Invariants for One-Counter Automata with Disequality Tests

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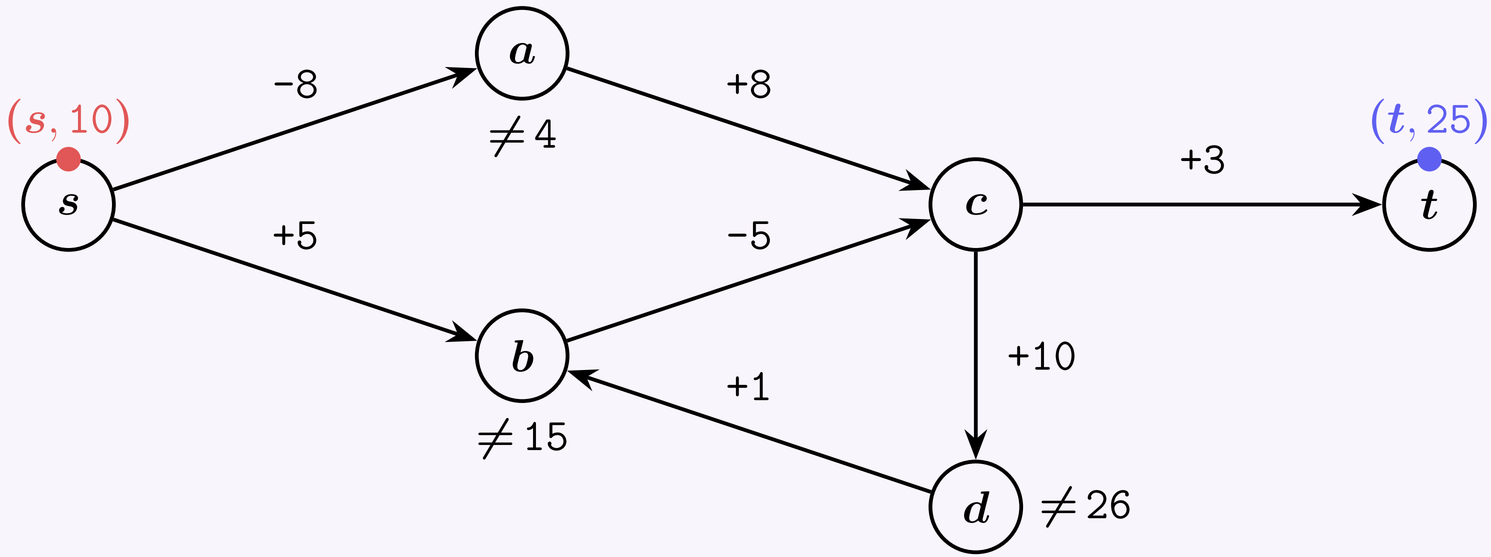
One-Counter Automata with Disequality Tests



A **configuration** consists of current state and counter value that respects the disequality tests.

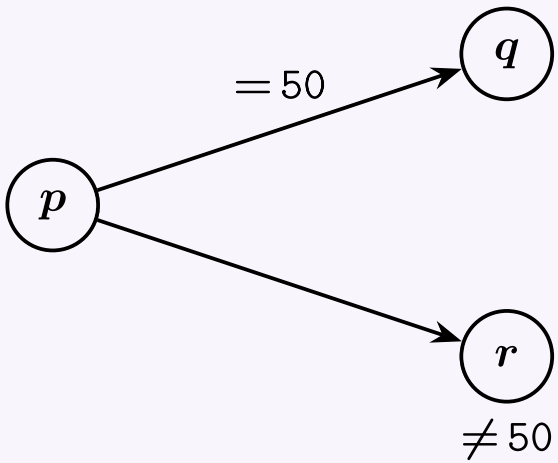
A **run** is a sequence of configurations.

Reachability: is there a run from **the initial configuration** to **the target configuration** ?

Motivation

Original motivation. Reachability in one-counter automata with disequality tests can be used for model checking in LTL extended with *flat freeze operators*. [Demri and Sangnier '10]
[Lechner, Mayr, Ouaknine, Pouly, and Worrell '18]

Standalone motivation. Equality and disequality tests allow for *if–then–else* conditionals.



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Standalone motivation. Equality and disequality tests allow for *if − then − else* conditionals.

Theorem. Reachability in one-counter automata (with equality tests) is NP-complete.

[Haase, Kreutzer, Ouaknine, and Worrell '09]

Simulating disequality tests with equality tests is inefficient.

Theorem. Reachability in one-counter automata with $\leq$ tests is PSPACE-complete.

[Fearnley and Jurdziński '13]

Our Main Contribution and Prior Work

Theorem. Reachability in one-counter automata (with equality tests) is NP-complete

[Haase, Kreutzer, Ouaknine, and Worrell '09]

Theorem. Coverability (and boundedness) in one-counter automata with disequality tests is in P.

[Almagor, Cohen, Pérez, Shirmohammadi, and Worrell '20]

"*The complexity of reaching a given configuration in this model is open, lying between* NP *and* PSPACE"

Theorem. Reachability in one-counter automata with disequality tests is in $\mathsf{coNP}^{\mathsf{NP}}$. [This paper]

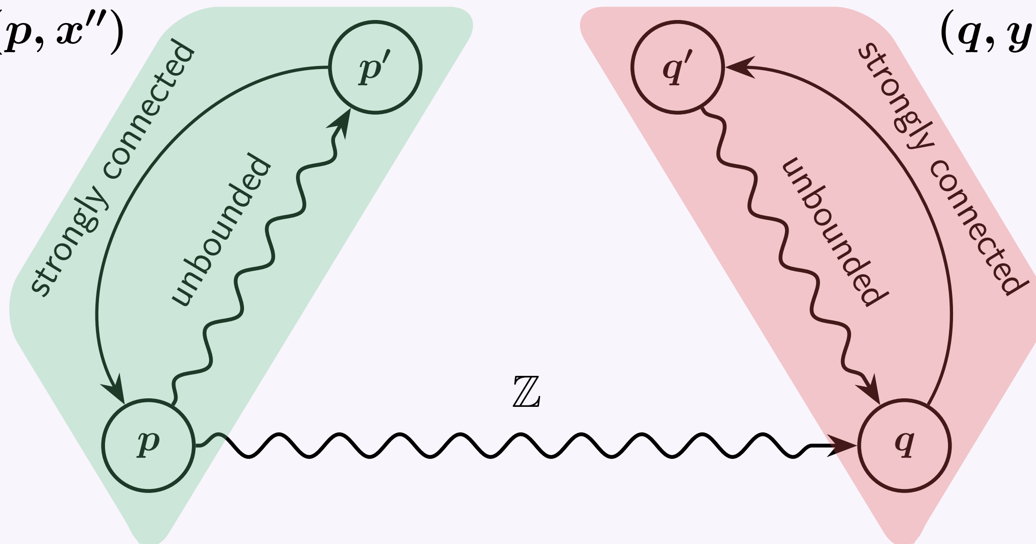
Reachability in Strongly Connected OCA with Disequality Tests

Case 1: (p, x) is unbounded and (q, y) is "reverse unbounded".

Claim: There is a run from (p, x) to (q, y) if and only if there is a $\mathbb{Z}$-run from (p, x) to (q, y).

$(p, x) \xrightarrow{*} (p', x') \xrightarrow{*} (p, x'')$

Positive cycle α.



$(q, y) \xleftarrow{*} (q', y') \xleftarrow{*} (q, y'')$

Negative cycle β.

$$(p, x) \xrightarrow{\alpha\,\alpha\,\cdots\,\alpha} (p, x + L) \xrightarrow[\mathbb{Z}]{*} (q, y + L) \xrightarrow{\beta\,\beta\,\cdots\,\beta} (q, y)$$

Reachability in Strongly Connected OCA with Disequality Tests

Case 1: (p, x) is unbounded and (q, y) is "reverse unbounded".

Claim: There is a run from (p, x) to (q, y) if and only if there is a $\mathbb{Z}$-run from (p, x) to (q, y).

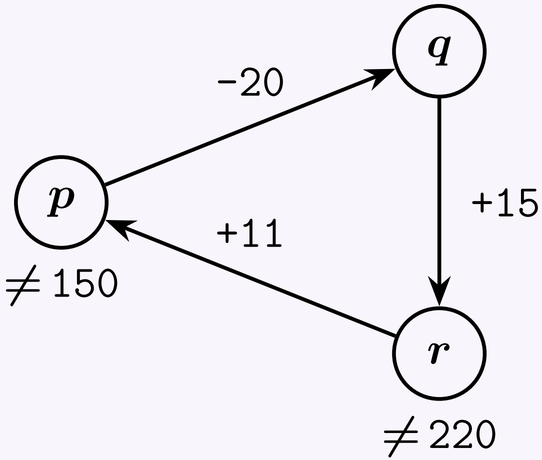
$\implies$ Case 1 is in NP.

Case 2: (p, x) is bounded (symmetric to (q, y) is "reverse bounded").

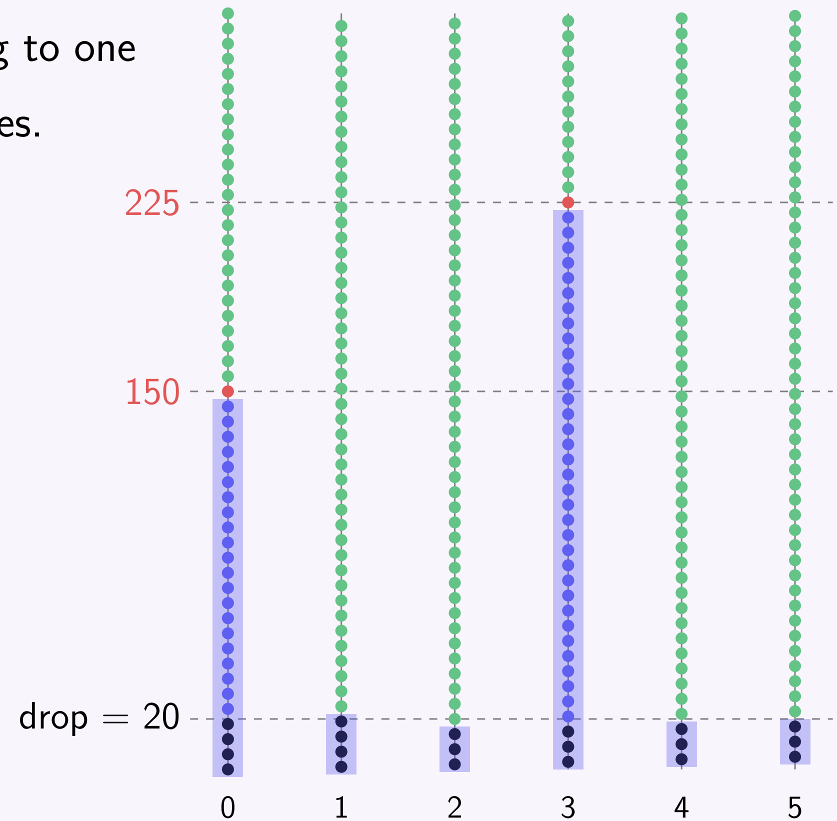
Mission for the remainder of this presentation: Case 2 is in $\mathsf{coNP}^{\mathsf{NP}}$.

Setting up an Invariant for Reachability

For each state, pick a cycle* with effect $e > 0$. In the example, for state p, $e = 6$.



Configurations belong to one of $e = 6$ many classes.



Key: Black = cannot afford cycle.

Red = violate disequality test.

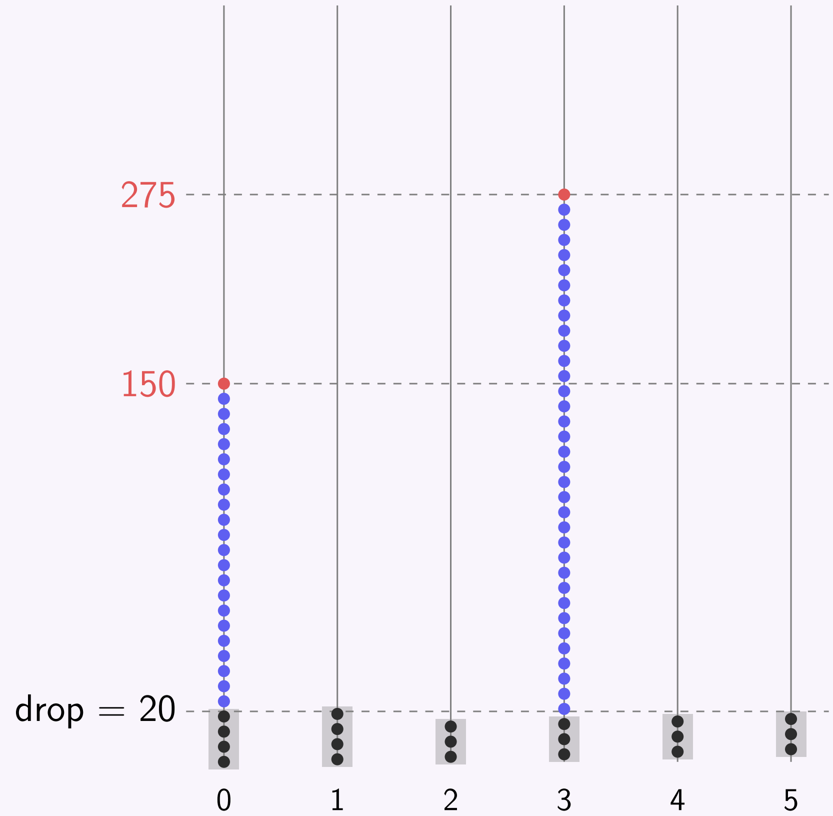
Green = unbounded.

Blue = bounded (by a disequality test).

Reachability Properties Below and in Chains

Reachability under the drop of all cycles can be checked in NP.

"Pessimistic Reachability"



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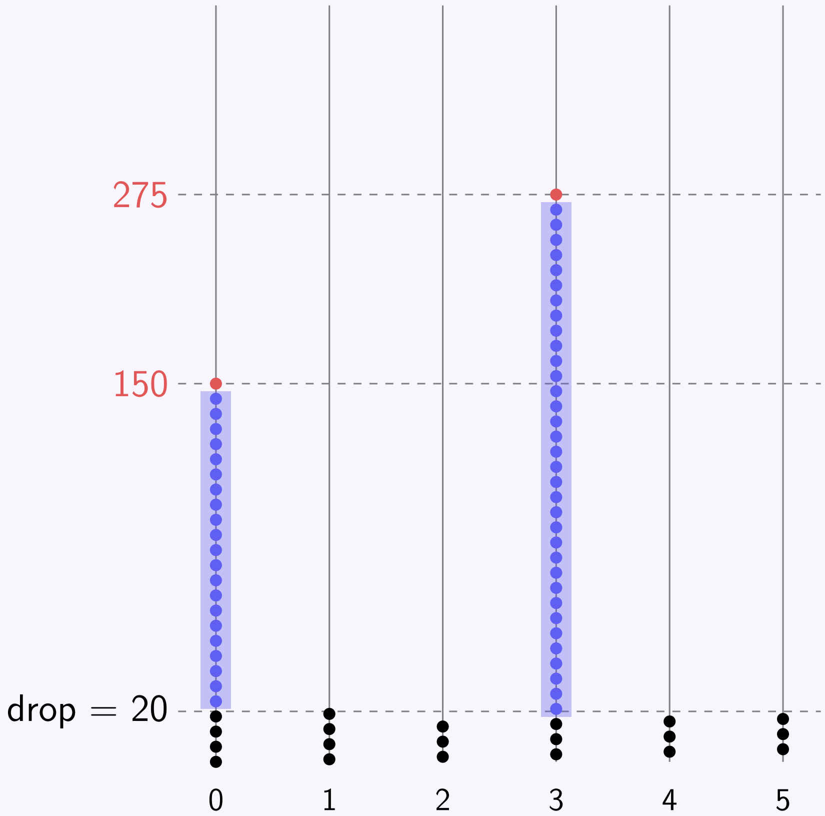
Bounded chains are upwards closed.

Let $\boldsymbol{R} = \{$ reachable points in bounded chains $\}$.



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Let $\boldsymbol{R} = \{$ reachable points in bounded chains $\}$.

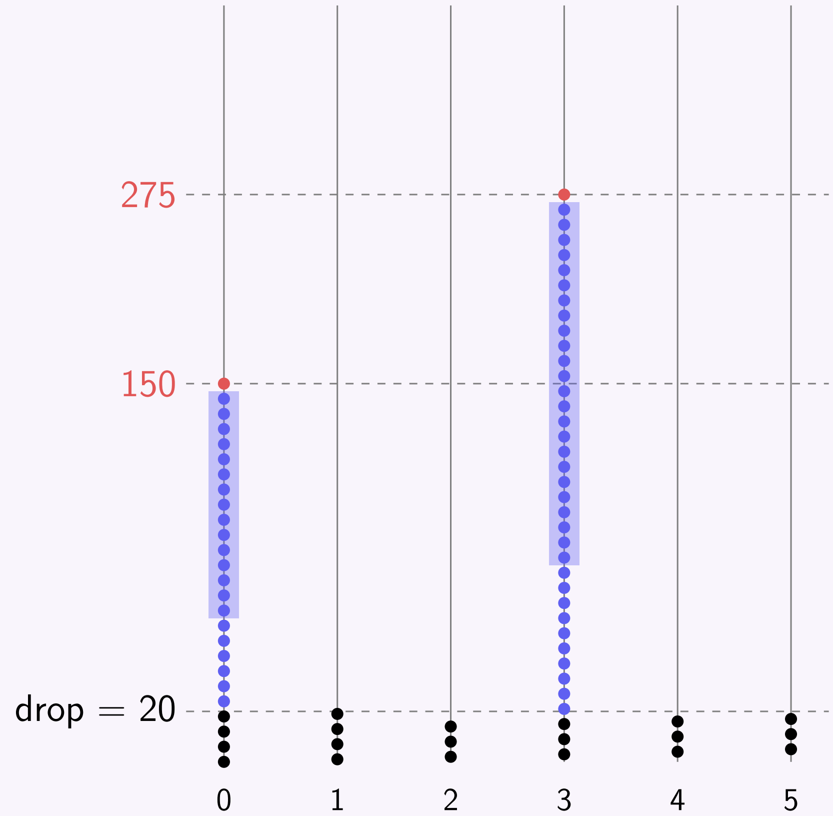
Suppose $\boldsymbol{R} = \{(\boldsymbol{p}, 60), (\boldsymbol{p}, 66), \ldots, (\boldsymbol{p}, 144)\} \cup \{(\boldsymbol{p}, 81), (\boldsymbol{p}, 89), \ldots, (\boldsymbol{p}, 219)\}$.

$\boldsymbol{R}$ has a concise description:

lowest configuration maximum cycle iteration count

$\boldsymbol{R} = \mathsf{AP}((\boldsymbol{p}, 60), \underline{+6}, 14) \cup \mathsf{AP}((\boldsymbol{p}, 81), +6, 23)$.

cycle effect



Invariants that Witness Non-reachability

Theorem. Let $\mathcal{A}$ is a strongly connected one-counter automata with disequality tests.

Let (p, x), (q, y) be two configurations and suppose (p, x) is bounded in $\mathcal{A}$.

Then, there *does not exist* a run from (p, x) to (q, y) in $\mathcal{A}$ if and only if

(Cond1) $(p, x) \in R$,

(Cond2) $(q, y) \notin \text{Post}^*_-(R)$, and

reachable via configurations under the drop of all cycles (pessimistic)

(Cond3) $\text{Post}(\text{Post}^*_-(R)) \cap \{\text{ bounded chains }\} \subseteq R$.

Theorem. Non-reachability in strongly connected one-counter automata with disequality tests is in NP^{NP}.

Proof idea. Guess R concisely. Check violation of (Cond1), or (Cond2), or (Cond3) in coNP.

Remember that $\text{NP}^{\text{coNP}} = \text{NP}^{\text{NP}}$.

Corollary. Reachability in strongly connected one-counter automata with disequality tests is in coNP^{NP}.

Invariants for One-Counter Automata with Disequality Tests

Corollary. Reachability in strongly connected one-counter automata with disequality tests is in $\mathsf{coNP}^{\mathsf{NP}}$.

[This presentation]

Theorem. Reachability in one-counter automata with disequality tests is in $\mathsf{coNP}^{\mathsf{NP}}$. [In the paper]

Theorem. Reachability in one-counter automata with *equality and disequality tests* is in $\mathsf{P}^{\mathsf{NP}^{\mathsf{NP}}}$.

[In the paper]

Thank You!

Presented by Henry Sinclair-Banks, University of Warwick, UK

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