Coverability in 2-VASS with One Unary Counter

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Aalborg, Denmark

Fun-Road-Trip Checklist

✓ at least one friend, and

✓ not have negative money!

Friends: 4

Money: 1000*kr*

Fun-Road-Trip Checklist

✓ at least one friend, and

✓ not have negative money!

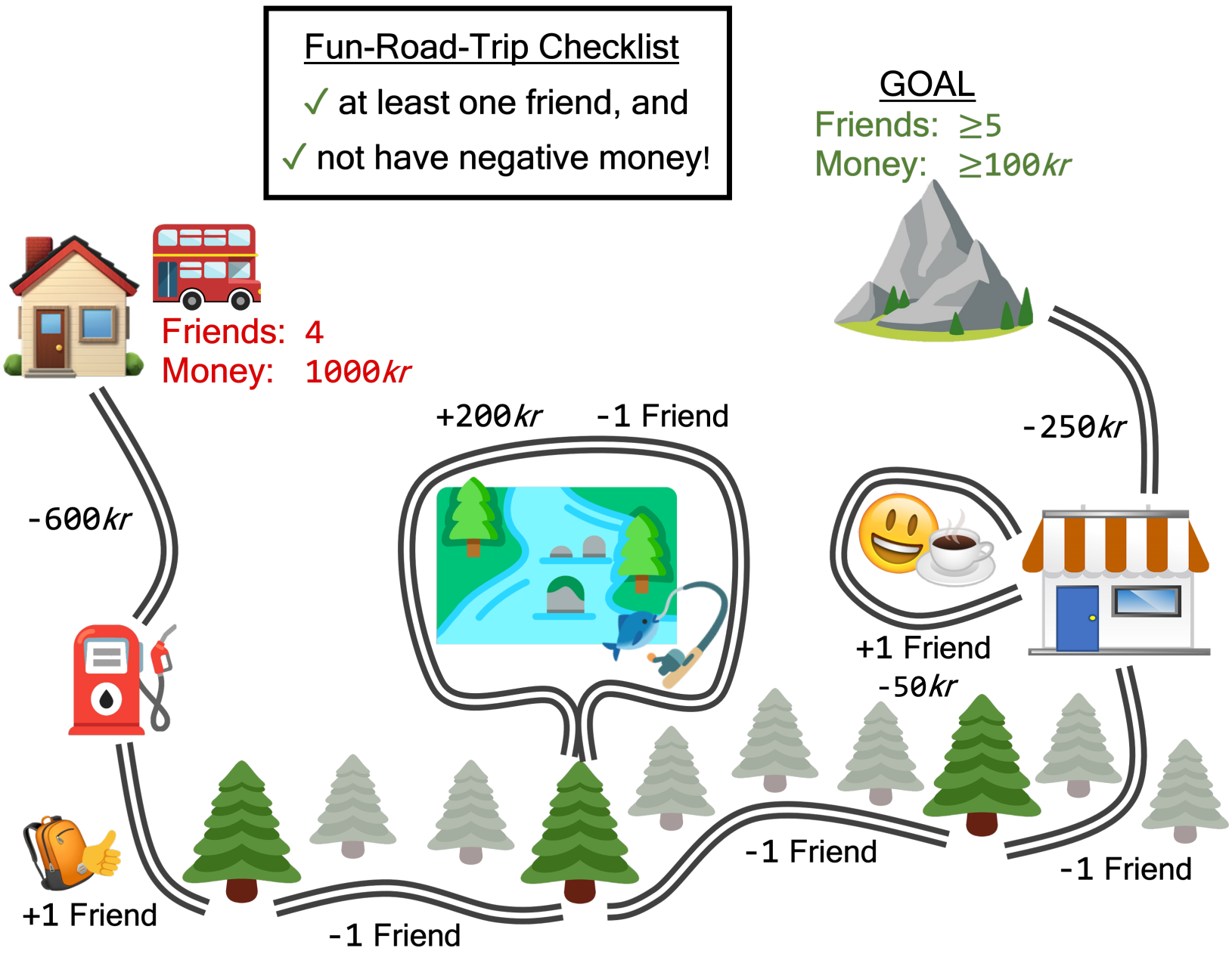
Friends: 4

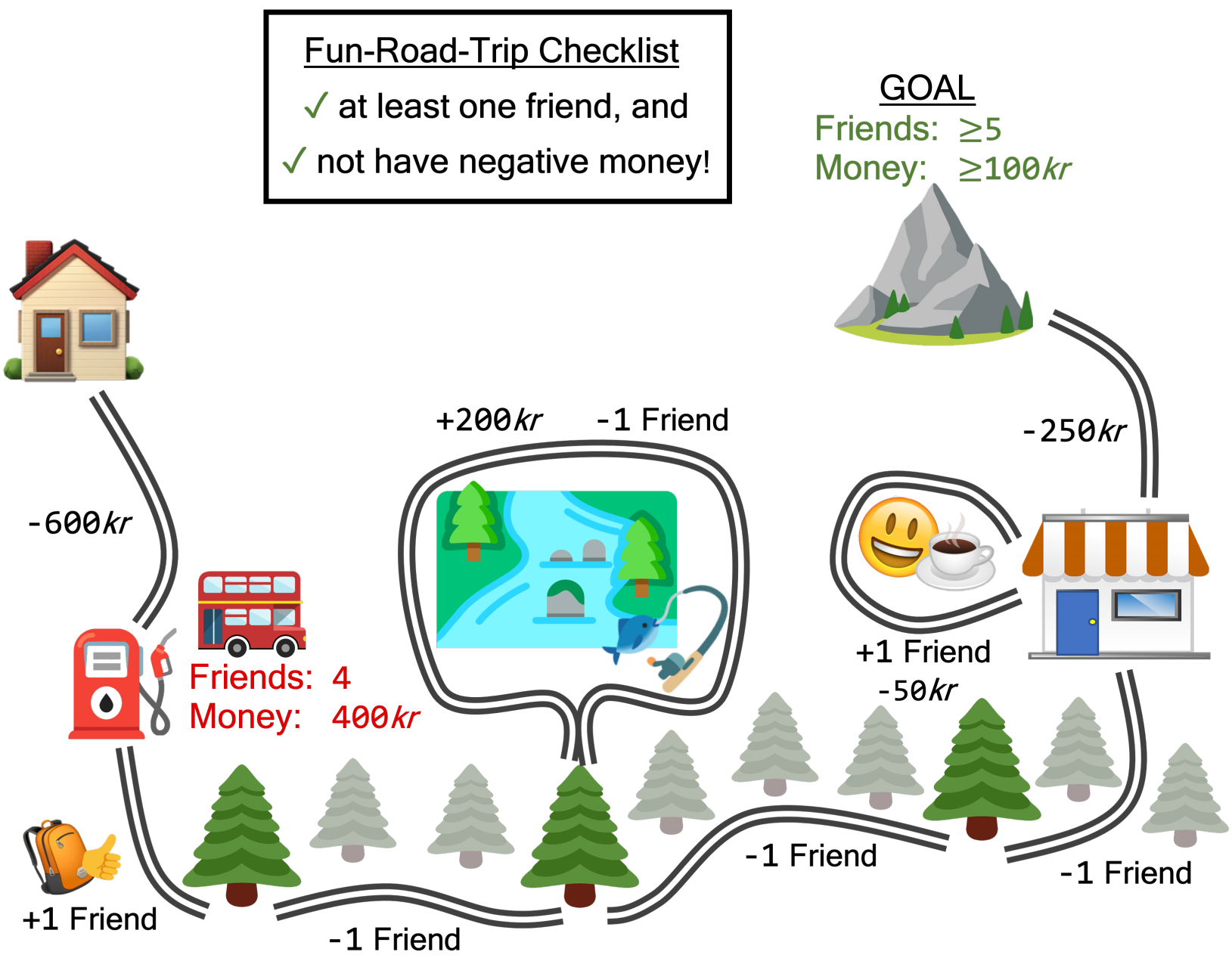
Money: 1000*kr*

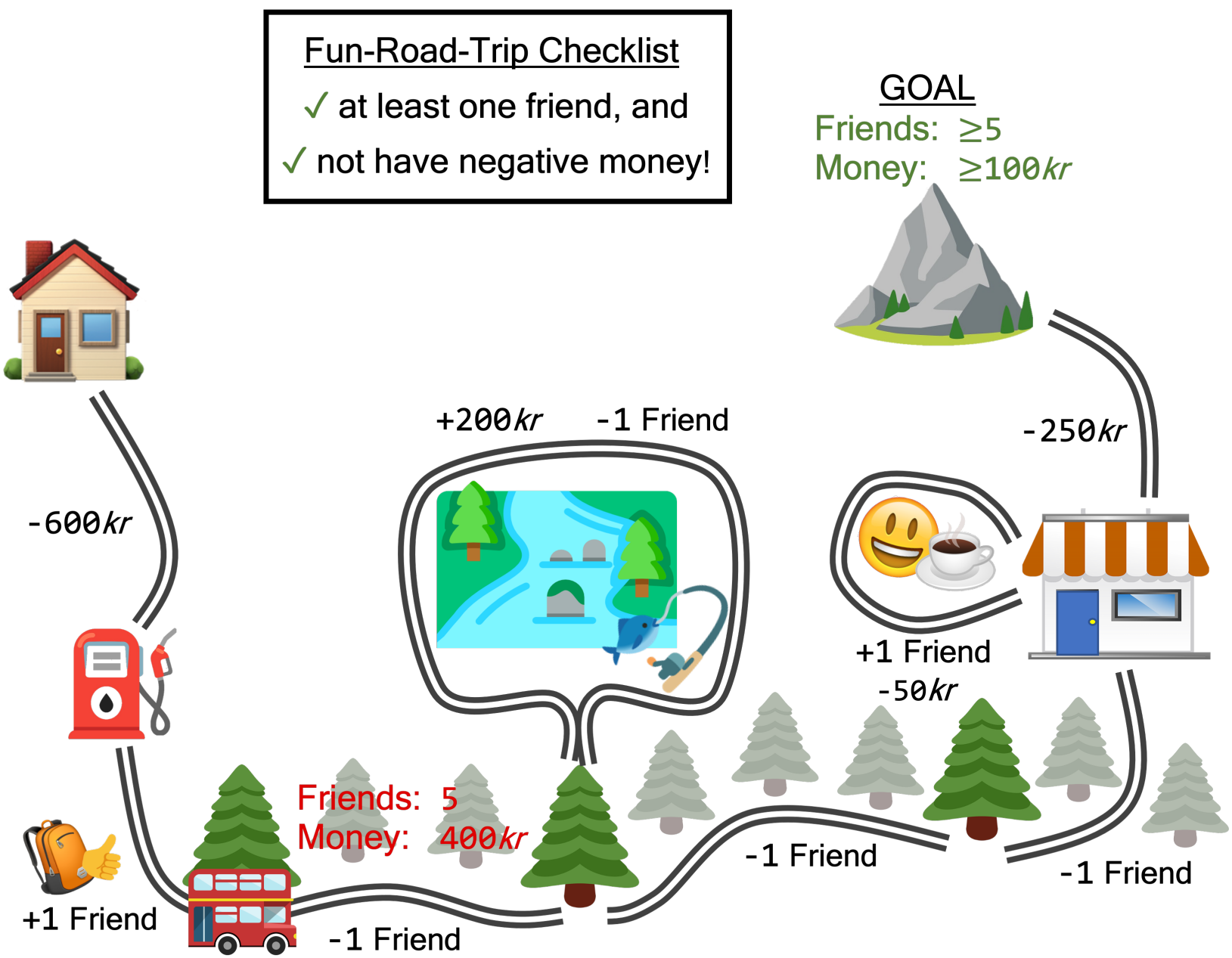
GOAL

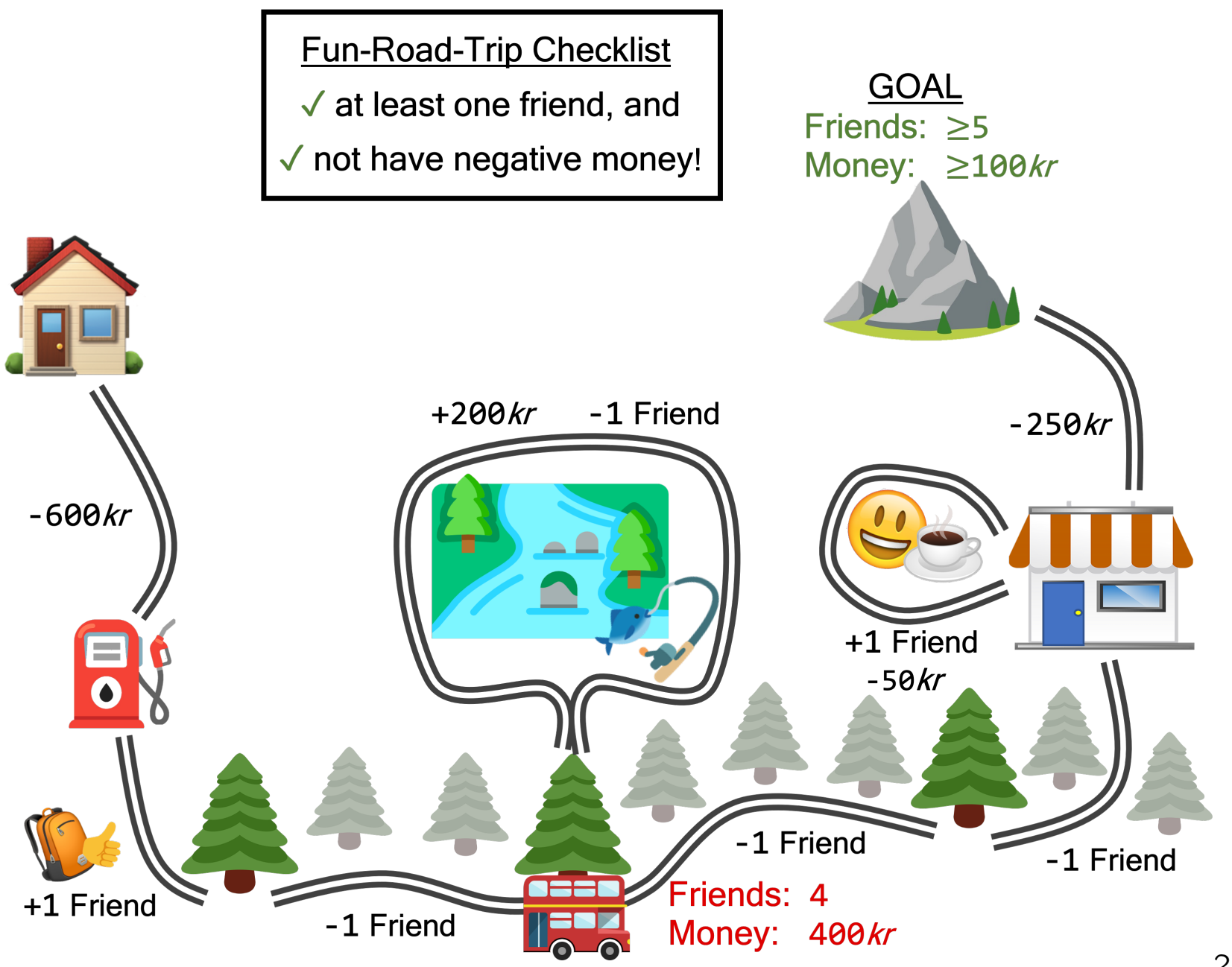
Friends: ≥5

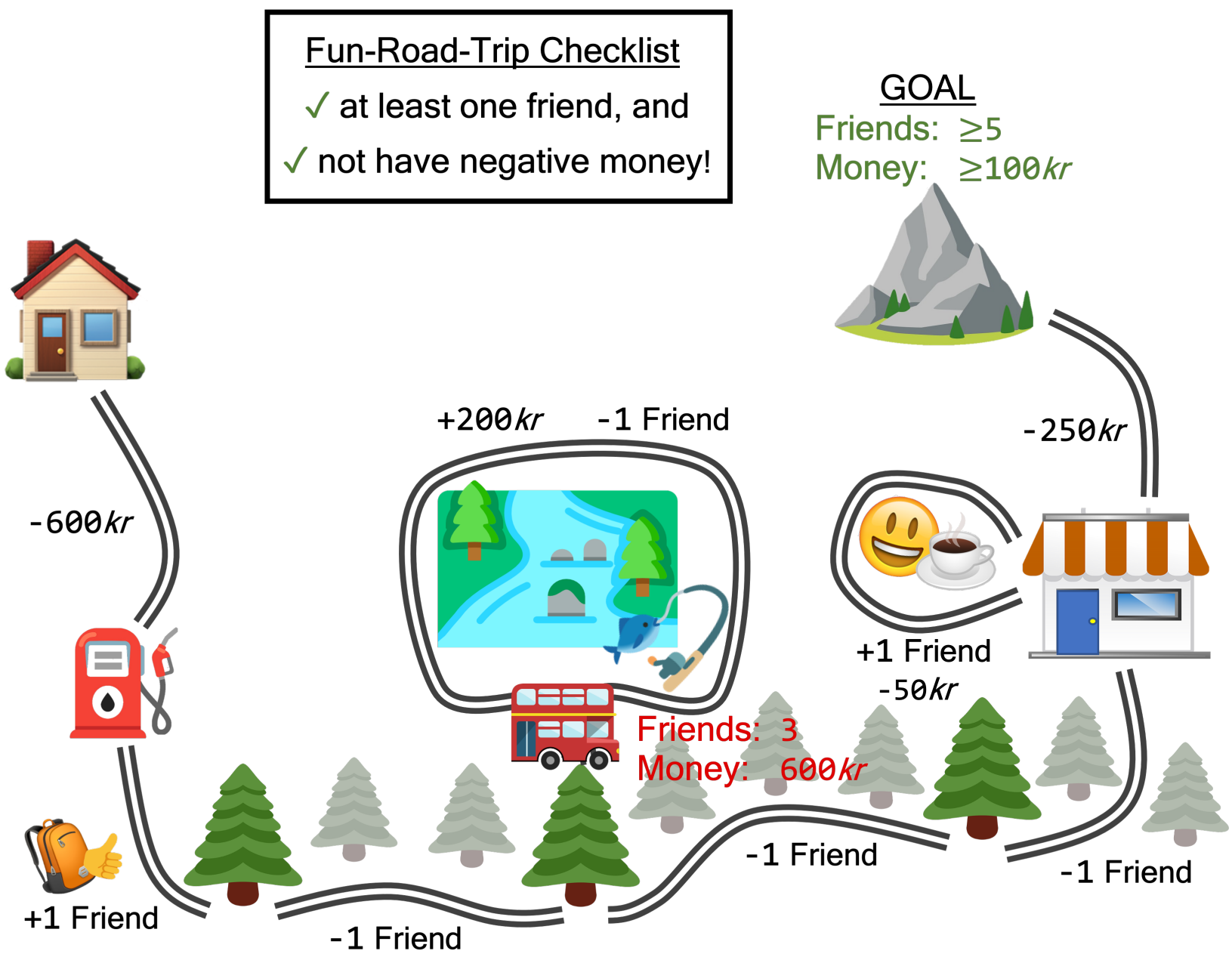
Money: ≥100*kr*

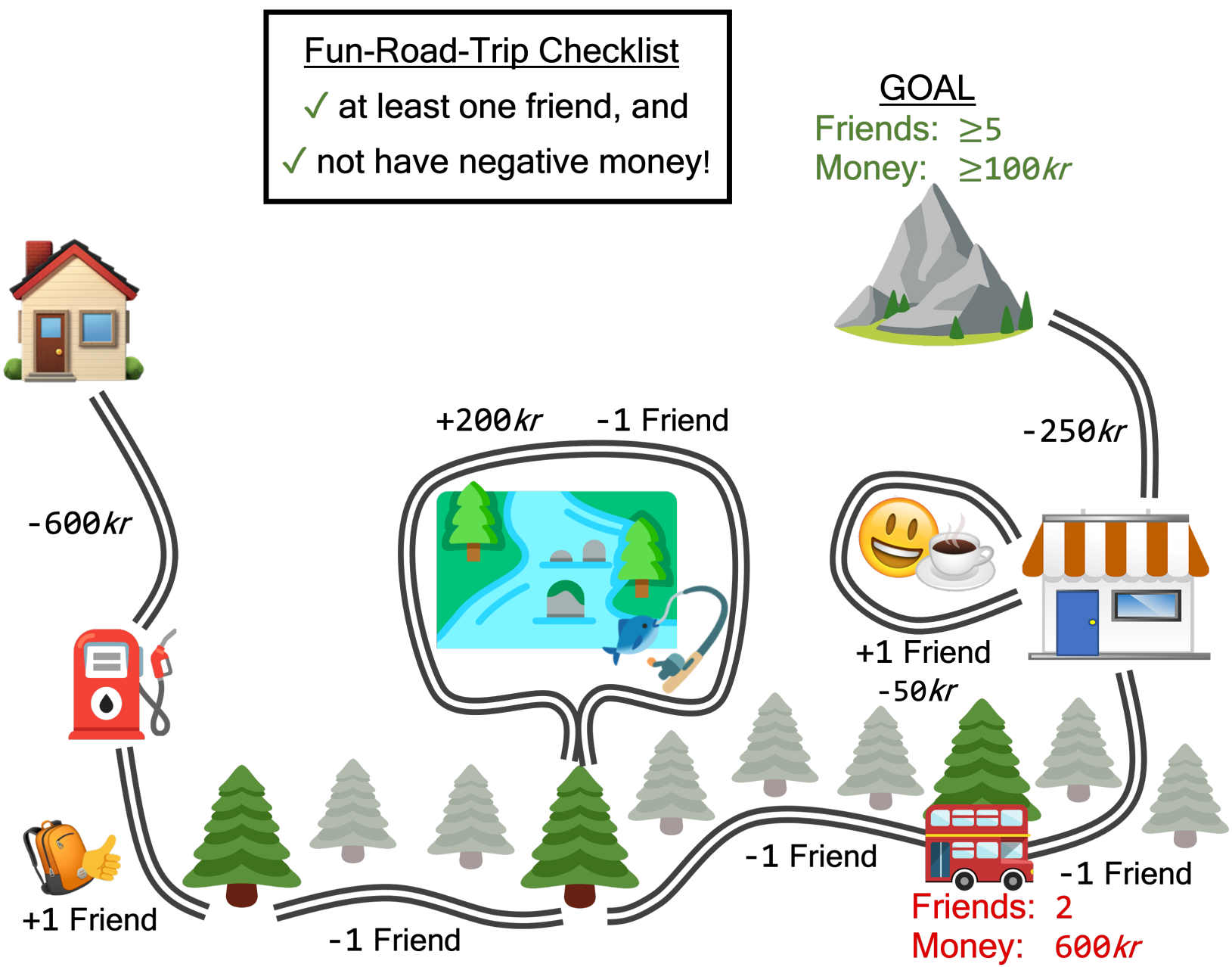

Fun-Road-Trip Checklist
✓ at least one friend, and
✓ not have negative money!
GOAL
Friends: ≥5
Money: ≥100kr
Friends: 4
Money: 1000kr
+200kr
-1 Friend
-250kr
-600kr
+1 Friend
-50kr
+1 Friend
-1 Friend
-1 Friend
-1 Friend

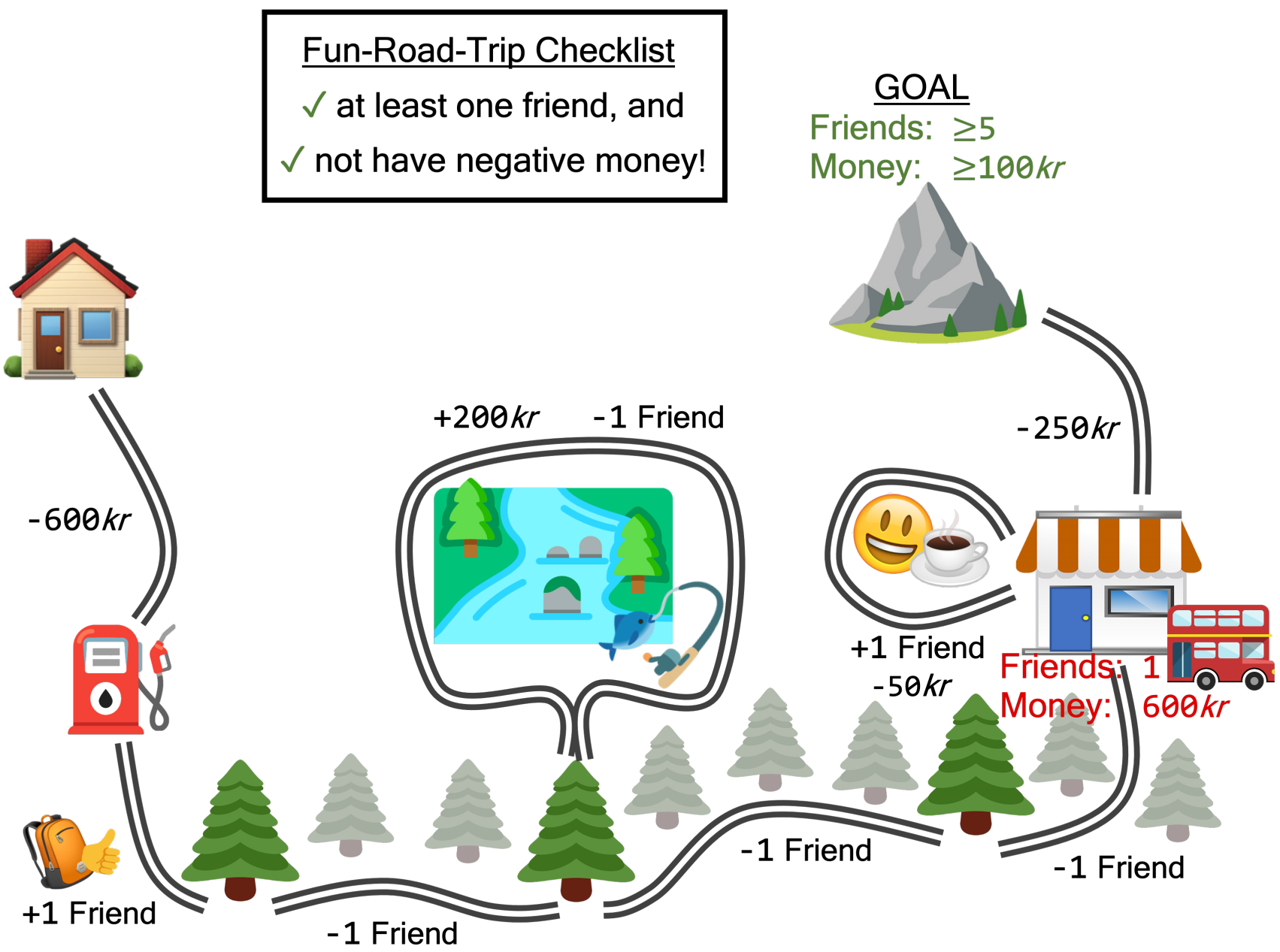

Fun-Road-Trip Checklist
✓ at least one friend, and
✓ not have negative money!
GOAL
Friends: ≥5
Money: ≥100kr
-600kr
+200kr
-1 Friend
-250kr
+1 Friend
-50kr
Friends: 4
Money: 400kr
+1 Friend
-1 Friend
-1 Friend
-1 Friend

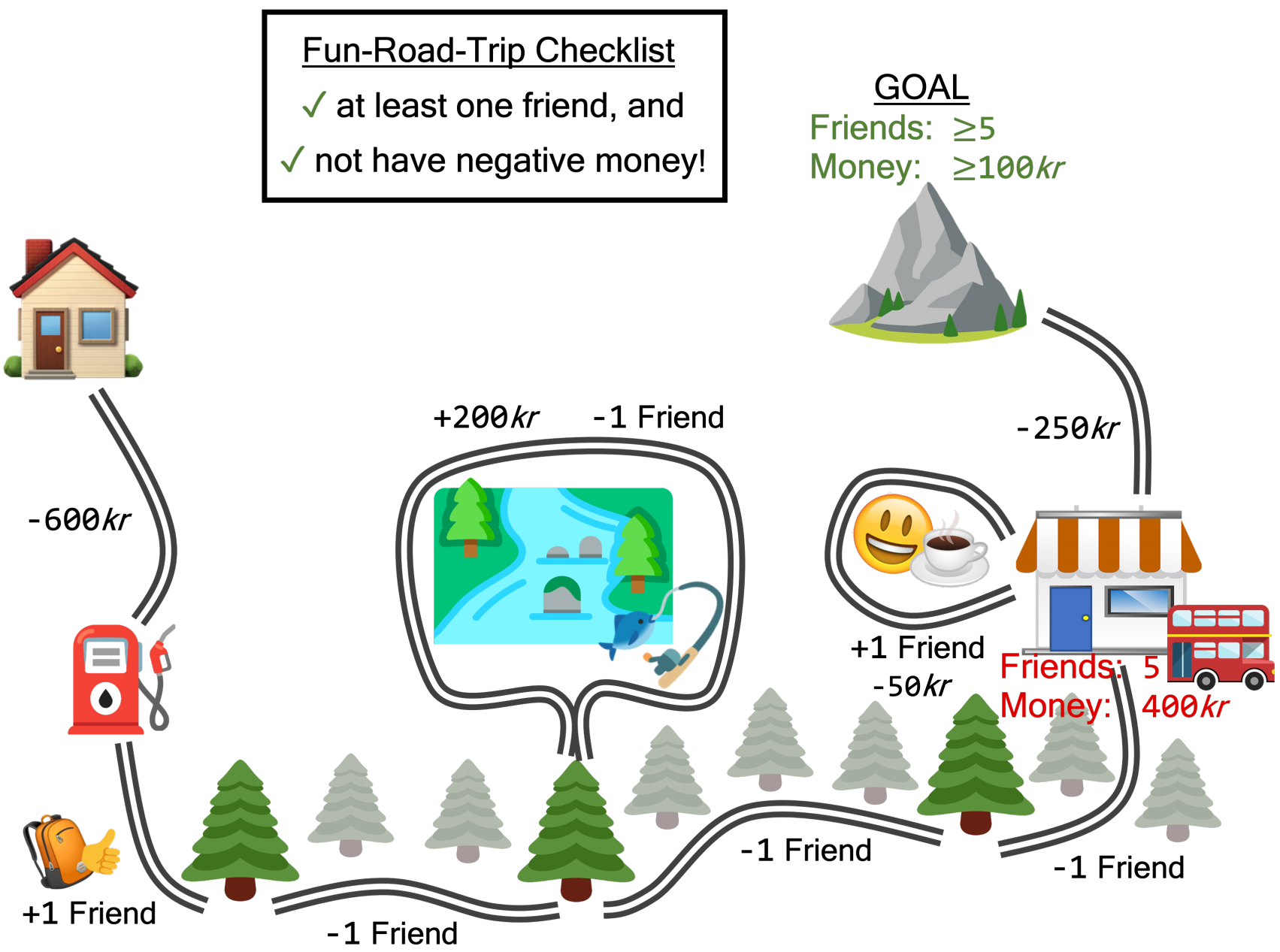

Fun-Road-Trip Checklist
✓ at least one friend, and
✓ not have negative money!
GOAL
Friends: ≥5
Money: ≥100kr
+200kr
-1 Friend
-250kr
-600kr
+1 Friend
-50kr
Friends: 5
Money: 400kr
+1 Friend
-1 Friend
-1 Friend
-1 Friend


Fun-Road-Trip Checklist
✓ at least one friend, and
✓ not have negative money!
GOAL
Friends: ≥5
Money: ≥100kr
-600kr
+200kr
-1 Friend
-250kr
+1 Friend
-50kr
+1 Friend
-1 Friend
-1 Friend
-1 Friend
Friends: 4
Money: 400kr


Fun-Road-Trip Checklist
✓ at least one friend, and
✓ not have negative money!
GOAL
Friends: ≥5
Money: ≥100kr
+200kr
-1 Friend
-250kr
-600kr
+1 Friend
-50kr
Friends: 3
Money: 600kr
+1 Friend
-1 Friend
-1 Friend
-1 Friend


Fun-Road-Trip Checklist
✓ at least one friend, and
✓ not have negative money!
GOAL
Friends: ≥5
Money: ≥100kr
-600kr
+200kr
-1 Friend
-250kr
+1 Friend
-50kr
+1 Friend
-1 Friend
-1 Friend
-1 Friend
Friends: 2
Money: 600kr


Fun-Road-Trip Checklist
✓ at least one friend, and
✓ not have negative money!
GOAL
Friends: ≥5
Money: ≥100kr
-600kr
+200kr
-1 Friend
-250kr
+1 Friend
-50kr
Friends: 1
Money: 600kr
+1 Friend
-1 Friend
-1 Friend
-1 Friend


Fun-Road-Trip Checklist
✓ at least one friend, and
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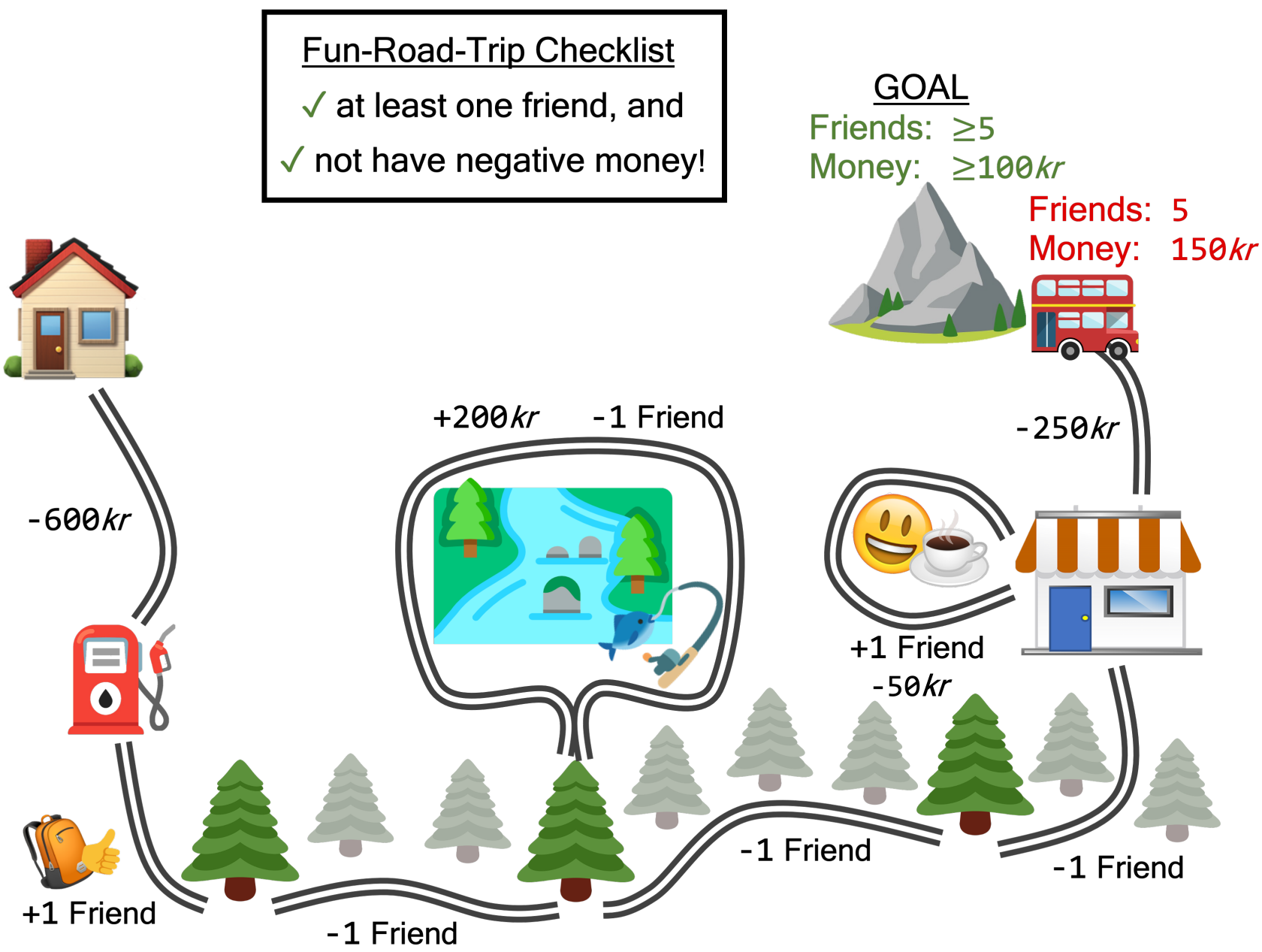
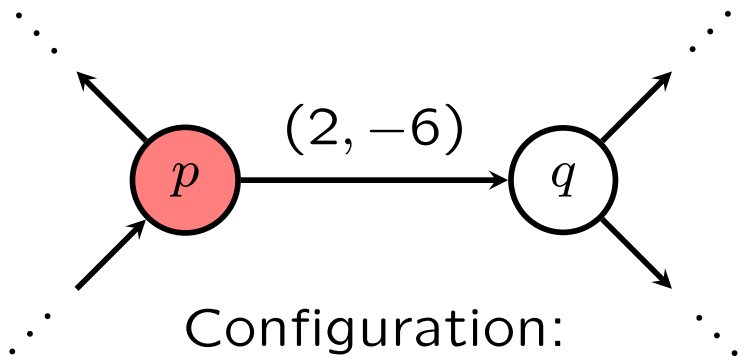
GOAL
Friends: ≥5
Money: ≥100kr
-600kr
+200kr
-1 Friend
-250kr
+1 Friend
-50kr
Friends: 5
Money: 400kr
+1 Friend
-1 Friend
-1 Friend
-1 Friend


Fun-Road-Trip Checklist
✓ at least one friend, and
✓ not have negative money!
GOAL
Friends: ≥5
Money: ≥100kr
Friends: 5
Money: 150kr
-600kr
+200kr
-1 Friend
-250kr
+1 Friend
-50kr
+1 Friend
-1 Friend
-1 Friend
-1 Friend

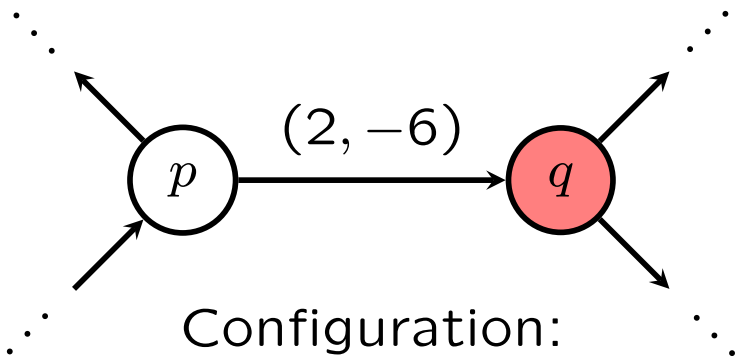
INTRODUCTION

Vector Addition Systems with States (2-VASS)



Configuration:

$p(5,8) \in Q \times \mathbb{N}^2$

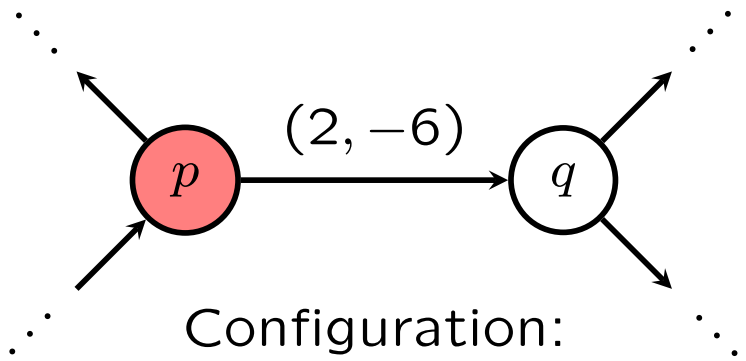


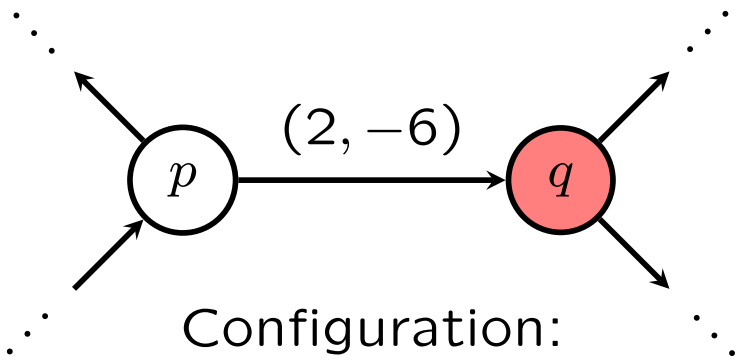
Configuration:

$q(7,2) \in Q \times \mathbb{N}^2$

INTRODUCTION

Vector Addition Systems with States (2-VASS)





Reachability does there exist a *run* in V from $p(\mathbf{u})$ to $q(\mathbf{v})$?

Coverability does there exist a *run* in V from $p(\mathbf{u})$ to $q(\mathbf{v}')$ for some $\mathbf{v}' \geq \mathbf{v}$?

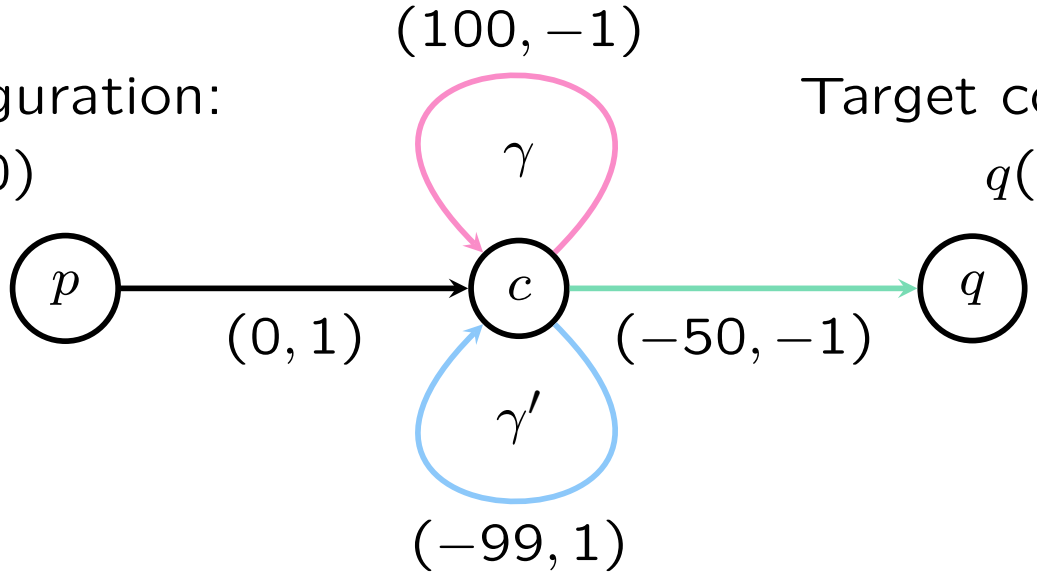
COVERABILITY EXAMPLE

Initial configuration:

$p(0,0)$

Target configuration:

$q(10,0)$



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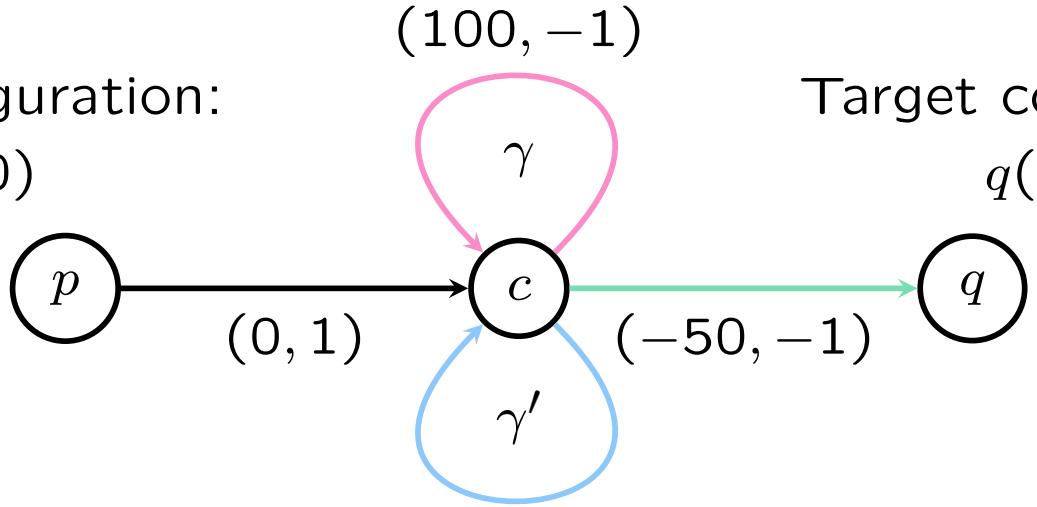
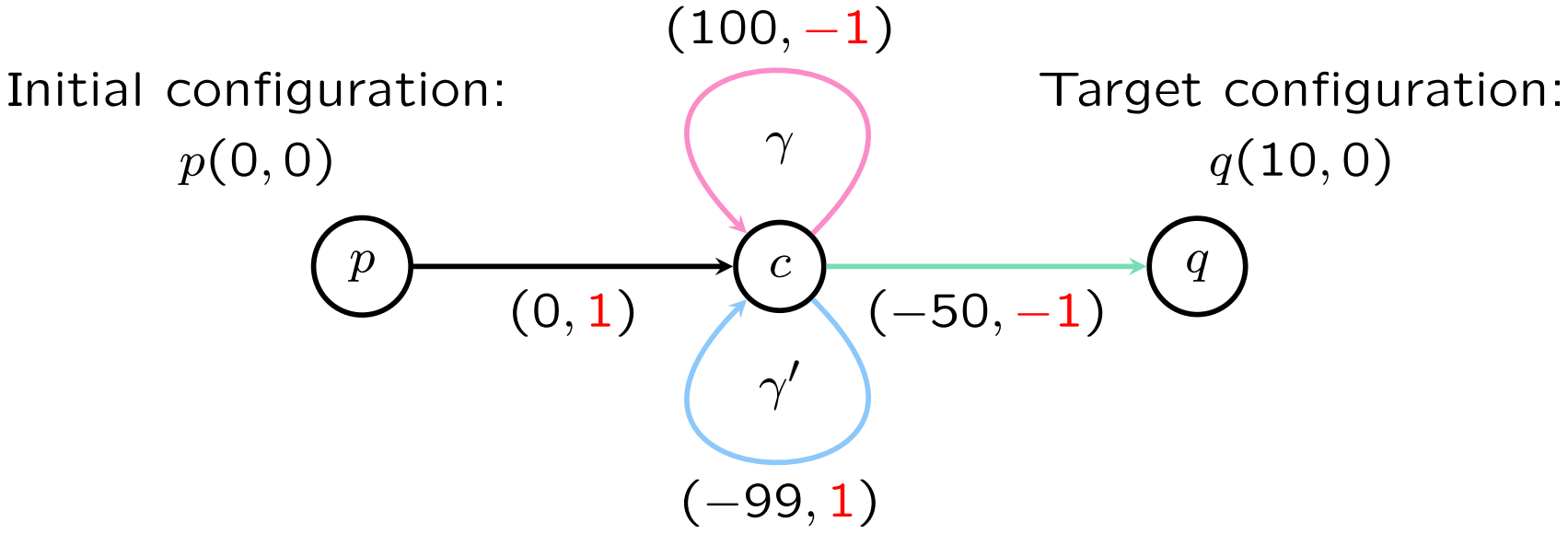
Target configuration:

$q(10,0)$



$$p(0,0) \longrightarrow c(0,1) \xrightarrow{\gamma} c(100,0) \xrightarrow{\gamma'} c(1,1) - \cdots$$
$$\cdots \xrightarrow{\gamma} c(101,0) \xrightarrow{\gamma'} c(2,1) - \cdots$$
$$\vdots$$
$$\cdots \xrightarrow{\gamma} c(159,0) \xrightarrow{\gamma'} c(60,1) \rightarrow q(10,0)$$
$$\cdots \xrightarrow{\gamma} c(160,0) \xrightarrow{\gamma'} c(61,1) - \cdots$$
$$\vdots$$

Coverability in 2-VASS with One Unary Counter



$$
\begin{array}{lllll}
p(0,0) \longrightarrow c(0,1) & \xrightarrow{\gamma} & c(100,0) & \xrightarrow{\gamma'} & c(1,1) \longrightarrow \cdots \\
\cdots & \xrightarrow{\gamma} & c(101,0) & \xrightarrow{\gamma'} & c(2,1) \longrightarrow \cdots \\
 & & \vdots & & \\
\cdots & \xrightarrow{\gamma} & c(159,0) & \xrightarrow{\gamma'} & c(60,1) \longrightarrow \cdots \\
\cdots & \xrightarrow{\gamma} & c(160,0) & \xrightarrow{\gamma'} & c(61,1) \longrightarrow \cdots \\
 & & \vdots & &
\end{array}
\qquad q(10,0)
$$

CONTRIBUTION

Binary **2-VASS coverability** is PSPACE-complete.

[Blondin, Finkel, Göller, Haase, and McKenzie '15]

Unary **2-VASS coverability** is NL-complete.

[Rackoff '78]

OUR CONTRIBUTION

Binary **2-VASS coverability** is PSPACE-complete.

[Blondin, Finkel, Göller, Haase, and McKenzie '15]

Theorem:

Coverability in 2-VASS with one unary counter is in NP.

Unary **2-VASS coverability** is NL-complete.

[Rackoff '78]

RESULTS

Theorem: Given a **2-VASS with one unary counter** V and suppose there exists a run in V from $p(\mathbf{u})$ to $q(\mathbf{v})$. Then there exists a compressed linear form path of *polynomial size* inducing a run from $p(\mathbf{u})$ to $q(\mathbf{v}')$ for some $\mathbf{v}' \geq \mathbf{v}$.

RESULTS

Theorem: Given a **2-VASS with one unary counter** V and suppose there exists a run in V from $p(\mathbf{u})$ to $q(\mathbf{v})$. Then there exists a compressed linear form path of *polynomial size* inducing a run from $p(\mathbf{u})$ to $q(\mathbf{v}')$ for some $\mathbf{v}' \geq \mathbf{v}$.

$\Rightarrow$ **Coverability in 2-VASS with one unary counter is in NP.**

... just guess and check compressed linear form paths.

CONCLUSION

Coverability in 2-VASS with one unary counter is in NP.

Unfortunately, we lack a matching NP-hard lower bound.

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Conjecture: coverability in P.

Future Work: is **reachability** also in NP?

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THANK YOU!

Presented by Henry Sinclair-Banks