#1

Henry's Early PhD Problems Presentation

Definitions:
- One Counter Nets (OCN)
- Vector Addition Systems with States (VASS)
- Semi-linear Sets.
- (Level 1/2) Polynomial Hierarchy (& NL)
- Decision Problems:
  - (compressed) Membership,
  - universality
  - Reachability / Coverability

Problems of interest:
- Open Complexity Gaps!
- 'New' variations of automata

#2

Semi-Linear Sets

$S = \bigcup_{i=1} L(b_i, P_i)$

> $b_i \in \mathbb{N}$ "base"

> $P_i = \{p_{i_1}, \dots, p_{i_{k_i}}\}$

(each $p_{ij} \in \mathbb{N}$) "periods"

Example: (linear set)

$b = 0$, $P = \{4, 6, 9, 20\}$

$L(b, P) = \{4, 6, 8, 9, 10, 12, \dots\}$

"chicken nuggets!"

One Counter Nets (OCN)

$A = (\Sigma, Q, S_0, \delta, F)$

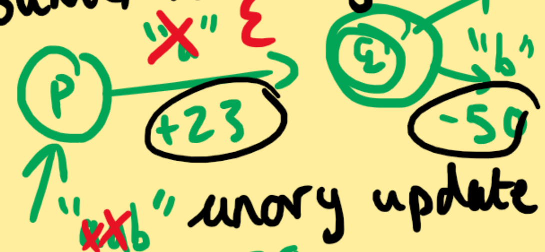
- Σ: input alphabet
- Q: States
- S_0: Starting State
- δ: transition function
- F: Final States.

$\delta \subseteq Q \times \Sigma \times \mathbb{N} \times Q$

Ex: $(p, \text{"a"}, 23, q)$

Example:

Counter non-neg.



binary. $\{\pm n \in \mathbb{N}\}$.

unary update: $\{-1, 0, +1\}$

Encodings: $\boxed{\text{unary}}$ in alph: $\Sigma = \{a\}$

binary in alph: $\Sigma = \{a, b\}$

Vector Addition Systems with States (VASS)

$V = (Q, T)$

- Q: States
- T: transitions.

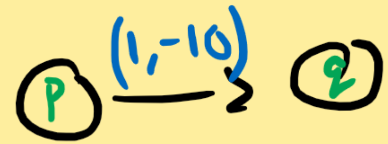
*k-dimensional (k-VASS)

$T \subseteq Q \times \mathbb{N}^k \times Q$

Ex: $(p, [x, y, \dots, z], q)$

Encodings: unary / binary

Example: $k = 2$



non-neg. counters.

#3

Polynomial Hierarchy (Lvl 1&2) ... by example

SAT

INPUT> C.N.F. Boolean Formula $\emptyset(x_1,\dots,x_n)$

QUESTION> $\exists \alpha_1,\dots,\alpha_n: \emptyset(\alpha_1,\dots,\alpha_n) = \text{TRUE}$?

Known to be "Level 1 complete" NP, coNP

SAT^2

INPUT: $\emptyset(x_1,\dots,x_n,y)$

QUESTION: $\forall\beta, \exists\alpha_1,\dots,\alpha_n: \emptyset(\alpha_1,\dots,\alpha_n,\beta)$ "TRUE"?

Known to be "Level 2 complete" NP^{coNP}, $coNP^{NP}$

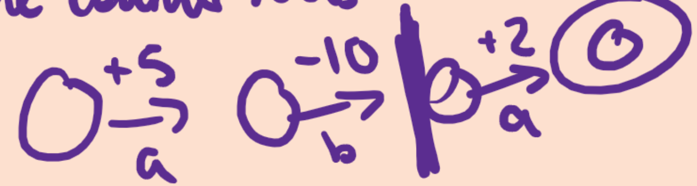
Decision Problem (Emptiness)

> INPUT: Language/Set/Automata ← X

> QUESTION: is $X = \emptyset$? (or $L(X) = \emptyset$...)

Examples: Semi-linear Sets – easy :)

One Counter Nets –



Nondeterministic Log Space (NL)

> • $O(\log(n))$ mem.

> • Bonus read-only for input

> non-det :)

#4

VASS Decision Problems

Configurations:

$q(\vec{v})$

$\vec{v} \in \mathbb{N}^k$

Coverability

INPUT > k-VASS V, States: p, q; Vectors: $\vec{v_0}$, $\vec{v}$

QUESTION >

"Does there exist some run r in V:

$$p(\vec{v_0}) \overset{r}{\rightsquigarrow} q(\vec{v}\,') \; ?$$

where $\vec{v}\,' \geq \vec{v}$ (componentwise)"

Complexity:

dimension (k) \ encoding	unary	binary
1	NL-C	NC^2 ($\subseteq P$)
2	↓	PSPACE-c
3	↓	↓
4	↓	↓
⋮		

Reachability

INPUT > k-VASS V, States: p, q; vectors: $\vec{v_0}$, $\vec{v}$

QUESTION >

"Does there exist some run r in V:

$$p(\vec{v_0}) \overset{r}{\rightsquigarrow} q(\vec{v}) \; ?"$$

Complexity

dimension (k) \ encoding	unary	binary
1	NL-C	NP-C
2	NL-C	PSPACE-c
3	NL-hard	PSPACE hard
4	↓	↓
⋮		
7	NP-hard ↓	

#5

OCN universality:

(unary input alphabet, binary counter updates)

INPUT> OCN $\mathcal{A}$

QUESTION> For all $\underline{w \in \Sigma^*}$, does $w \in L(\mathcal{A})$?

$w = a^n$

Lower Bound: coNP-hard

(inherited from NFA universality)

Nearly a new lower bound: $coNP^{NP}$:(

via Semi-linear Set (non-) universality is $coNP^{NP}$ complete.

Upper Bound: $coNP^{NP}$

> Non-universality needs:

- $w = a^n$: $\underline{a^n \notin L(\mathcal{A})}$

"non-membership" *

* This is actually "compressed (non-) membership"

Because n may be exponentially large, so need to write it in binary (a^n), not unary ($\underbrace{aa \dots aa}_{n}$).

#6

OCN Compressed Membership:

unary input alphabet, binary counter updates.

INPUT > OCN $\mathcal{A}$, integer n

QUESTION > Does $a^n \in L(\mathcal{A})$?

lower bound : NL-hard
(maybe P-hard?)

P?

upper bound : NP.

(from universality $\in coNP^{NP}$)

Idea: show non-universality using compressed membership.

Why?

If $a^n \in L(\mathcal{A}) \Rightarrow \exists$ run r : $s_0(0) \xrightarrow{r} t(x) : (x \geq 0)$

"linear form" : $s \to J_0\, \gamma_1^{e_1} J_1\, \gamma_2^{e_2} J_2 \dots J_{k-1}\, \gamma_k^{e_k} J_k \to t$

$|J_i| \leq |Q|$

$e_i \leq$ linear in $|n|$

$\underbrace{J_0 \dots J_k}_{k \text{ small } (|Q|^2)}$

Polysize!

NP-algo: - Guess L.F (run equiv. of r)

- Check it (*)

NP done