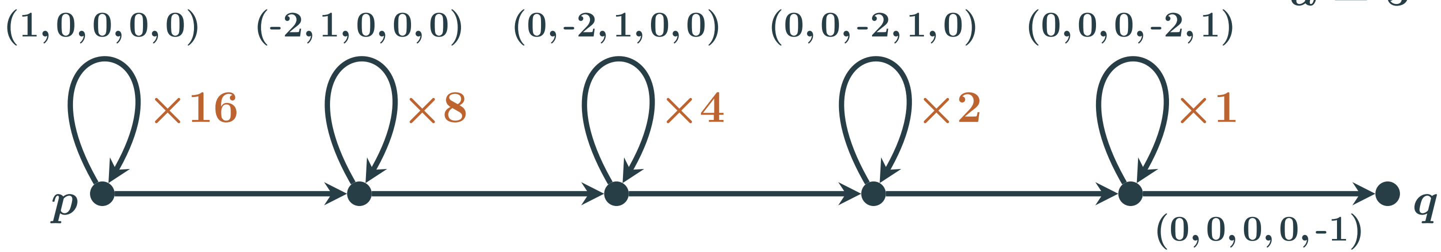
History and Complexity

Richard Lip

[Lipton '76]

Example of Long Coverability Runs

$d = 5$



Any coverability run from p to q has length $2^{\Omega(d)}$.

d is the dimension: number of components.

n is the size: number of states plus the absolute values of all updates (unary encoding).

History and Complexity


Richard Lipton

Theorem: Coverability in VASS requires $2^{\Omega(d)} \cdot \log(n)$-space. [Lipton '76]

Idea: find instances only admitting $n^{2^{\Omega(d)}}$ length runs. *"Lipton's construction"*


Charles Rackoff

Theorem: Coverability in VASS can be decided in $2^{\mathcal{O}(d \log d)} \cdot \log(n)$-space. [Rackoff '78]

Idea: argue that there are always $n^{2^{\mathcal{O}(d \log d)}}$ length runs. *"Rackoff's bound"*


Ernst Mayr

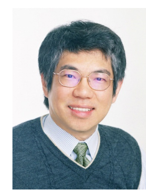

Albert Meyer

Open Problem

Improve these bounds.

[Mayr and Meyer '82]


Louis Rosier


Hsu-Chun Yen

Refined via a multiparameter analysis.

[Rosier and Yen '85]

d is the dimension: number of components.
n is the size: number of states plus the absolute values of all updates (unary encoding).

Contributions

Theorem: Coverability in VASS is always witnessed by $n^{2^{\mathcal{O}(d)}}$ length runs.

$\implies$ Coverability in VASS can be decided in $2^{\mathcal{O}(d)} \cdot \log(n)$-space. **OPTIMAL!**

$\implies$ Coverability in VASS can be decided in $n^{2^{\mathcal{O}(d)}}$-time. **CONDITIONALLY OPTIMAL!**

Theorem: Assuming ETH, coverability in VASS requires $n^{2^{\Omega(d)}}$-time.

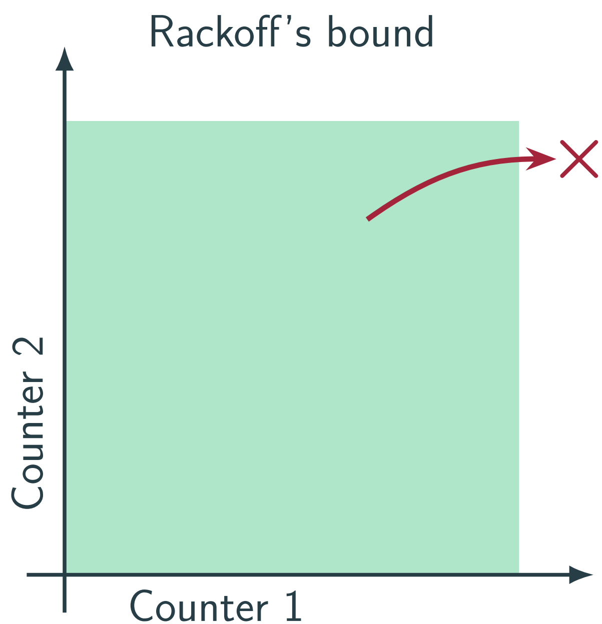
Theorem: Under the k–cycle hypothesis, coverability in VASS requires $n^{2-o(1)}$-time, for $d = 2$.

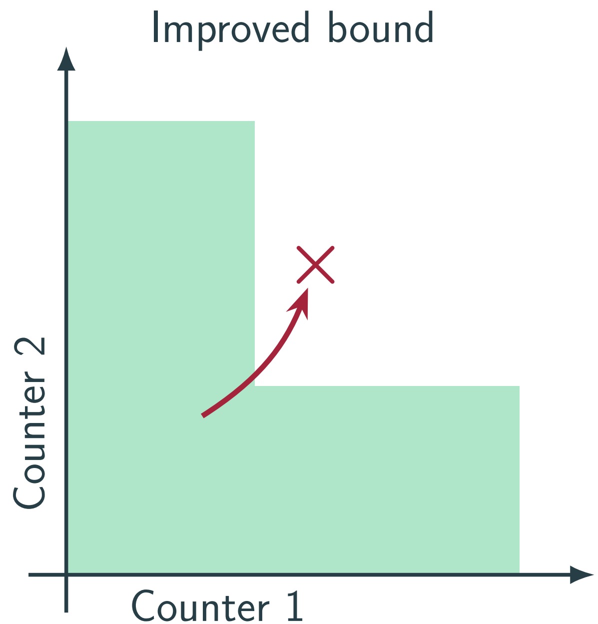
Theorem: Under the k–hyperclique hypothesis, coverability in *linearly bounded* VASS requires $n^{d-2-o(1)}$-time.

Improving Rackoff's Upper Bound

Theorem: Coverability in VASS is always witnessed by $n^{2^{\mathcal{O}(d)}}$ length runs. [this paper]

Idea: Carefully use Rackoff's bounding technique with sharper counter value bounds.



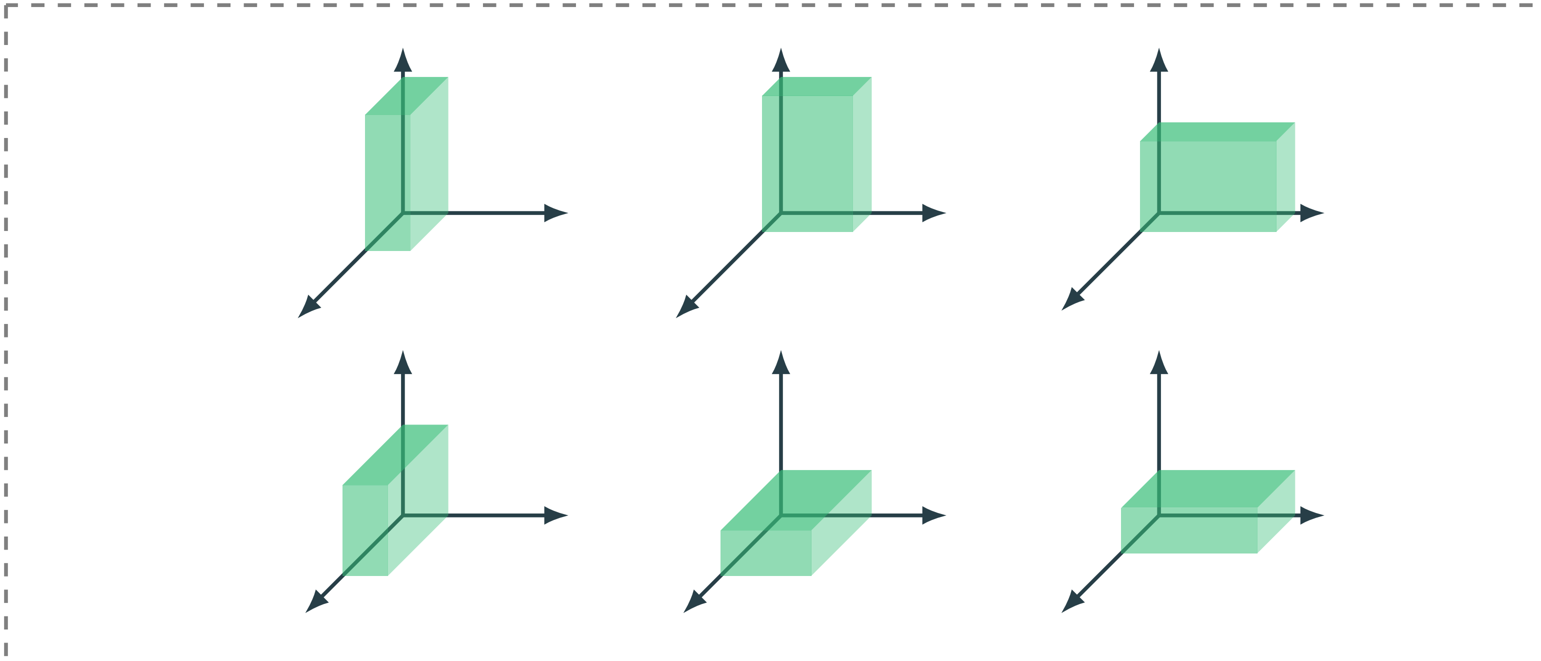


Improving Rackoff's Upper Bound

Theor

[this paper]

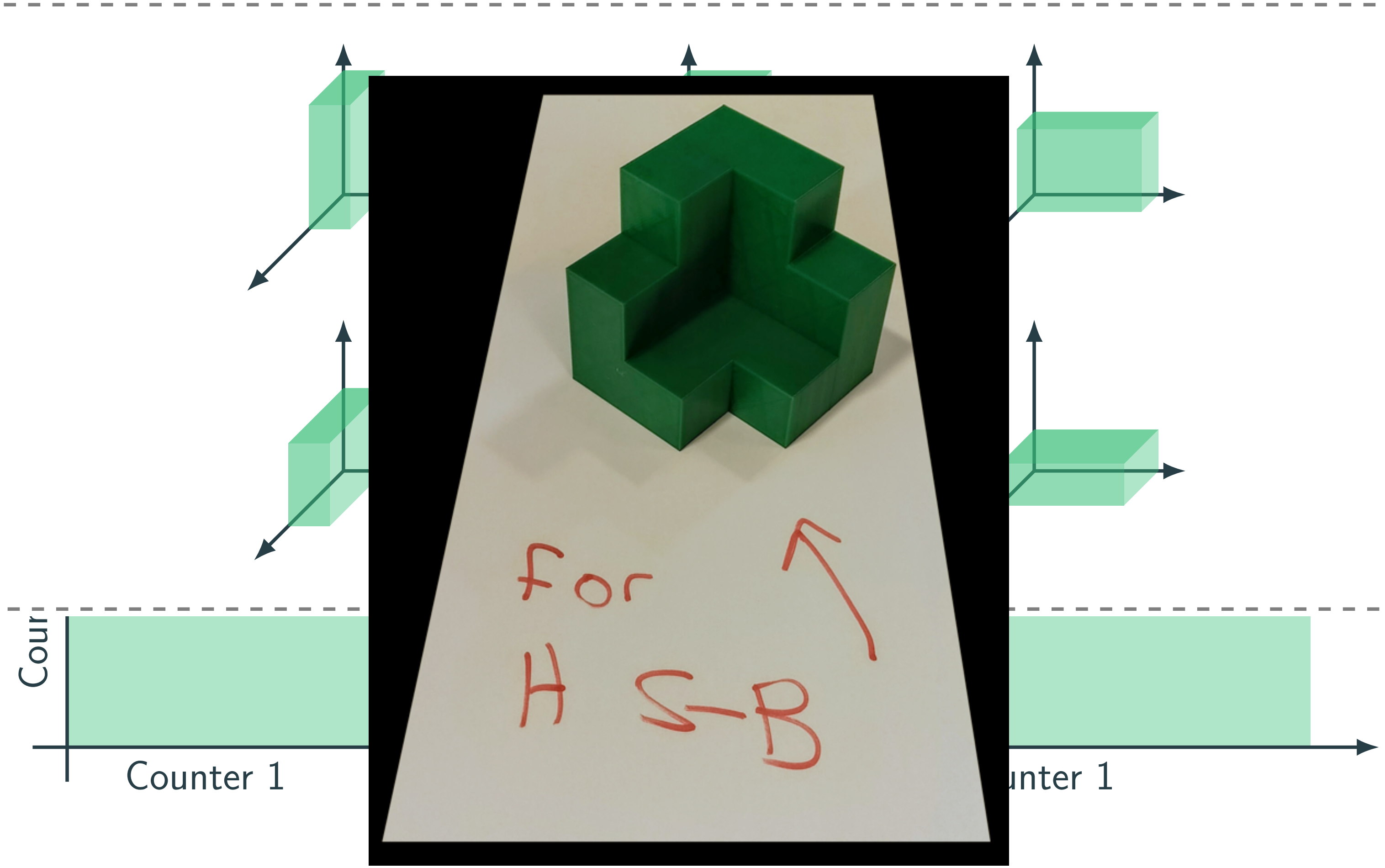
Idea:



Improving Rackoff's Upper Bound

Theor [this paper]

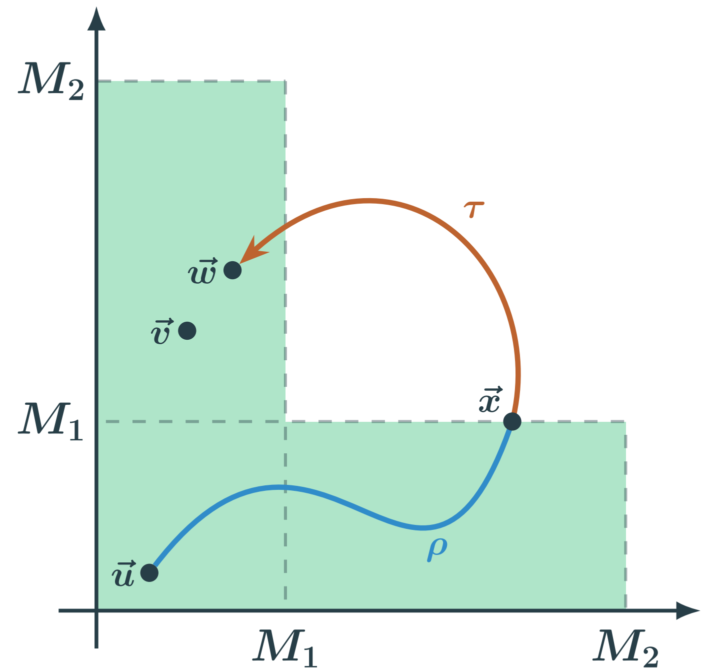
Idea:



Bounding the Length of Coverability Runs

Consider the shortest coverability run $\vec{u} \xrightarrow{\pi} \vec{w}$, where $\vec{w} \geq \vec{v}$.

Split π at first "non-thin" configuration: $\vec{u} \xrightarrow{\rho} \vec{x} \xrightarrow{\tau} \vec{w}$.



ρ is the *thin part* of the run, its length is bounded by the number of thin configurations.

$$len(\rho) \leq d!(M_1 \cdot M_2 \cdot \ldots \cdot M_d)$$

τ is the *tail* of the run, at least one component had a large value at $\vec{x}$, so can then be 'ignored'.

$$len(\tau) \leq M_d$$

$$len(\pi) = len(\rho) + len(\tau) \leq 2 \cdot d!(M_1 \cdot M_2 \cdot \ldots \cdot M_d) \leq n^{2^{\mathcal{O}(d)}}$$

Conditionally Optimal Time Bound

$\implies$ Coverability in VASS can be decided in $n^{2^{\mathcal{O}(d)}}$-time.

CONDITIONALLY OPTIMAL!

Theorem: Assuming ETH, coverability in VASS requires $n^{2^{\Omega(d)}}$-time. [this paper]

Exponential Time Hypothesis

$\Downarrow$

There are no $n^{o(k)}$-time algorithms for finding a k-clique in a graph.

Idea: Reduce the problem of finding a $k = 2^d$-clique in a graph to coverability in VASS.

Coverability in VASS Revisited: Improving Rackoff's Bound to Obtain Conditional Optimality

Theorem: Coverability in VASS is always witnessed by $n^{2^{\mathcal{O}(d)}}$ length runs. [this paper]

$\Longrightarrow$ Coverability in VASS can be decided in $2^{\mathcal{O}(d)} \cdot \log(n)$-space.

$\Longrightarrow$ Coverability in VASS can also be decided in $n^{2^{\mathcal{O}(d)}}$-time.

Theorem: Assuming ETH, coverability in VASS requires $n^{2^{\Omega(d)}}$-time. [this paper]

Open problem: Can coverability in VASS with $d = 1$ be decided in $o(n^2)$-time?

Thank You!

Presented by Henry Sinclair-Banks, University of Warwick, UK

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