The Complexity of Coverability in Fixed Dimension VASS with Various Encodings

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CHAPTER ZERO

INTRODUCTION

Warm-up Example

Problem Statement

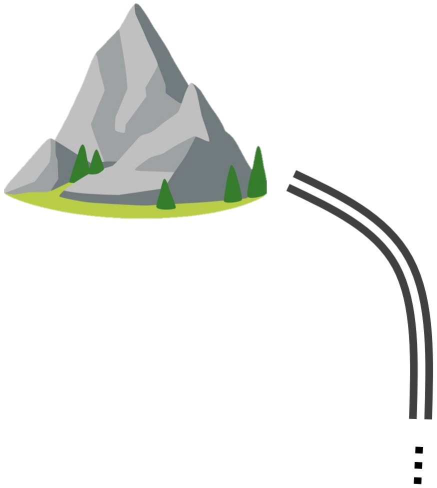
Background and Motivation

Fun-Road-Trip Checklist

✓ always at least one friend, and

✓ never negative money!

Friends: 4
Money: €100



__Fun-Road-Trip Checklist__

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__GOAL__

Friends: ≥ 5
Money: ≥ €10

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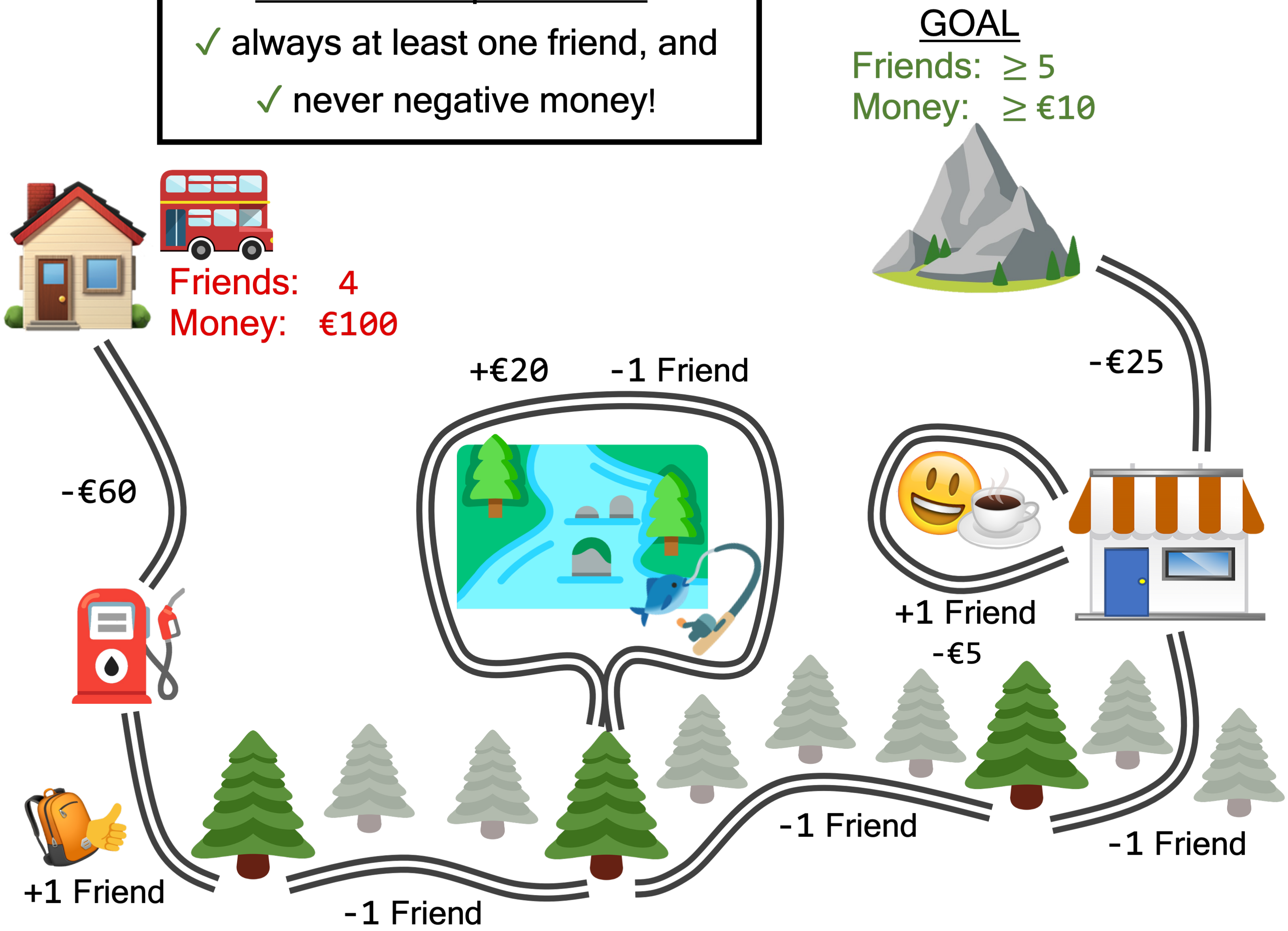

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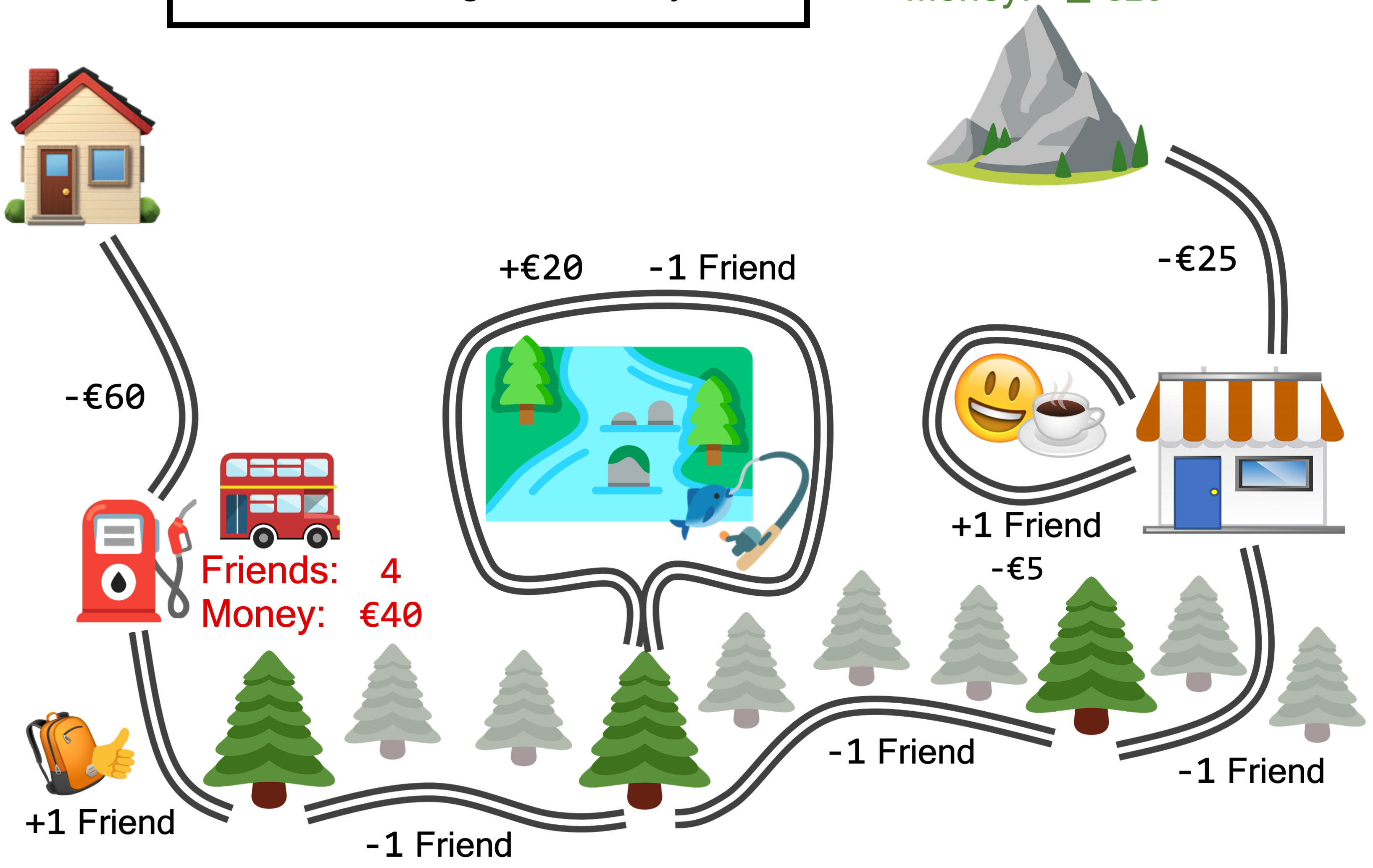

-€60

Friends: 4

Money: €40

+1 Friend

-1 Friend

+€20

-1 Friend

-1 Friend

+1 Friend

-€5

-1 Friend

-€25

Fun-Road-Trip Checklist

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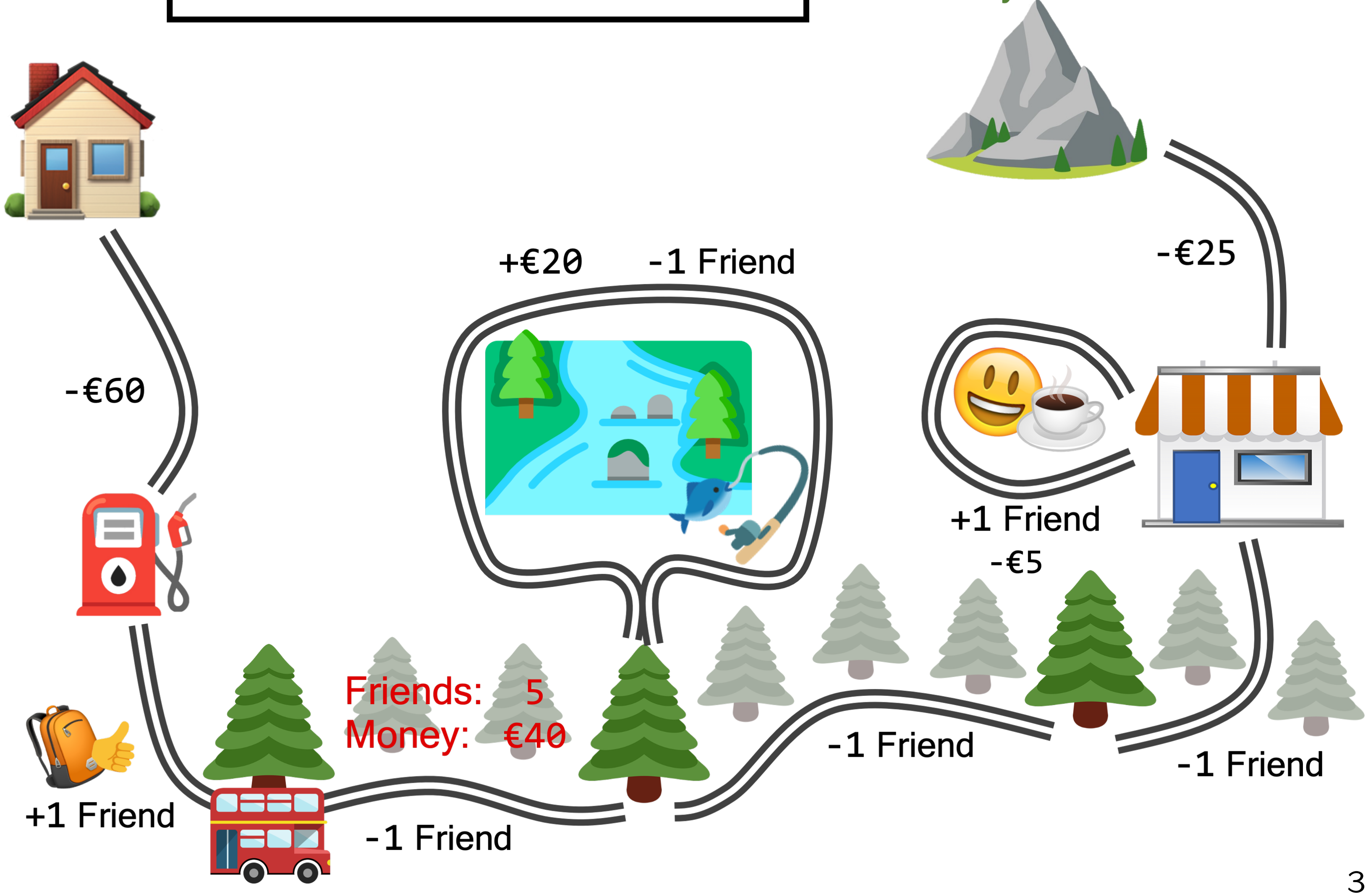

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Friends: 5

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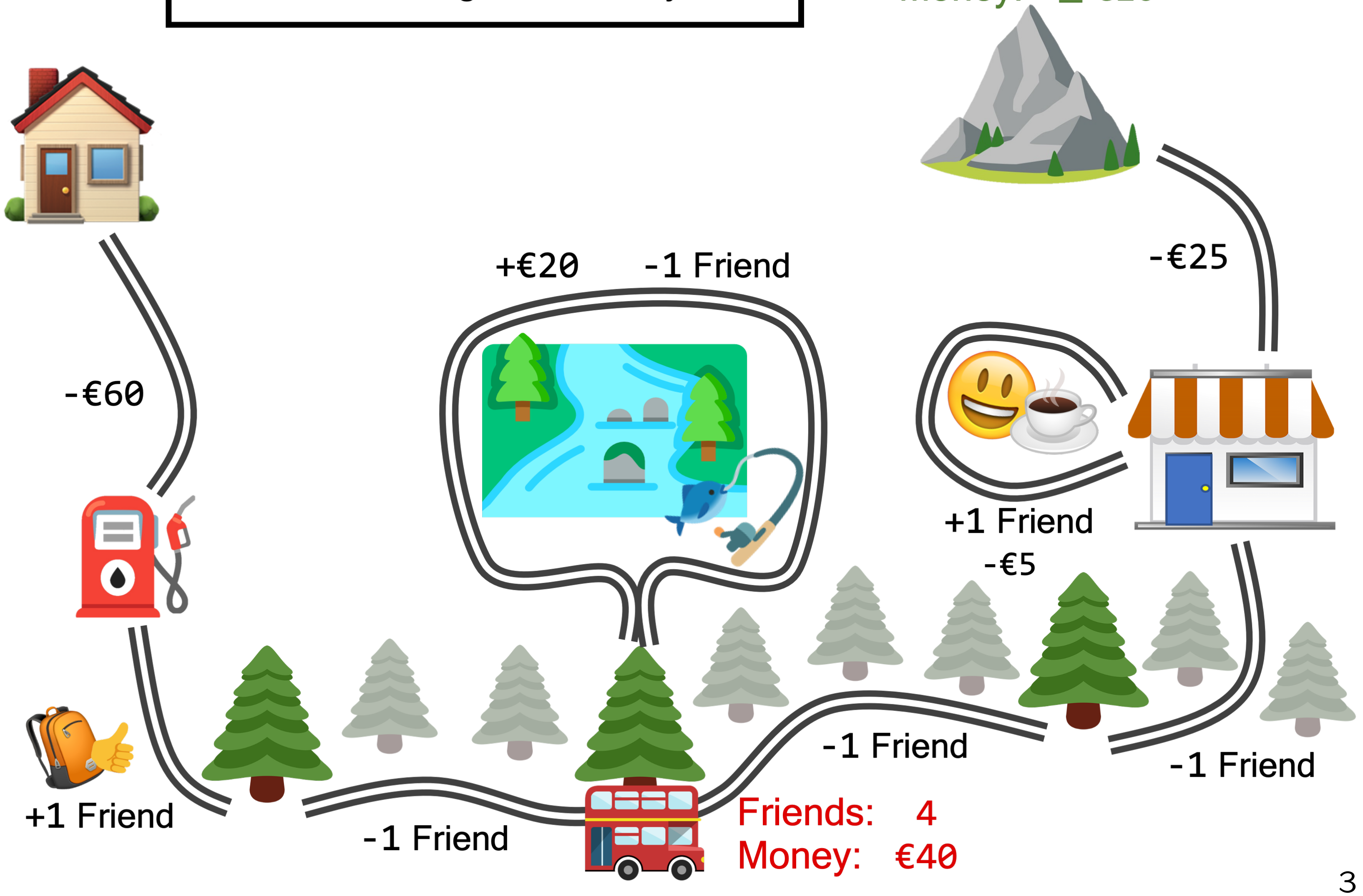

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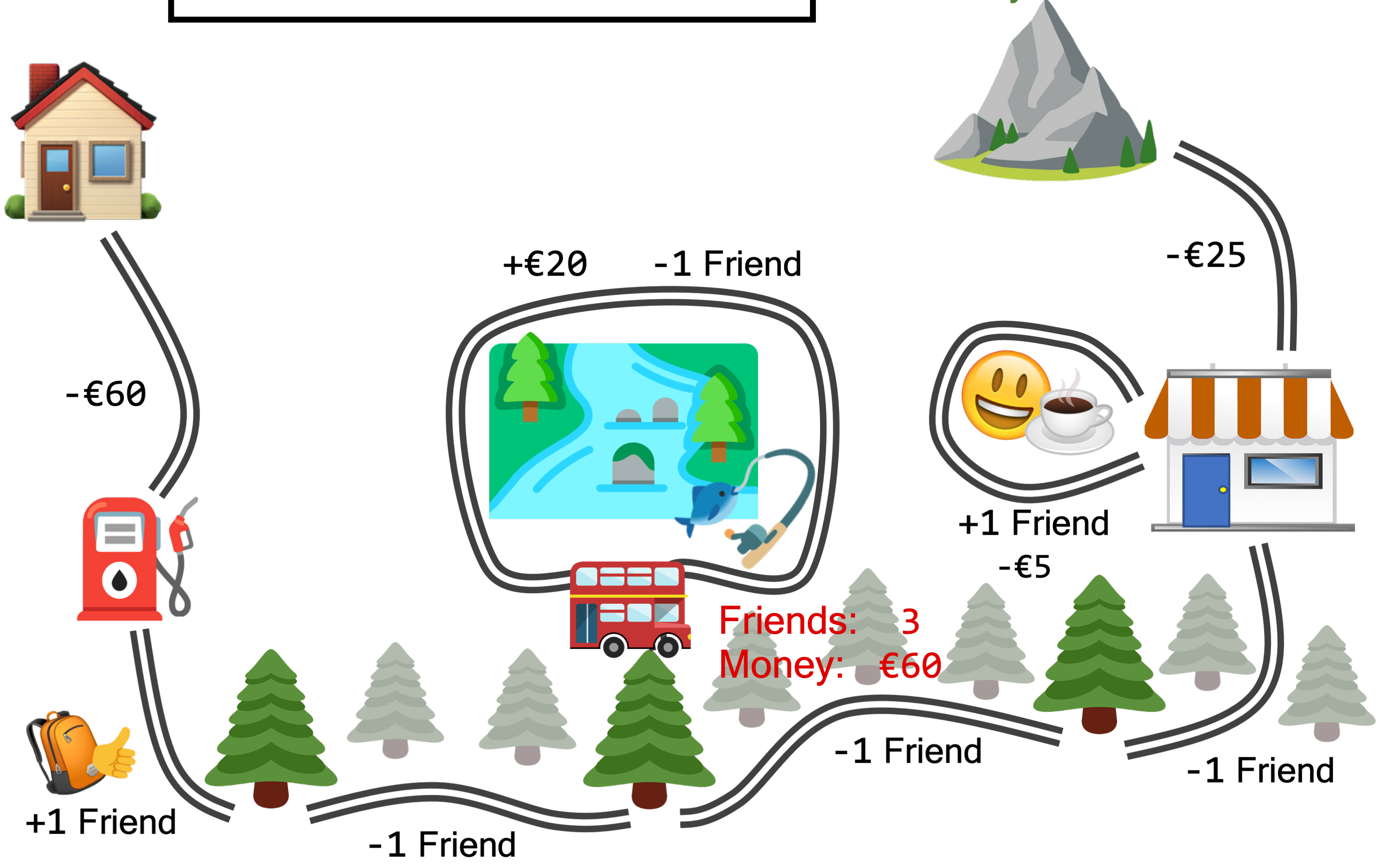

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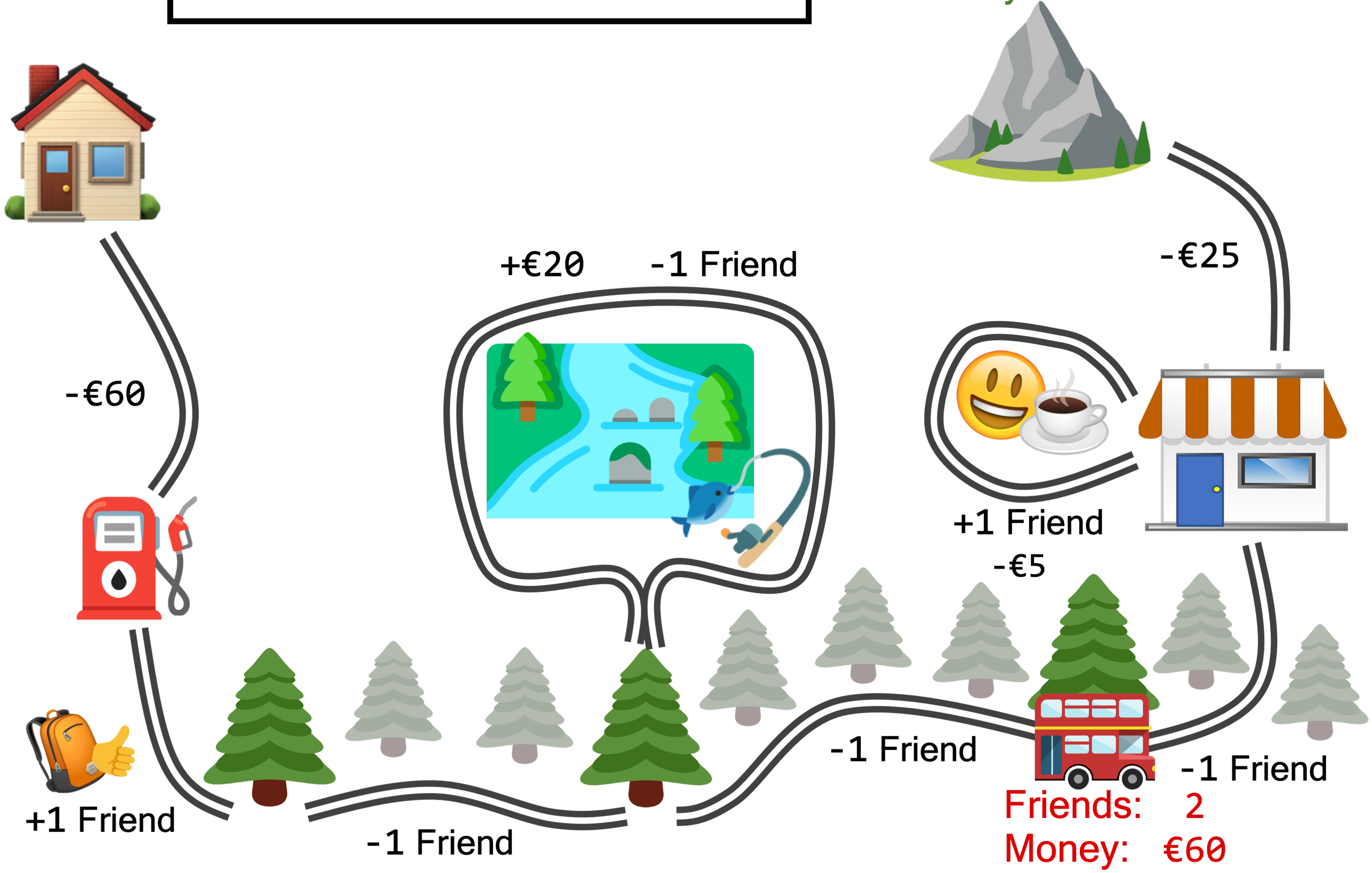

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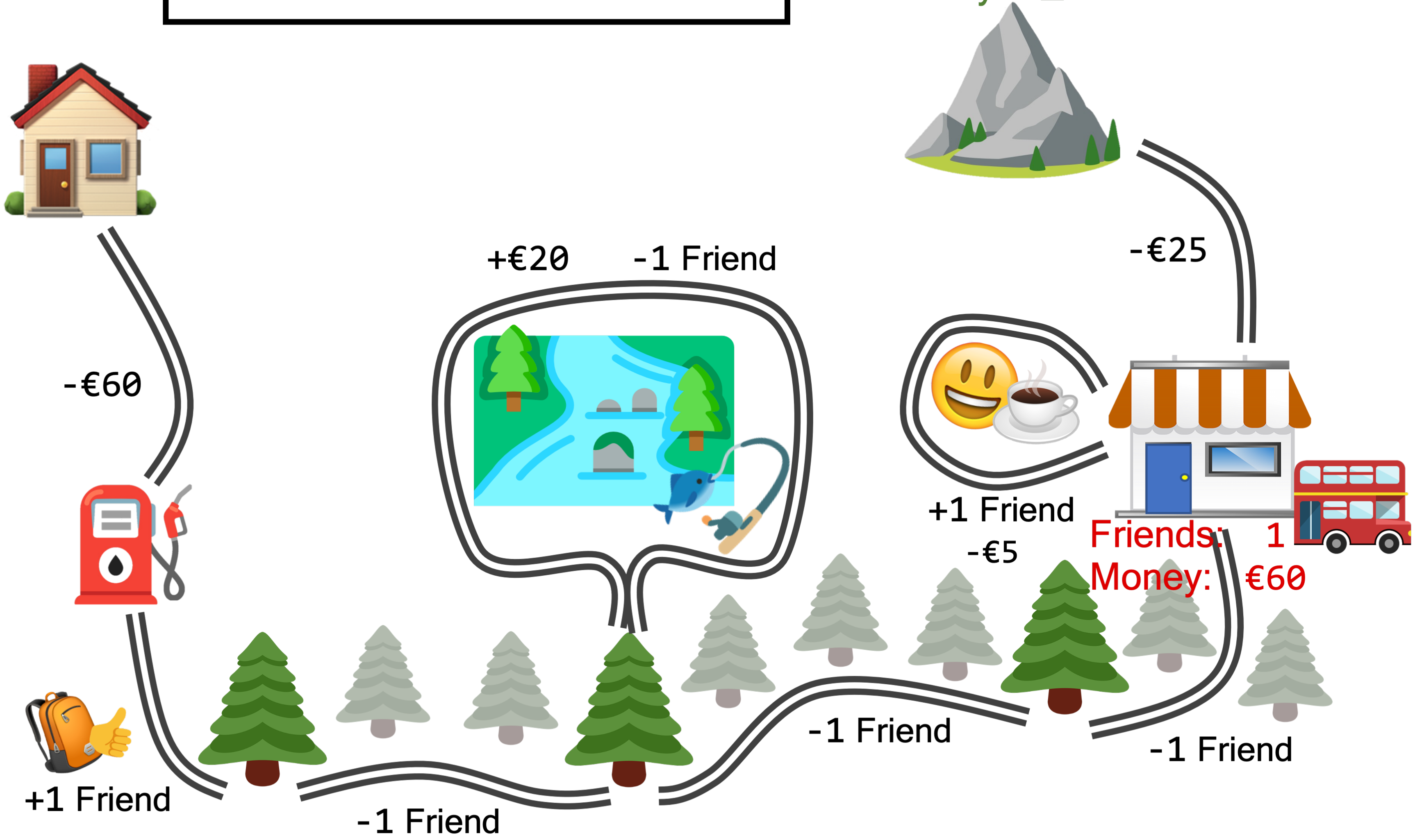

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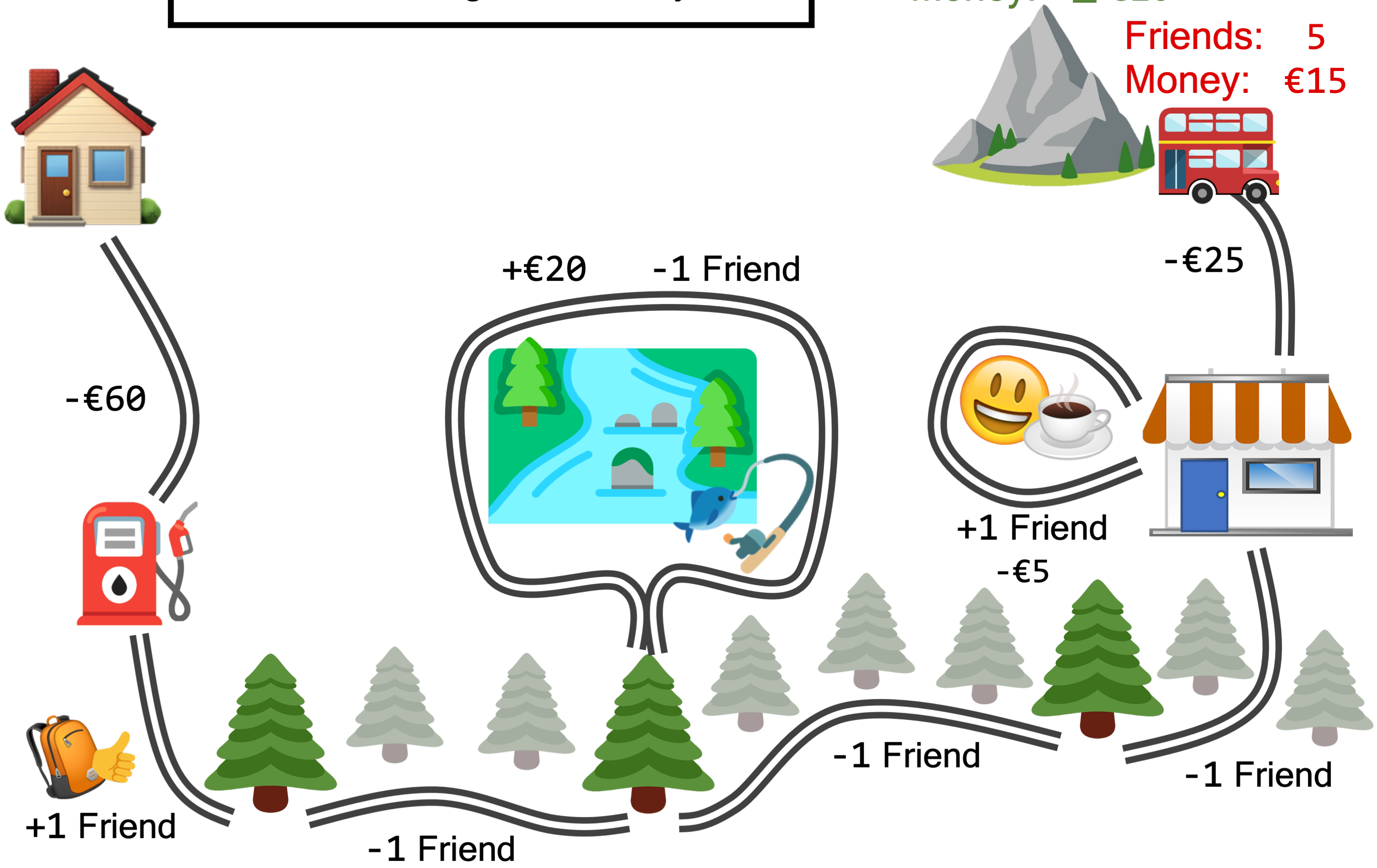


Friends: 5

Money: €15

-€60

+€20

-1 Friend

-€25

+1 Friend

-€5

+1 Friend

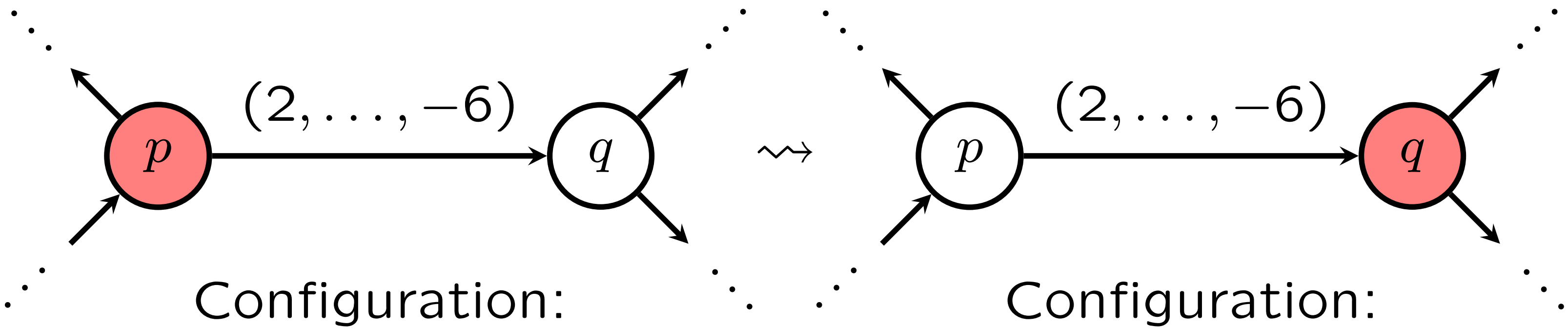
-1 Friend

-1 Friend

-1 Friend

PROBLEM STATEMENT

Vector Addition Systems with States (k-VASS)



Configuration: $p(5, \ldots, 8) \in Q \times \mathbb{N}^k$

Configuration: $q(7, \ldots, 2) \in Q \times \mathbb{N}^k$

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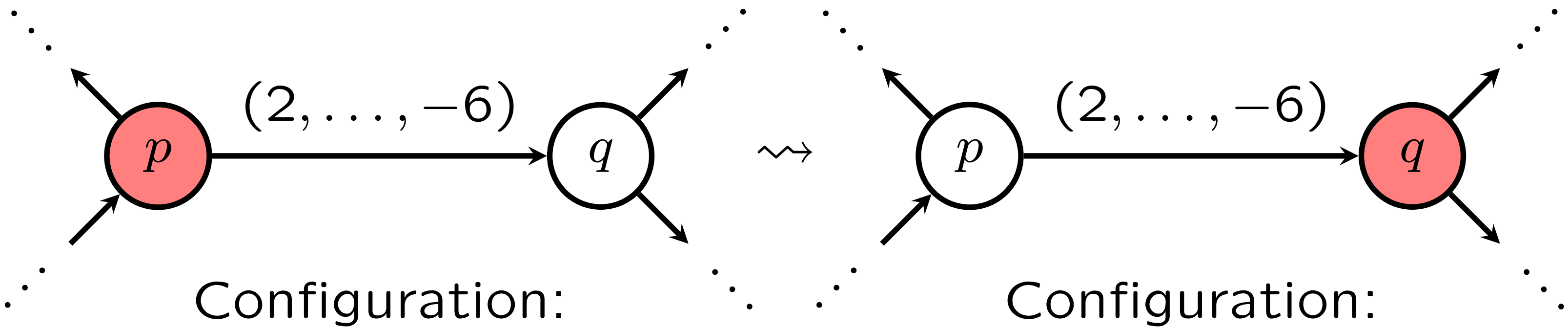

Configuration: $p(5, \ldots, 8) \in Q \times \mathbb{N}^k$

Configuration: $q(7, \ldots, 2) \in Q \times \mathbb{N}^k$

Reachability does there exist a *run* from $p(\vec{\mathbf{u}})$ to $q(\vec{\mathbf{v}})$?

Coverability does there exist a *run* from $p(\vec{\mathbf{u}})$ to $q(\vec{\mathbf{w}})$ for some $\vec{\mathbf{w}} \geq \vec{\mathbf{v}}$?

BACKGROUND

Coverability in non-fixed dimension VASS is EXPSPACE-complete, regardless of the encoding. [Lipton '76] [Rackoff '78]

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Coverability in binary 1-VASS is in NC^2.
[Almagor, Cohen, Pérez, Shirmohammadi, and Worrell '20]

TABLE OF RESULTS

		Number of unary counters		
		0	1	≥ 2
Number of binary counters	0	NL-complete [*folklore*]	NL-complete [Valiant and Paterson ’75]	NL-complete [Rackoff ’78]
	1			
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MOTIVATION

Petri nets are an equivalent model of computation.

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Related problems (with asymmetric treatment of counters):

Coverability in 1-VASS with a pushdown stack is PSPACE-hard and is decidable. [Leroux, Sutre, and Totzke '15]
[Englert, Hofman, Lasota, Lazic, Leroux, and Straszyński '20]

Reachability in 2-VASS where one counter can be zero-tested is PSPACE-complete. [Leroux and Sutre '20]

CHAPTER ONE

UPPER BOUNDS

Our Contribution

The Overall Approach

Technique: “Polynomially Many Short Cycles”

OUR CONTRIBUTION

Theorem: Coverability in 2-VASS with one binary counter and one unary counter is in NP. [*our result*]

OUR CONTRIBUTION

Theorem: Coverability in 2-VASS with two binary counters is PSPACE-complete. [Blondin, Finkel, Göller, Haase, and McKenzie '15]

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Theorem: Coverability in 2-VASS with two unary counters is NL-complete. [Rackoff '78]

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FINDING SMALL WITNESSES

Start with a path $\pi = \tau_0 \, \gamma_1^{e_1} \, \tau_1 \, \cdots \, \tau_{k-1} \, \gamma_k^{e_k} \, \tau_k$ such that all paths τ_i and cycles γ_i are short, and $p(\vec{\mathbf{u}}) \xrightarrow{\pi} q(\vec{\mathbf{v}})$.

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Obtain a path $\rho = \tau_0\,\sigma_1^{e_1}\,\tau_1 \,\cdots\, \tau_{k-1}\,\sigma_k^{e_k}\,\tau_k$ such that there are polynomially many distinct short cycles σ_i, and $p(\vec{\mathbf{u}}) \xrightarrow{\rho} q(\vec{\mathbf{w}})$ where $\vec{\mathbf{w}} \geq \vec{\mathbf{v}}$.

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Obtain a *polynomial size compressed linear form run* ς such that $p(\vec{\mathbf{u}}) \xrightarrow{\varsigma} q(\vec{\mathbf{x}})$ where $\vec{\mathbf{x}} \geq \vec{\mathbf{w}} \geq \vec{\mathbf{v}}$.

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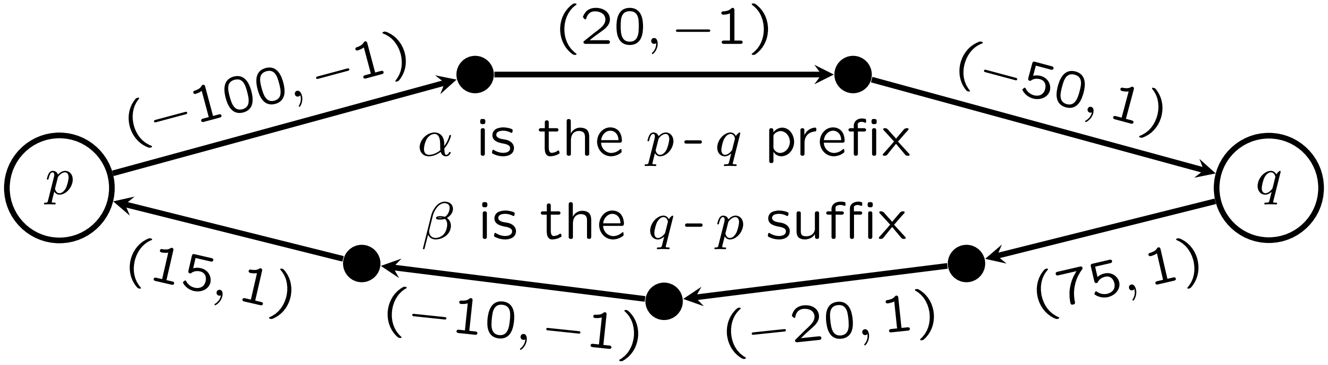
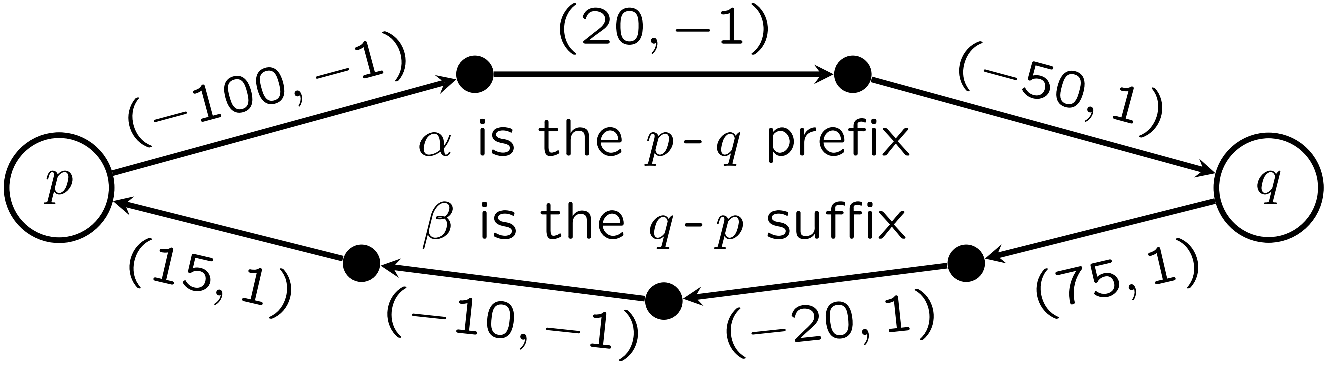
CHARACTERISATION OF A SHORT CYCLE



α is the p-q prefix

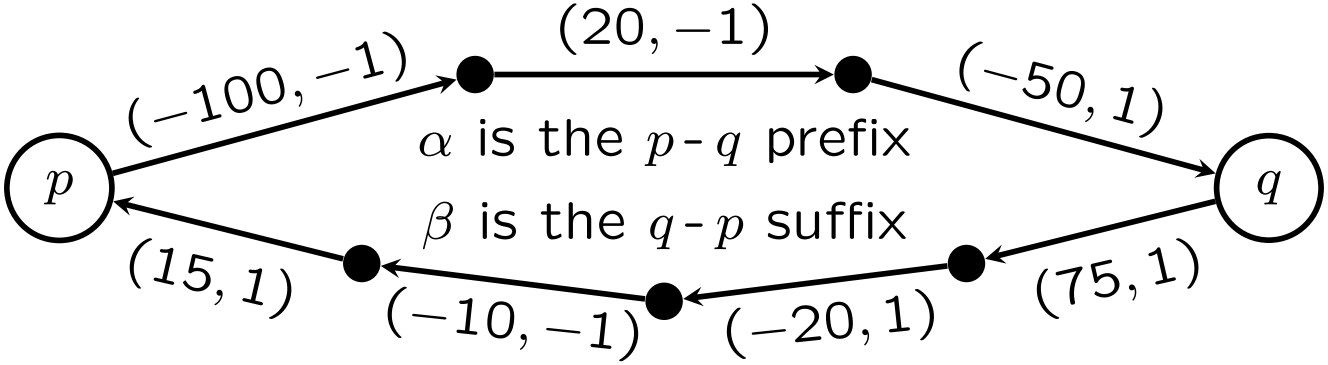
β is the q-p suffix

CHARACTERISATION OF A SHORT CYCLE



(a) Start-end state p,

CHARACTERISATION OF A SHORT CYCLE



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(a) Start-end state p,

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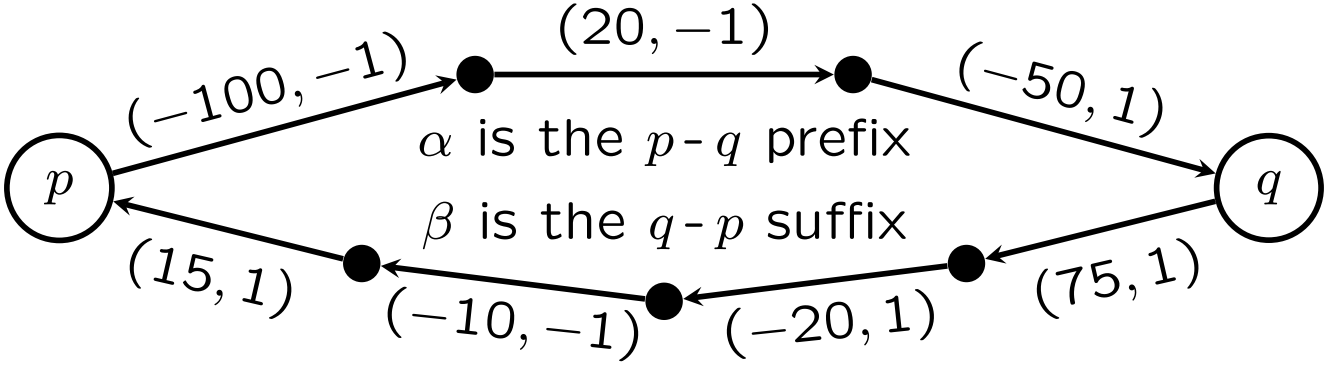

α is the p-q prefix

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(a) Start-end state p,

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(c) Length of α is 3,

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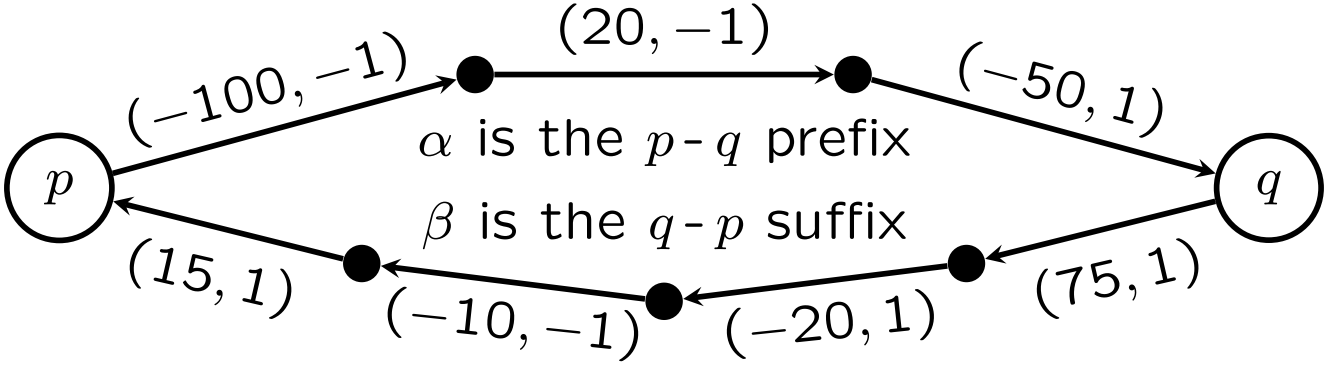

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(a) Start-end state p,

(b) State where minimum binary effect observed is q,

(c) Length of α is 3,

(d) Length of β is 4,

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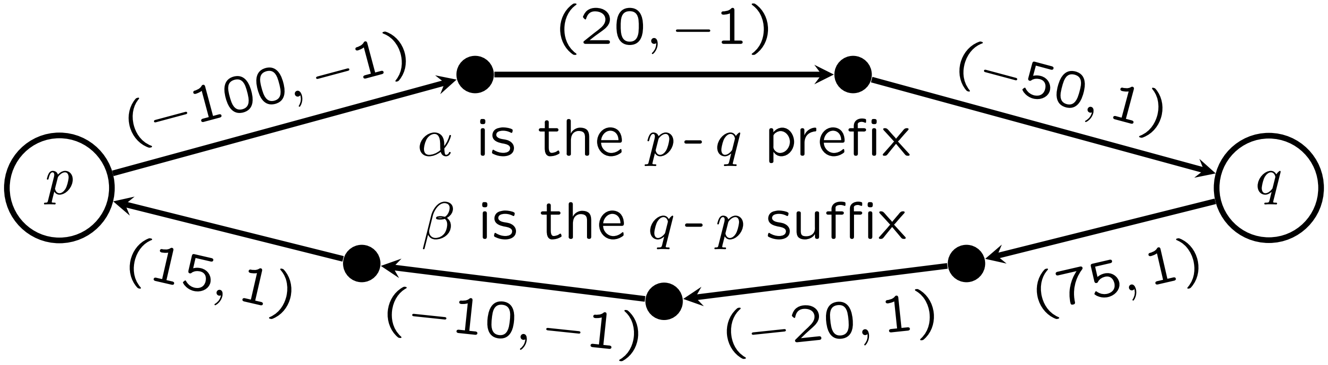


α is the p-q prefix

β is the q-p suffix

(a) Start-end state p,

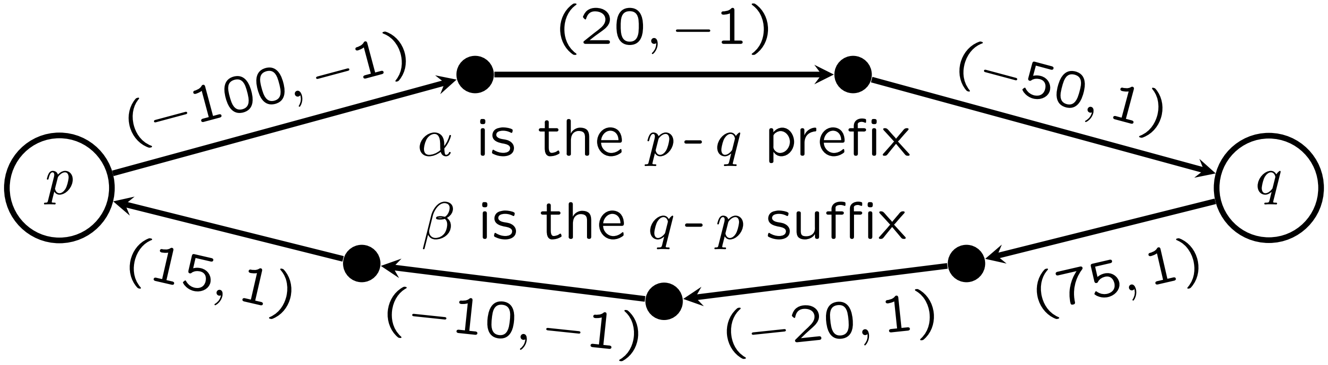
(b) State where minimum binary effect observed is q,

(c) Length of α is 3,

(d) Length of β is 4,

(e) Unary effect of α is -1,

CHARACTERISATION OF A SHORT CYCLE



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(f) Unary effect of β is 2,

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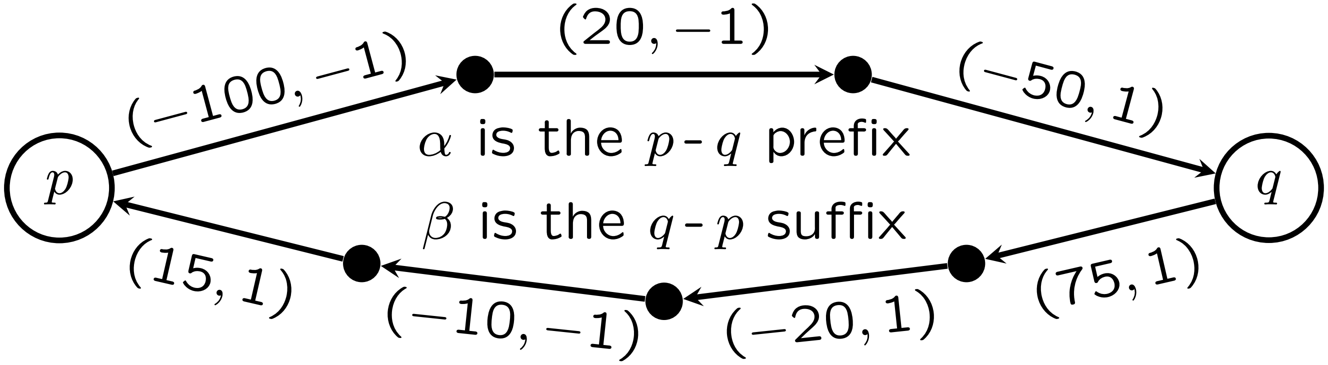

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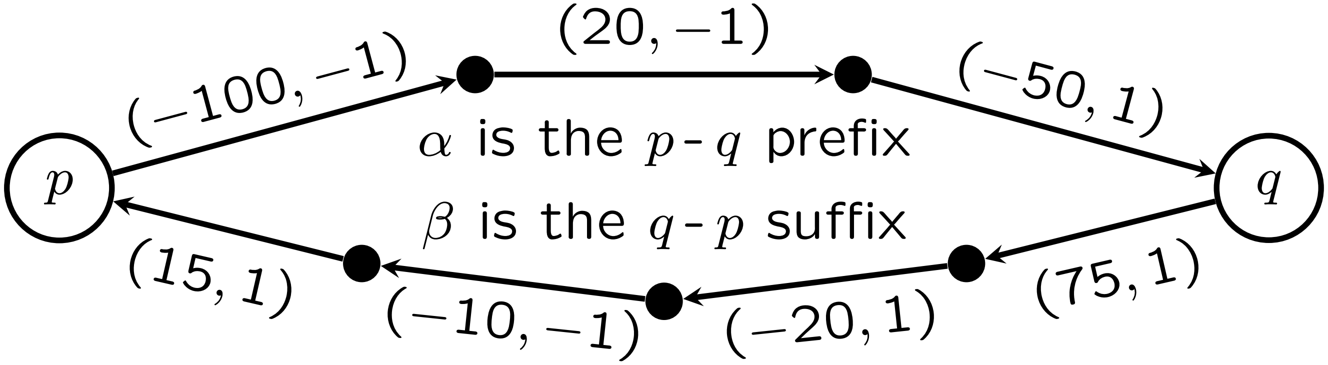
(d) Length of β is 4,

(e) Unary effect of α is -1,

(f) Unary effect of β is 2,

(g) Minimum unary effect over α is -2,

CHARACTERISATION OF A SHORT CYCLE



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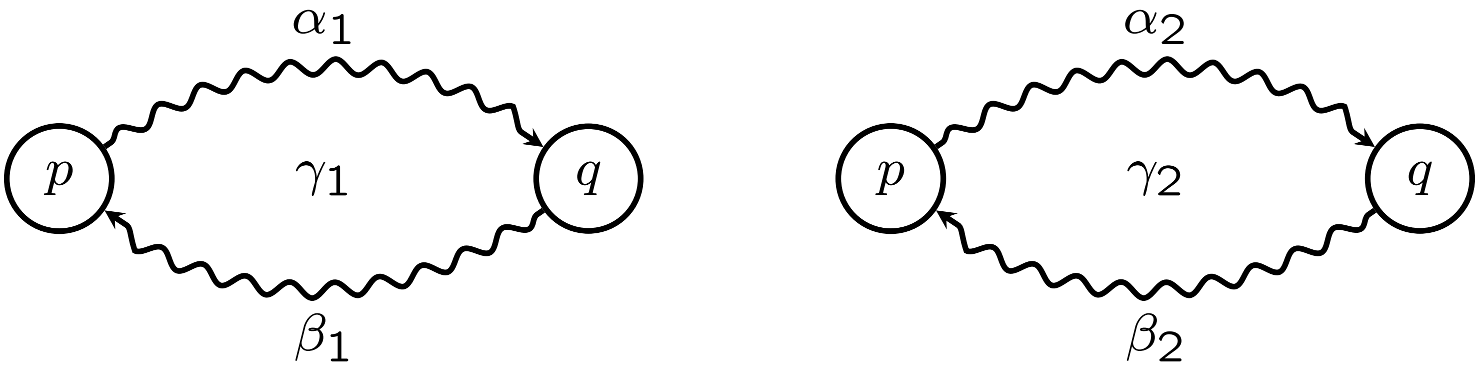
(g) Minimum unary effect over α is -2, and

(h) Minimum unary effect over β is 0.

SHORT CYCLE REPLACEMENT



Suppose γ_1 and γ_2 have the same characterisation and consider $\sigma = \alpha_i\,\beta_j$ where $i, j \in \{1, 2\}$ selected for greatest binary effect.

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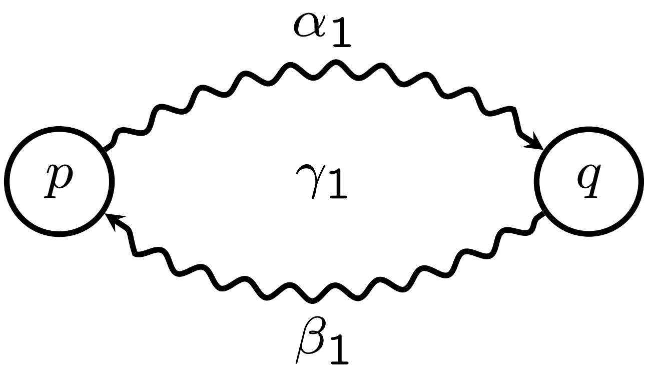

Suppose γ_1 and γ_2 have the same characterisation and consider $\sigma = \alpha_i \, \beta_j$ where $i, j \in \{1, 2\}$ selected for greatest binary effect.

Idea: replace all iterations of γ_1 and γ_2 in a run with iterations of σ, the run remains executable and has at least the effect.

$$\pi = \tau_1 \, \gamma_1 \, \tau_2 \, \gamma_2 \, \tau_3 \;\rightsquigarrow\; \rho = \tau_1 \, \sigma \, \tau_2 \, \sigma \, \tau_3$$

If $p(\vec{\mathbf{u}}) \xrightarrow{\pi} q(\vec{\mathbf{v}})$, then $p(\vec{\mathbf{u}}) \xrightarrow{\rho} q(\vec{\mathbf{w}})$ and $\vec{\mathbf{w}} \geq \vec{\mathbf{v}}$.

NUMBER OF CHARACTERISATIONS

(a) Start-end state,

(b) State where minimum binary effect observed,

(c) Length of the prefix α,

(d) Length of the suffix β,

(e) Unary effect of the prefix α,

(f) Unary effect of the suffix β,

(g) Minimum unary effect over the prefix α, and

(h) Minimum unary effect over the suffix β.

How many different characterisations?

NUMBER OF CHARACTERISATIONS

(a) Start-end state, $|\mathbf{Q}|$

(b) State where minimum binary effect observed,

(c) Length of the prefix α,

(d) Length of the suffix β,

(e) Unary effect of the prefix α,

(f) Unary effect of the suffix β,

(g) Minimum unary effect over the prefix α, and

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How many different characterisations?

NUMBER OF CHARACTERISATIONS

(a) Start-end state, $|\mathbf{Q}|$

(b) State where minimum binary effect observed, $|\mathbf{Q}|$

(c) Length of the prefix α,

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(e) Unary effect of the prefix α,

(f) Unary effect of the suffix β,

(g) Minimum unary effect over the prefix α, and

(h) Minimum unary effect over the suffix β.

How many different characterisations?

NUMBER OF CHARACTERISATIONS

(a) Start-end state,	$\mathbf{|Q|}$
(b) State where minimum binary effect observed,	$\mathbf{|Q|}$
(c) Length of the prefix α,	$\mathbf{|Q| + 1}$
(d) Length of the suffix β,	
(e) Unary effect of the prefix α,	
(f) Unary effect of the suffix β,	
(g) Minimum unary effect over the prefix α, and	
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(b) State where minimum binary effect observed, $|\mathbf{Q}|$

(c) Length of the prefix α, $|\mathbf{Q}| + 1$

(d) Length of the suffix β, $|\mathbf{Q}| + 1$

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(c) Length of the prefix α,	$\mathbf{\lvert Q\rvert + 1}$
(d) Length of the suffix β,	$\mathbf{\lvert Q\rvert + 1}$
(e) Unary effect of the prefix α,	$\mathbf{2\lvert Q\rvert + 1}$
(f) Unary effect of the suffix β,	
(g) Minimum unary effect over the prefix α, and	
(h) Minimum unary effect over the suffix β.	

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(a) Start-end state,	$\mathbf{\|Q\|}$
(b) State where minimum binary effect observed,	$\mathbf{\|Q\|}$
(c) Length of the prefix α,	$\mathbf{\|Q\| + 1}$
(d) Length of the suffix β,	$\mathbf{\|Q\| + 1}$
(e) Unary effect of the prefix α,	$\mathbf{2\|Q\| + 1}$
(f) Unary effect of the suffix β,	$\mathbf{2\|Q\| + 1}$
(g) Minimum unary effect over the prefix α, and	
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(a) Start-end state,	$\mathbf{\lvert Q\rvert}$
(b) State where minimum binary effect observed,	$\mathbf{\lvert Q\rvert}$
(c) Length of the prefix α,	$\mathbf{\lvert Q\rvert + 1}$
(d) Length of the suffix β,	$\mathbf{\lvert Q\rvert + 1}$
(e) Unary effect of the prefix α,	$\mathbf{2\lvert Q\rvert + 1}$
(f) Unary effect of the suffix β,	$\mathbf{2\lvert Q\rvert + 1}$
(g) Minimum unary effect over the prefix α, and	$\mathbf{\lvert Q\rvert + 1}$
(h) Minimum unary effect over the suffix β.	

How many different characterisations?

NUMBER OF CHARACTERISATIONS

(a) Start-end state,	$\mathbf{\lvert Q\rvert}$
(b) State where minimum binary effect observed,	$\mathbf{\lvert Q\rvert}$
(c) Length of the prefix α,	$\mathbf{\lvert Q\rvert + 1}$
(d) Length of the suffix β,	$\mathbf{\lvert Q\rvert + 1}$
(e) Unary effect of the prefix α,	$\mathbf{2\lvert Q\rvert + 1}$
(f) Unary effect of the suffix β,	$\mathbf{2\lvert Q\rvert + 1}$
(g) Minimum unary effect over the prefix α, and	$\mathbf{\lvert Q\rvert + 1}$
(h) Minimum unary effect over the suffix β.	$\mathbf{\lvert Q\rvert + 1}$

How many different characterisations?

NUMBER OF CHARACTERISATIONS

(a) Start-end state,	$\mathbf{\lvert Q\rvert}$
(b) State where minimum binary effect observed,	$\mathbf{\lvert Q\rvert}$
(c) Length of the prefix α,	$\mathbf{\lvert Q\rvert + 1}$
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How many different characterisations?

$$\leq \mathbf{\lvert Q\rvert^2(\lvert Q\rvert + 1)^4(2\lvert Q\rvert + 1)^2}$$

TECHNIQUE

“Polynomially Many Short Cycles”

The cycle replacement idea gives runs witnessing coverability that only contain one short cycle (that may be iterated many times) for each characterisation.

There are a polynomial number of different characterisations.

Conclusion: no more than a polynomial number of distinct short cycles need exist in any executable run witnessing coverability.

CHAPTER TWO

LOWER BOUNDS

Combinations of Encodings

Open Problems and Our Contributions

Technique: “Dual Counters”

COMPLEXITY OF COVERABILITY

Various Encodings	Binary encoded counter updates	Unary encoded counter updates
Binary encoded initial and target vectors	$k \geq 2$: PSPACE-complete $k = 1$: only gap between NL and in NC^2	
Unary encoded initial and target vectors		$k \geq 1$: NL-complete No complexity gaps.

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Various Encodings	Binary encoded counter updates	Unary encoded counter updates
Binary encoded initial and target vectors	$k \geq 2$: PSPACE-complete $k = 1$: only gap between NL and in NC^2	$k \geq 4$: **NP-hard** $k \geq 8$: **PSPACE-hard** Many complexity gaps!
Unary encoded initial and target vectors	Reduces from above: *New initial and final states, add initial vector at start, and subtract target vector at end. Ask coverability to and from* $\vec{0}$.	$k \geq 1$: NL-complete No complexity gaps.

OPEN PROBLEMS

Problem: Coverability in k-VASS with k unary counters and binary encoded initial and target vectors.

OPEN PROBLEMS

~~**Problem:** Coverability in k-VASS with k unary counters and binary encoded initial and target vectors.~~

Problem: Binary coverability in unary k-VASS.

OPEN PROBLEMS

~~**Problem:** Coverability in k-VASS with k unary counters and binary encoded initial and target vectors.~~

Problem: Binary coverability in unary k-VASS.

Complexity Gaps:

$k = 1$: NL-hard and in NC^2.

$k = 2$: NL-hard and in NP.

$k = 3$: NL-hard and in PSPACE.

$4 \leq k \leq 7$: NP-hard and in PSPACE.

$k \geq 8$: PSPACE-complete.

HARDNESS OF REACHABILITY

Theorem*: Unary reachability in unary 3-VASS is NP-hard.

[Czerwiński and Orlikowski '22+]

Proof approach: reduce from SAT.

HARDNESS OF REACHABILITY

Theorem*: Unary reachability in unary 3-VASS is NP-hard.
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Proof approach: reduce from SAT.

Theorem: Unary reachability in unary 5-VASS is PSPACE-hard.
[Czerwiński and Orlikowski '22]

Proof approach: reduce from reachability in exponentially bounded two-counter automata.

OUR CONTRIBUTIONS

Theorem*: Binary coverability in unary 4-VASS is NP-hard.

[our result]

Proof approach: reduce from unary reachability in unary 3-VASS using "dual counters" technique.

OUR CONTRIBUTIONS

Theorem*: Binary coverability in unary 4-VASS is NP-hard.

[*our result*]

Proof approach: reduce from unary reachability in unary 3-VASS using "dual counters" technique.

Theorem: Binary coverability in unary 8-VASS is PSPACE-hard.

[*our result*]

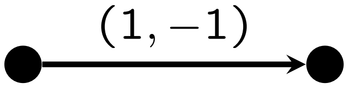
Proof approach: reduce from unary reachability in unary 5-VASS using "dual counters" technique.

TECHNIQUE

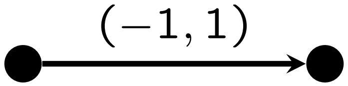
“Dual Counters”

Consider a unary counter c, define its dual counter d such that

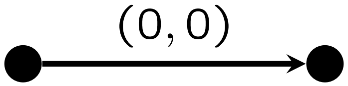
- Whenever c increments, d decrements:



- Whenever c decrements, d increments:



- Whenever c holds its value, so does d:



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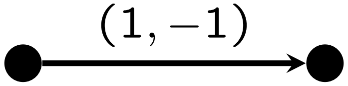

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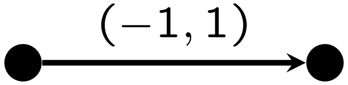

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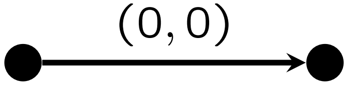

If c is initialised with u, then d is initialised with $M-u$, where M is at least the maximum possible value that c can observe.

Coverability targets $c \geq v$ and $d \geq M-v$ implies $c = v$ must hold.

REDUCTION CHALLENGES

Unary reachability in unary 3-VASS is NP-hard, so by taking all dual counters, binary coverability in unary 6-VASS is NP-hard.

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Similarly, unary reachability in unary 5-VASS is PSPACE-hard, so by taking all dual counters, binary coverability in unary 10-VASS is PSPACE-hard.

REDUCTION CHALLENGES

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Similarly, unary reachability in unary 5-VASS is PSPACE-hard, so by taking all dual counters, binary coverability in unary 10-VASS is PSPACE-hard.

Which dual counters are really necessary?

CONCLUSION

Theorem: Coverability in 2-VASS with one binary counter and one unary counter is in NP. [*our result*]

Open Problem: Is reachability also in NP?

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Open Problem: is there a $k < 4$ such that binary coverability in unary k-VASS is NP-hard?

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CONCLUSION

Theorem: Coverability in 2-VASS with one binary counter and one unary counter is in NP. *[our result]*

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THANK YOU!

Presented by Henry Sinclair-Banks, University of Warwick

For OFCOURSE, MPI−SWS, Kaiserslautern

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