Coverability in 2-VASS with One Unary Counter

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Germany

Highlights'22
30th June 2022
Paris, France

Fun-Road-Trip Checklist

✓ always at least one friend, and

✓ never negative money!

Friends: 4

Money: €100

Fun-Road-Trip Checklist

✓ always at least one friend, and

✓ never negative money!

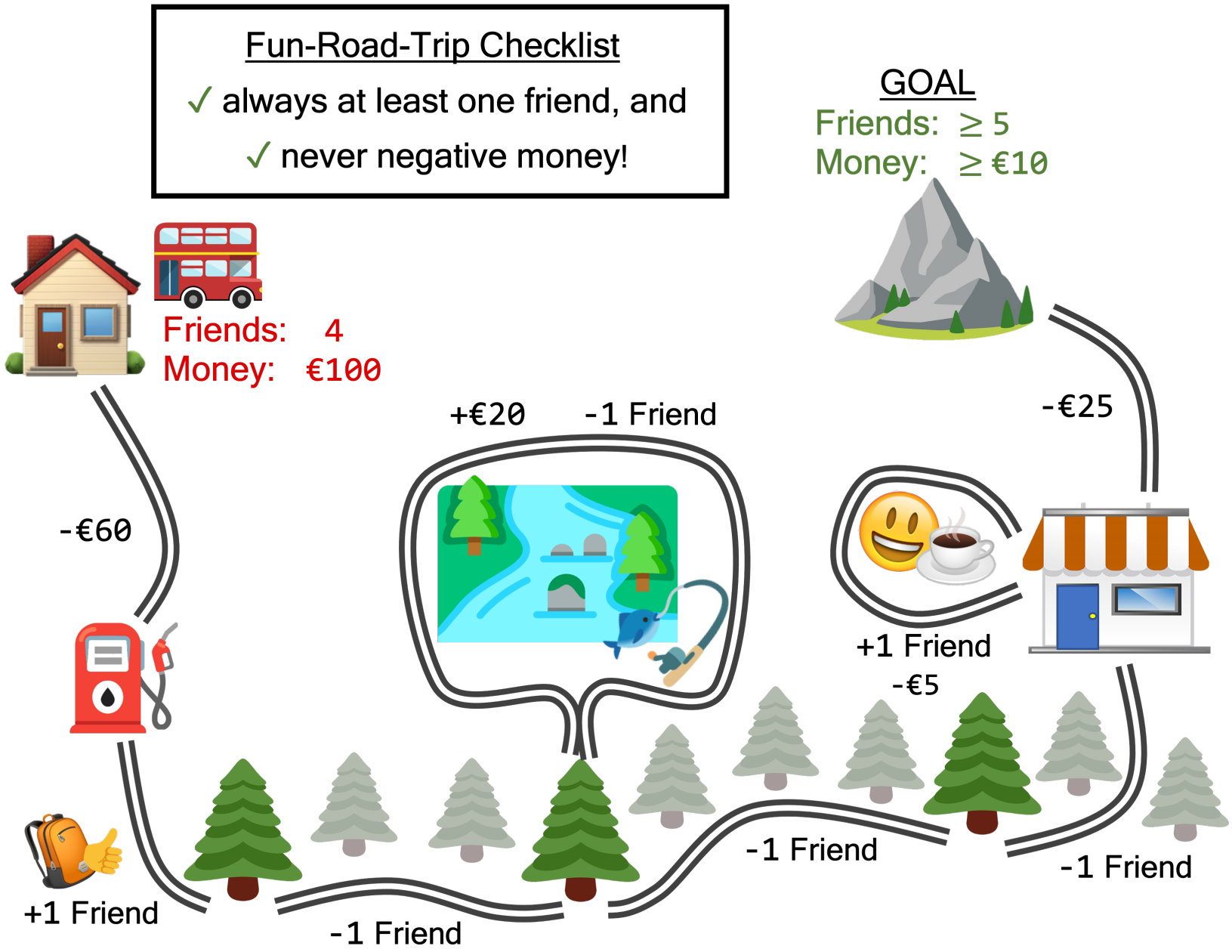
GOAL

Friends: ≥ 5

Money: ≥ €10

Friends: 4

Money: €100


Fun-Road-Trip Checklist
✓ always at least one friend, and
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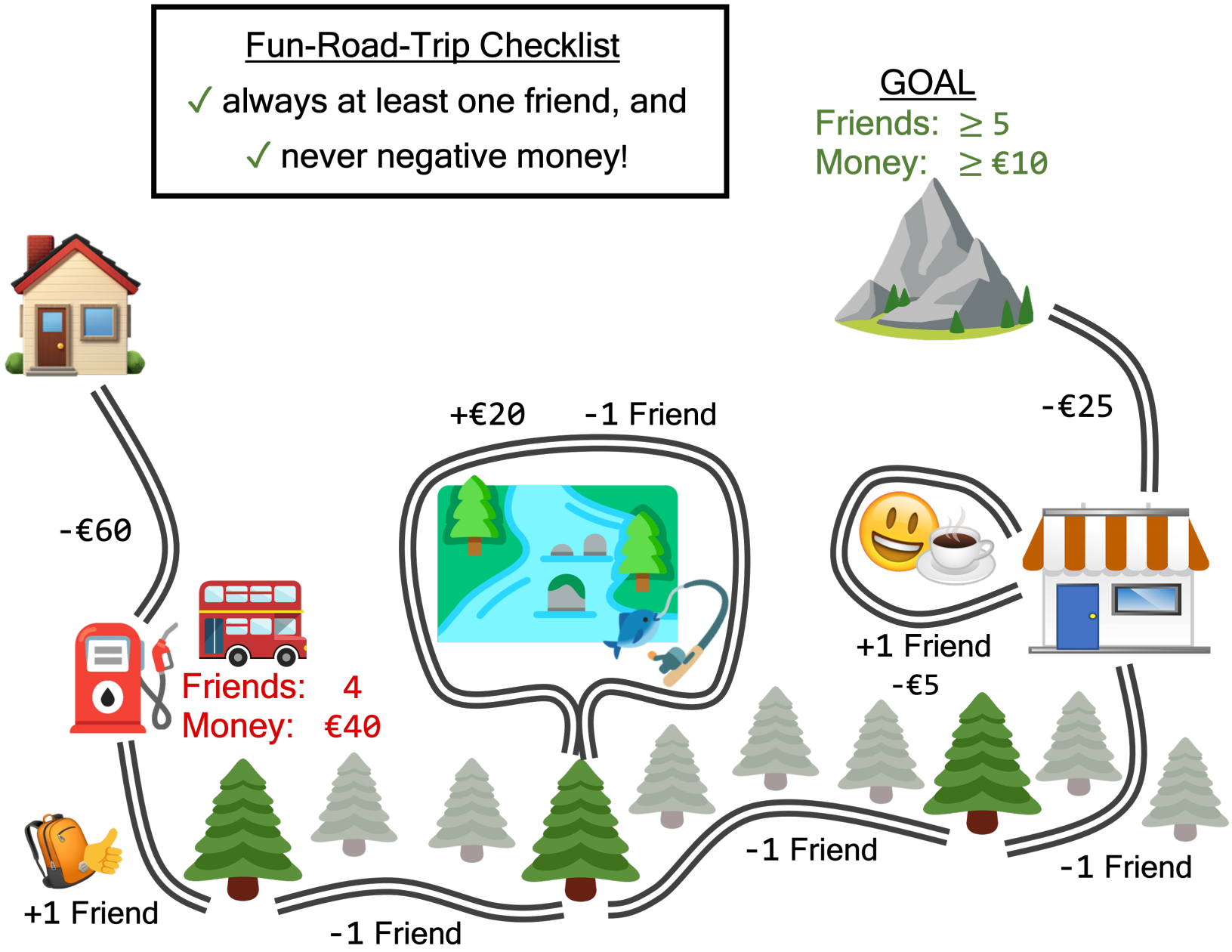
GOAL
Friends: ≥ 5
Money: ≥ €10
Friends: 4
Money: €100
+€20
-1 Friend
-€25
-€60
+1 Friend
-€5
+1 Friend
-1 Friend
-1 Friend
-1 Friend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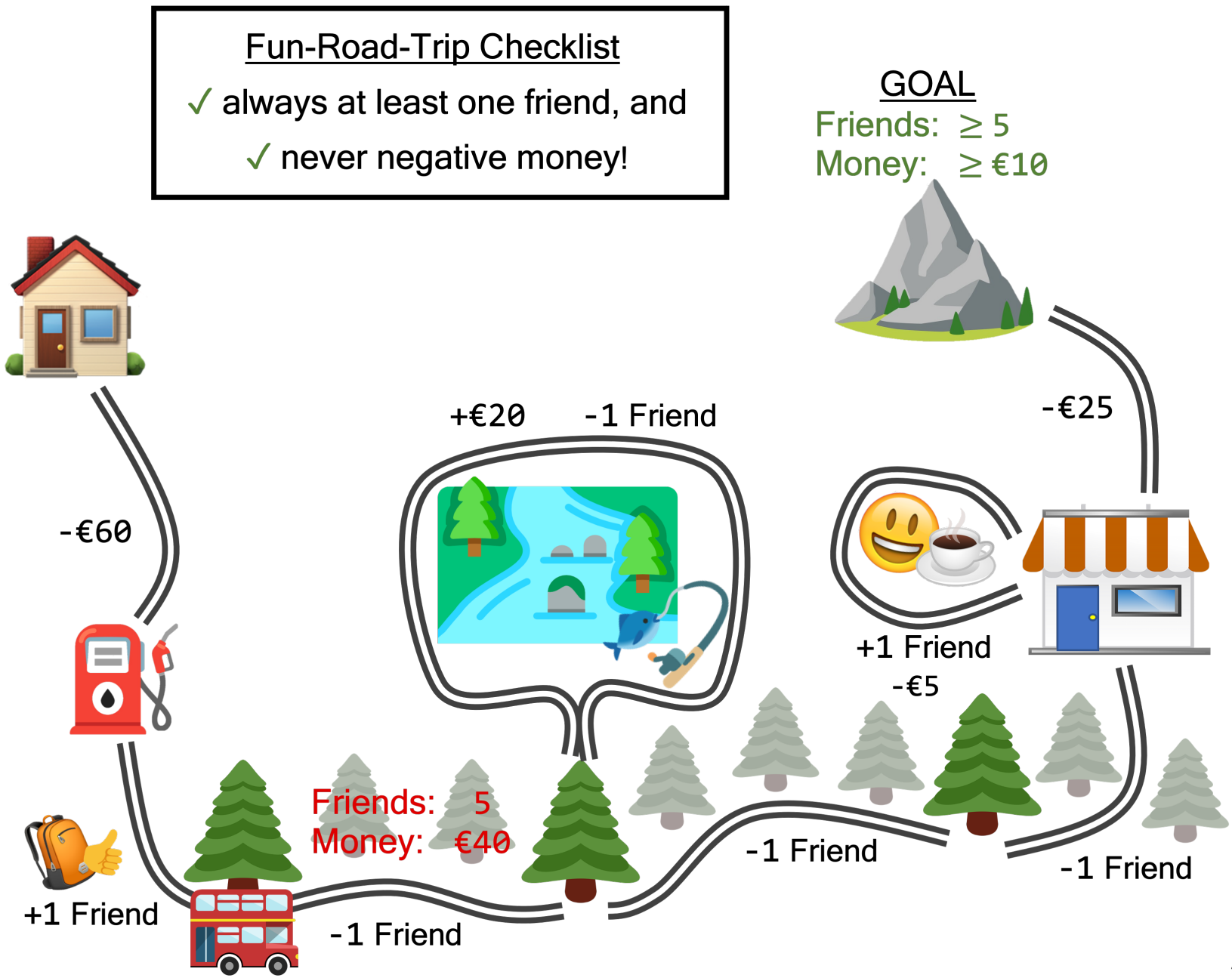
+€20
-1 Friend
-€25
+1 Friend
-€5
Friends: 4
Money: €40
+1 Friend
-1 Friend
-1 Friend
-1 Friend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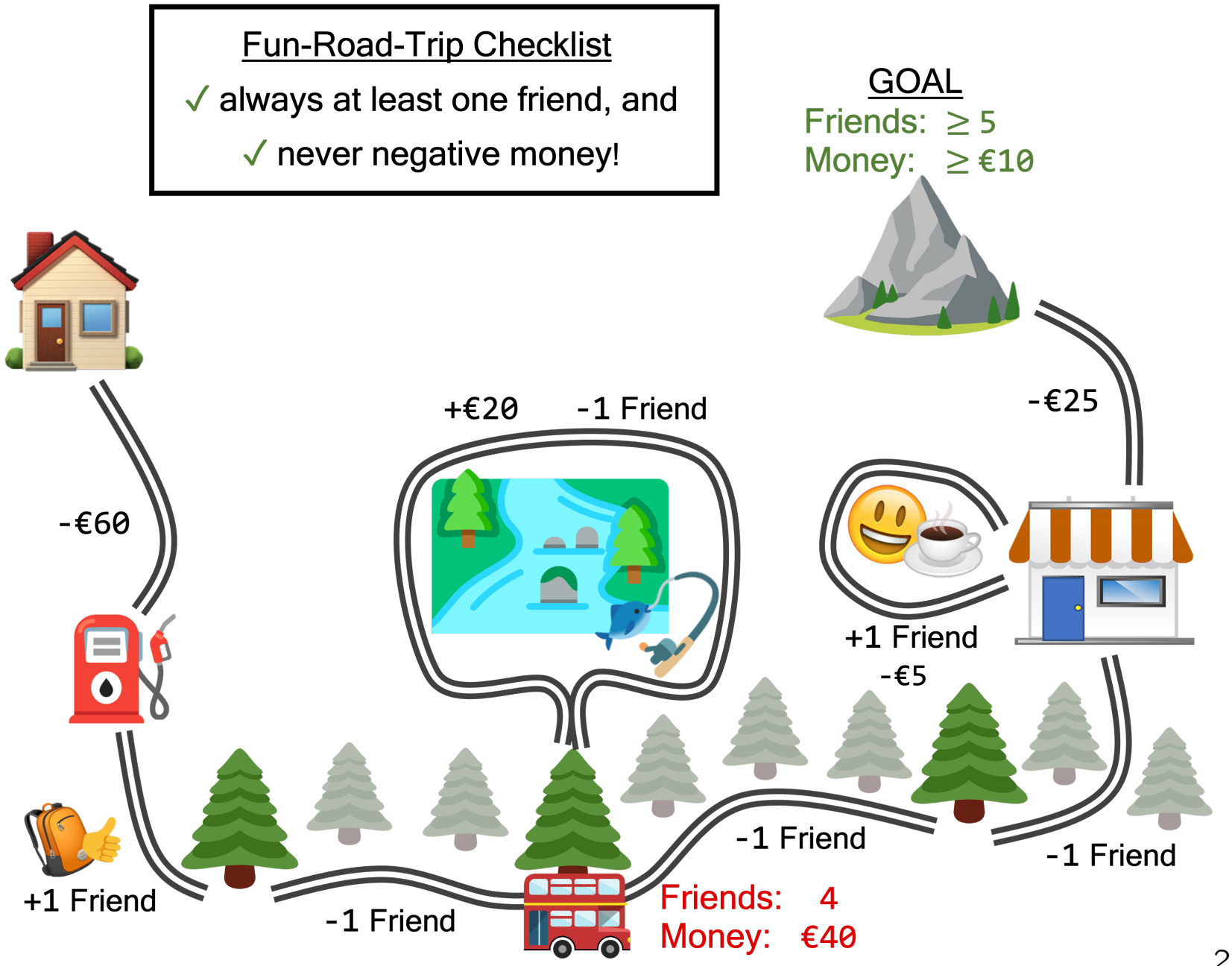
-€60
+€20
-1 Friend
-€25
+1 Friend
-€5
Friends: 5
Money: €40
+1 Friend
-1 Friend
-1 Friend
-1 Friend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-1 Friend
-1 Friend
-1 Friend
Friends: 4
Money: €40

Fun-Road-Trip Checklist

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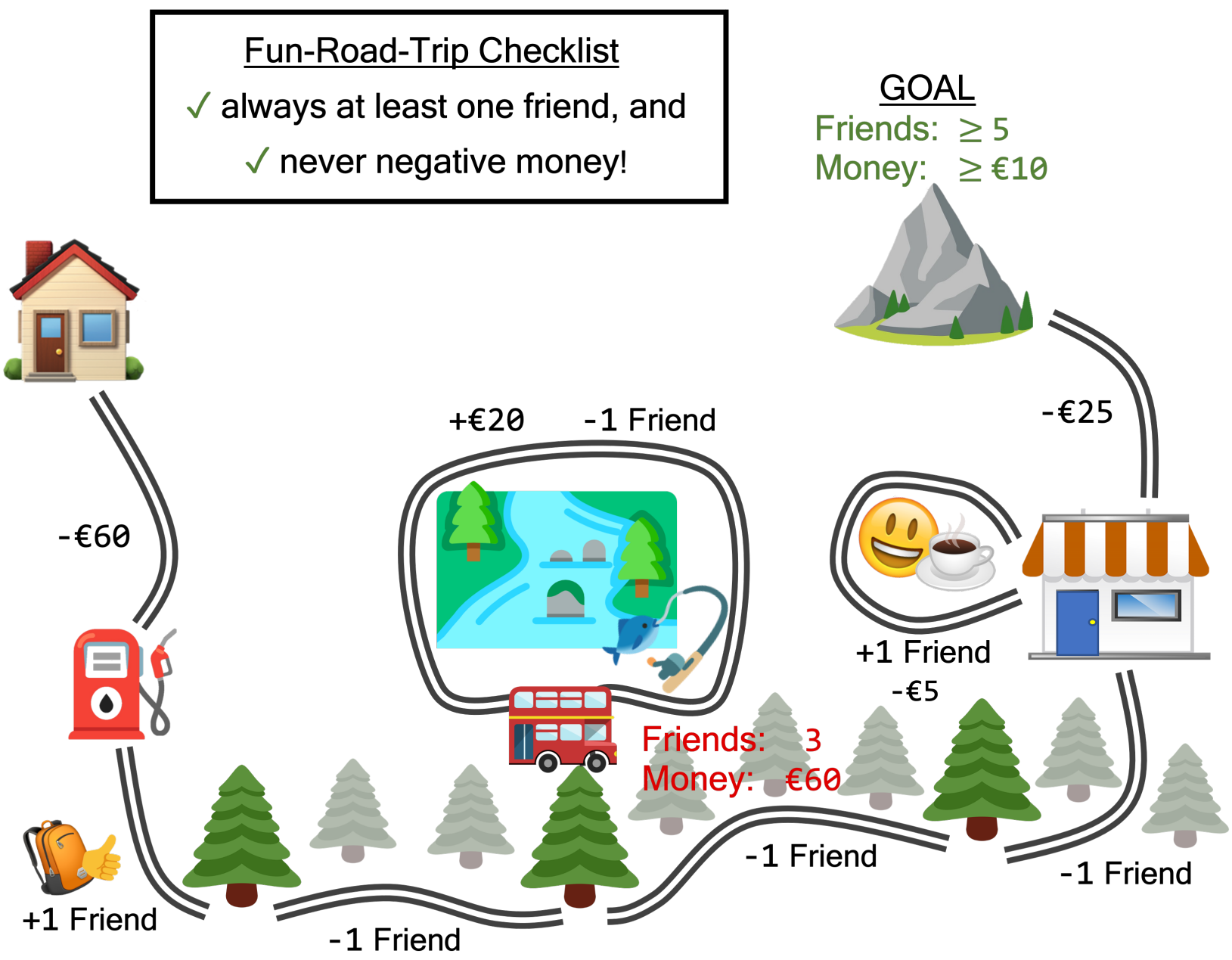

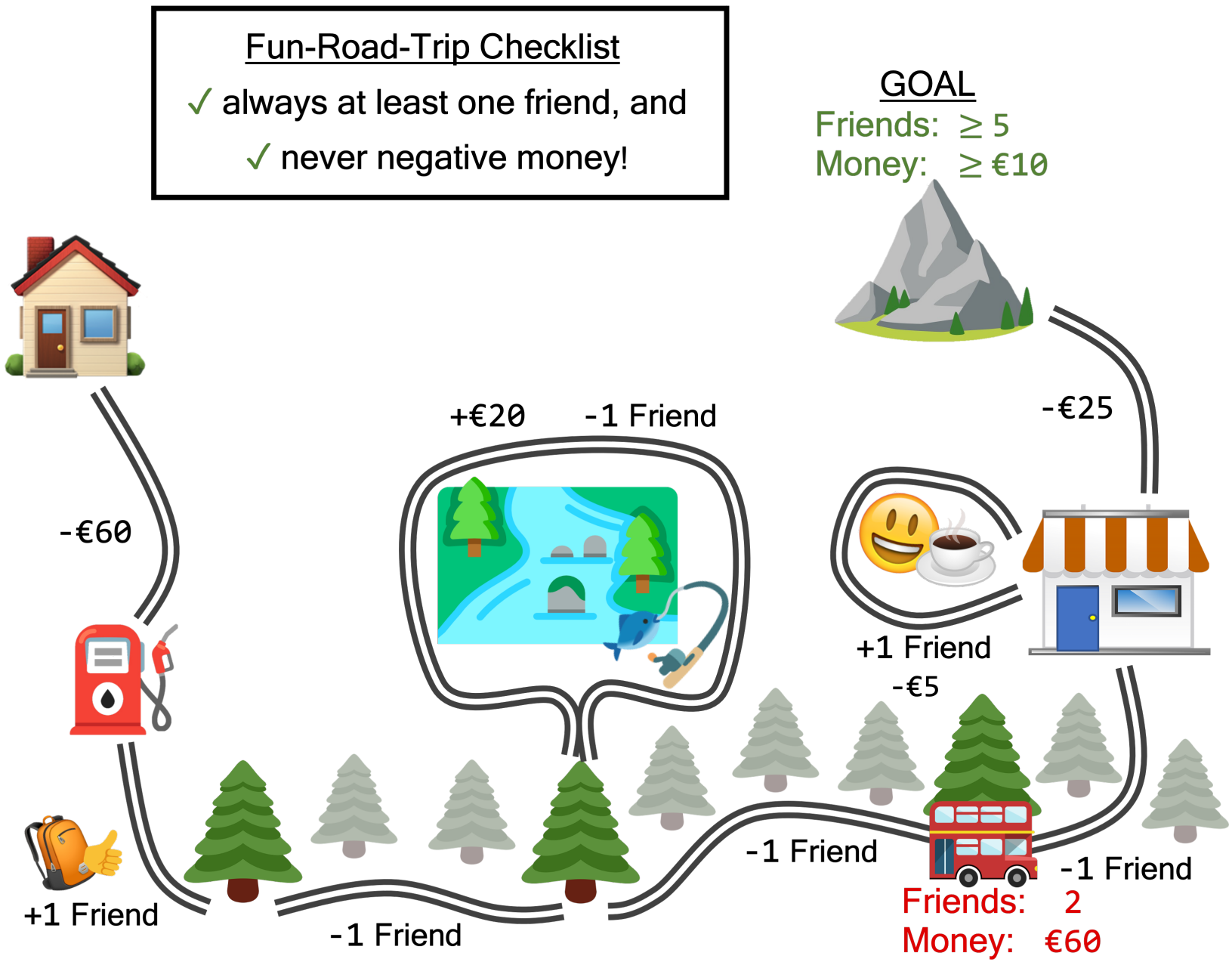
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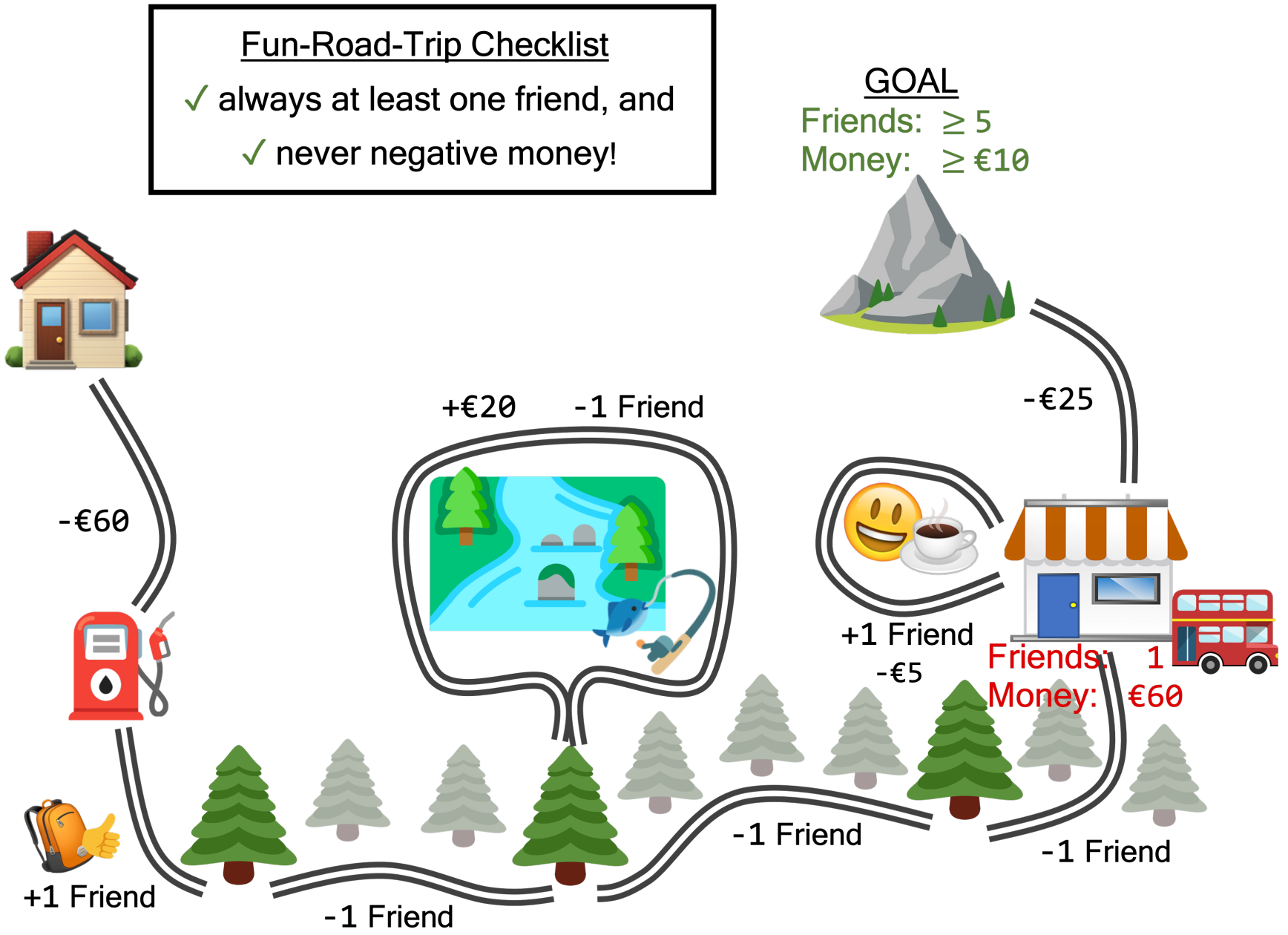
-€60
+€20
-1 Friend
-€25
+1 Friend
-€5
+1 Friend
-1 Friend
-1 Friend
-1 Friend
Friends: 2
Money: €60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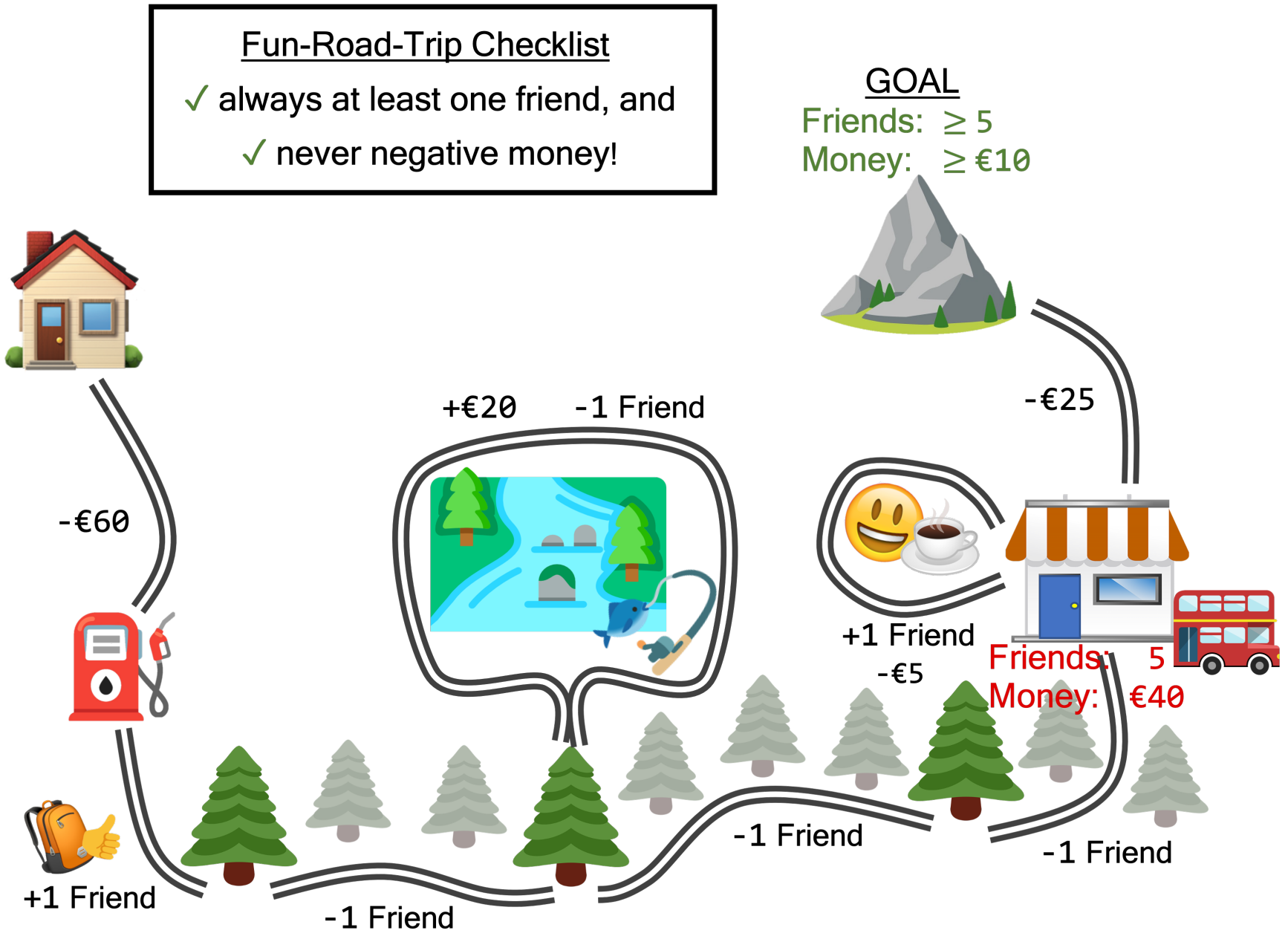
GOAL
Friends: ≥ 5
Money: ≥ €10
-€60
+€20
-1 Friend
-€25
+1 Friend
-€5
Friends: 1
Money: €60
+1 Friend
-1 Friend
-1 Friend
-1 Friend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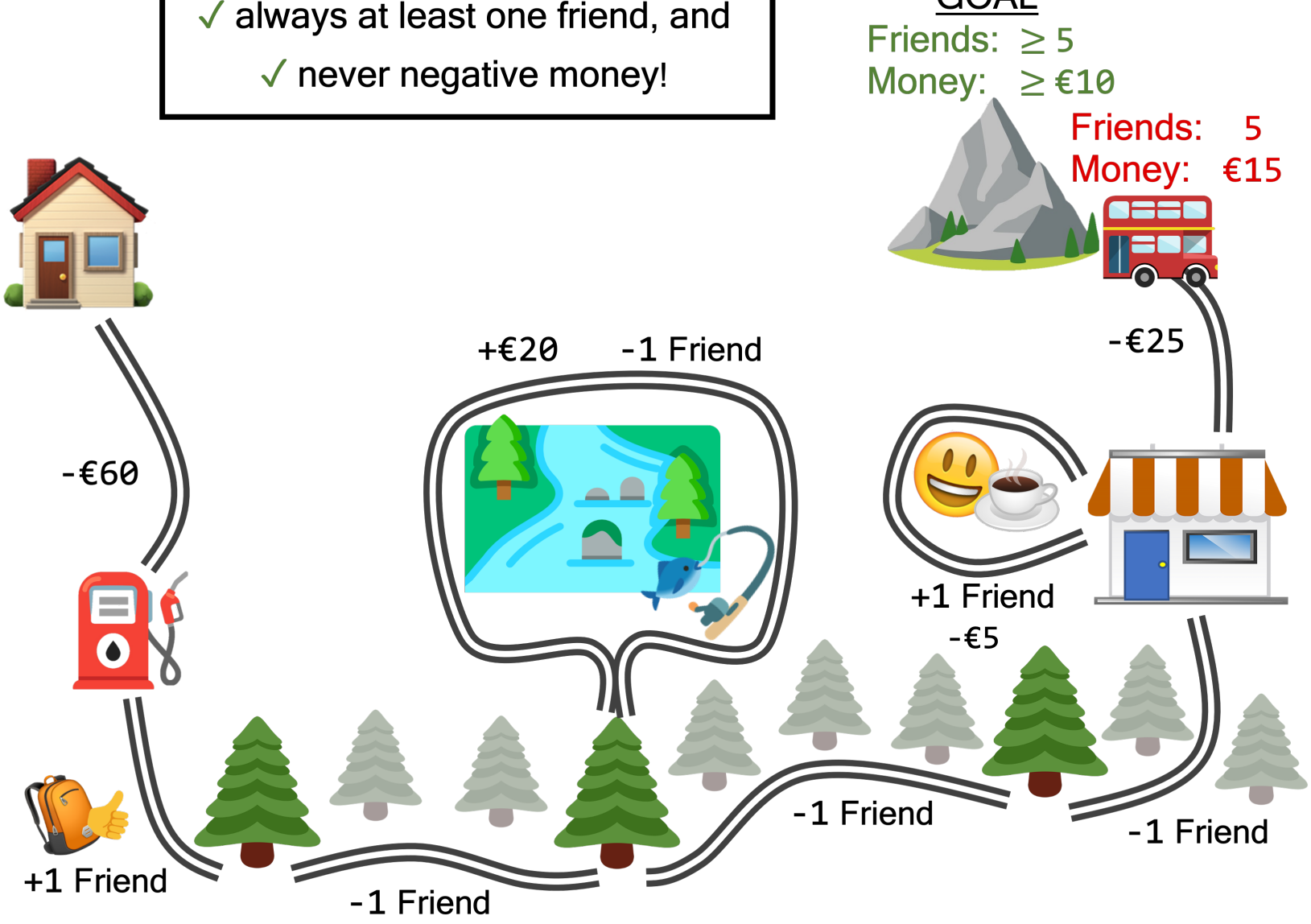
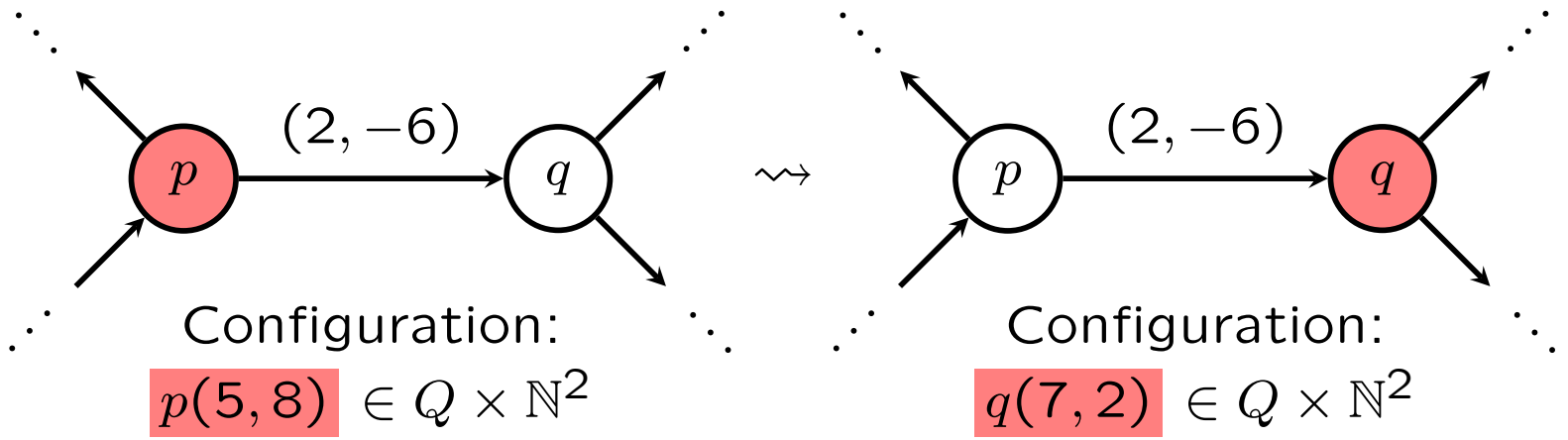


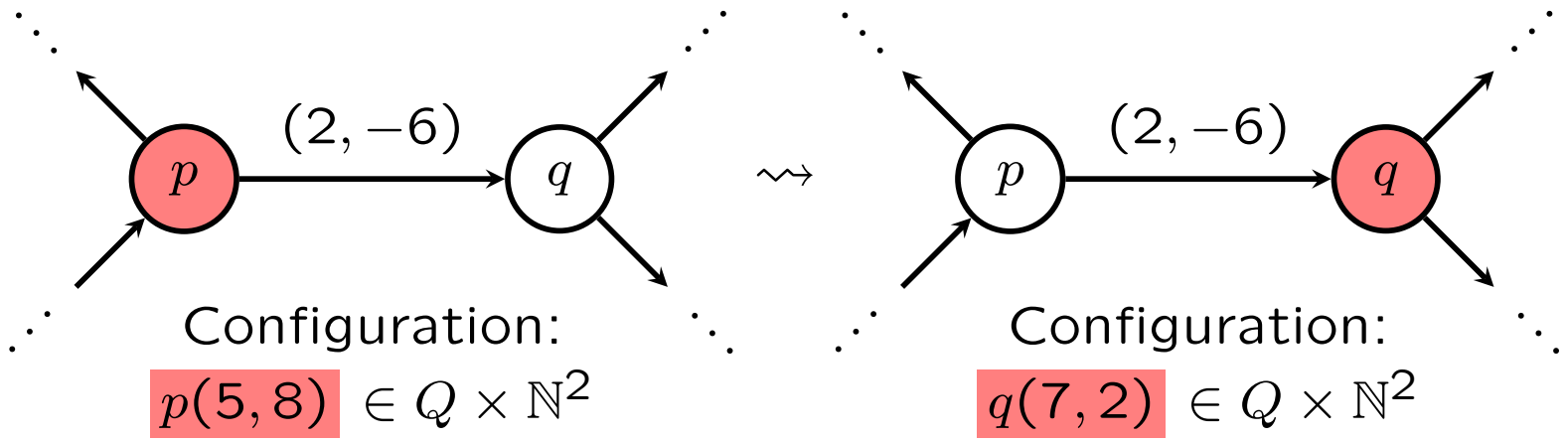
INTRODUCTION

Vector Addition Systems with States (2-VASS)



INTRODUCTION

Vector Addition Systems with States (2-VASS)



Reachability does there exist a *run* from $p(\mathbf{u})$ to $q(\mathbf{v})$?

Coverability does there exist a *run* from $p(\mathbf{u})$ to $q(\mathbf{v}')$ for some $\mathbf{v}' \geq \mathbf{v}$?

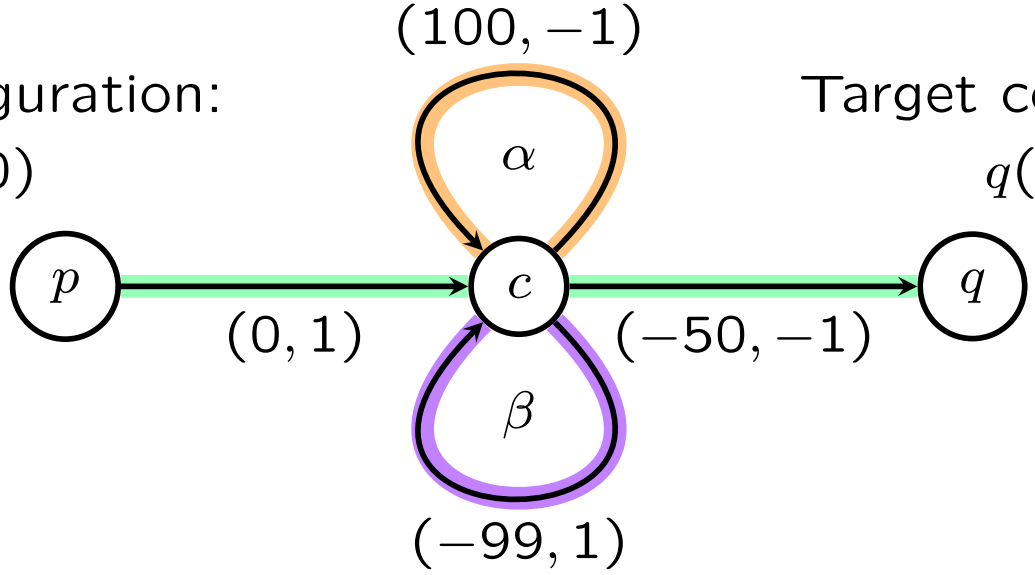
COVERABILITY EXAMPLE

Initial configuration:

$p(0,0)$

Target configuration:

$q(10,0)$



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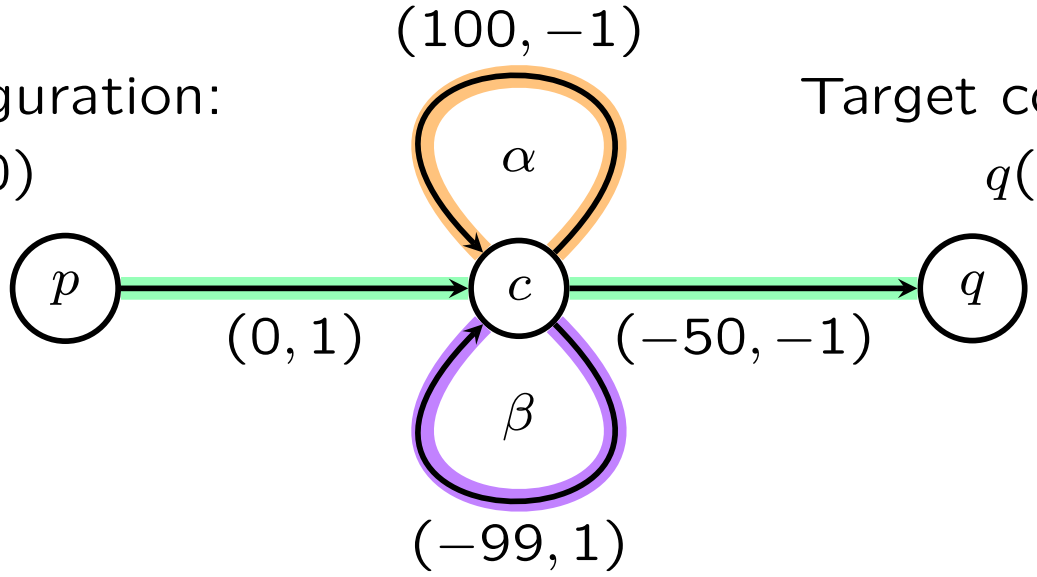
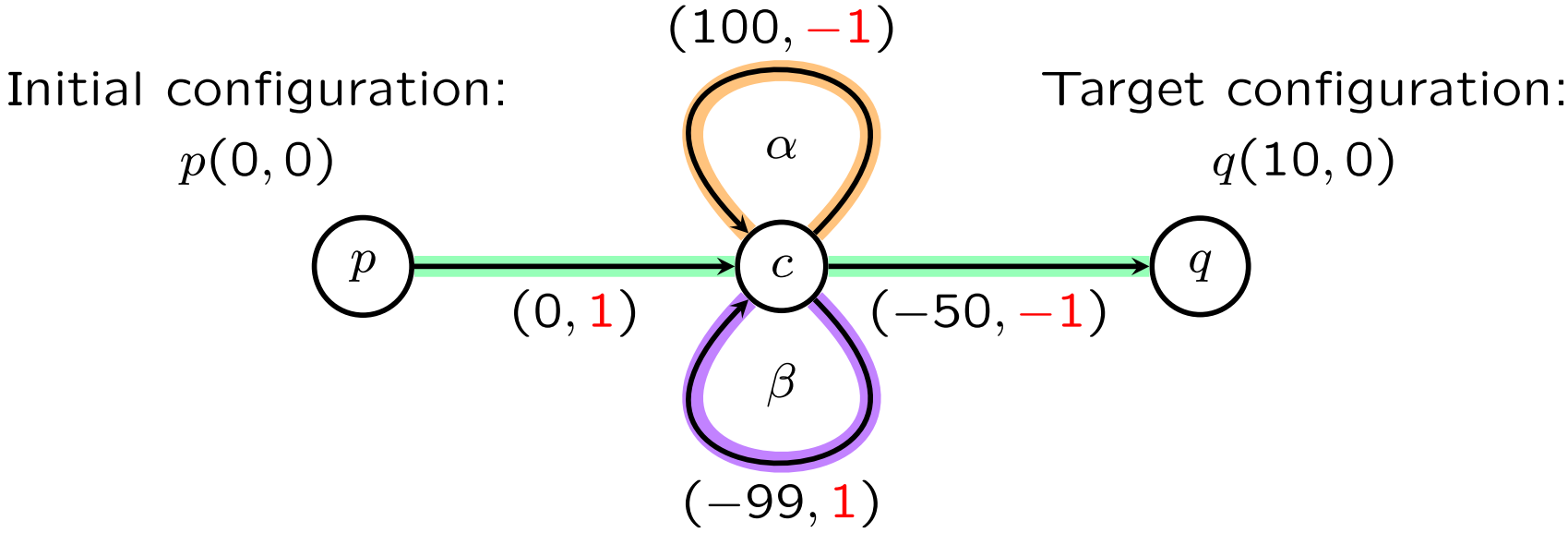


$$
\begin{array}{lllll}
p(0,0) \longrightarrow c(0,1) & \xrightarrow{\alpha} & c(100,0) & \xrightarrow{\beta} & c(1,1) \longrightarrow \cdots \\
\cdots & \xrightarrow{\alpha} & c(101,0) & \xrightarrow{\beta} & c(2,1) \longrightarrow \cdots \\
 & & \vdots & & \\
\cdots & \xrightarrow{\alpha} & c(159,0) & \xrightarrow{\beta} & c(60,1) \longrightarrow \cdots \\
\cdots & \xrightarrow{\alpha} & c(160,0) & \xrightarrow{\beta} & c(61,1) \longrightarrow \cdots \\
 & & \vdots & &
\end{array}
\qquad \nearrow q(10,0)
$$

Coverability in 2-VASS with One Unary Counter



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 & & \vdots & &
\end{array}
\qquad \longrightarrow q(10,0)
$$

CONTRIBUTION

Theorem: [Blondin, Finkel, Göller, Haase, and McKenzie '15]

Coverability in 2-VASS with binary counters is PSPACE-complete.

Theorem: [Rackoff '78]

Coverability in 2-VASS with unary counters is NL-complete.

OUR CONTRIBUTION

Theorem: [Blondin, Finkel, Göller, Haase, and McKenzie '15]

Coverability in 2-VASS with binary counters is PSPACE-complete.

Theorem:

Coverability in 2-VASS with one unary counter is in NP.

Theorem: [Rackoff '78]

Coverability in 2-VASS with unary counters is NL-complete.

RESULTS

Theorem: Suppose there exists a run from from $p(\mathbf{u})$ to $q(\mathbf{v})$ in a given **2-VASS with one unary counter**, then there exists a *compressed linear form path of polynomial size* that induces a run from $p(\mathbf{u})$ to $q(\mathbf{v}')$ for some $\mathbf{v}' \geq \mathbf{v}$.

RESULTS

Theorem: Suppose there exists a run from from $p(\mathbf{u})$ to $q(\mathbf{v})$ in a given **2-VASS with one unary counter**, then there exists a *compressed linear form path of polynomial size* that induces a run from $p(\mathbf{u})$ to $q(\mathbf{v}')$ for some $\mathbf{v}' \geq \mathbf{v}$.

$\Rightarrow$ **Coverability in 2-VASS with one unary counter is in NP.**

... just guess and check compressed linear form paths.

CONCLUSION

Coverability in 2-VASS with one unary counter is in NP.

Unfortunately, we lack a matching lower bound.

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Conjecture: coverability in P.

Future Work: is **reachability** also in NP?

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Conjecture: **coverability** in P.

Future Work: is **reachability** also in NP?

THANK YOU!

Presented by Henry Sinclair-Banks