Using Counter Machines to Find Cliques and Cycles in Graphs

Henry Sinclair-Banks

About a part of joint work with Marvin Künnemann, Filip Mazowiecki, Lia Schütze, and Karol Węgrzycki.

Warwick Postgraduate Colloquium in Computer Science
Theory and Foundations
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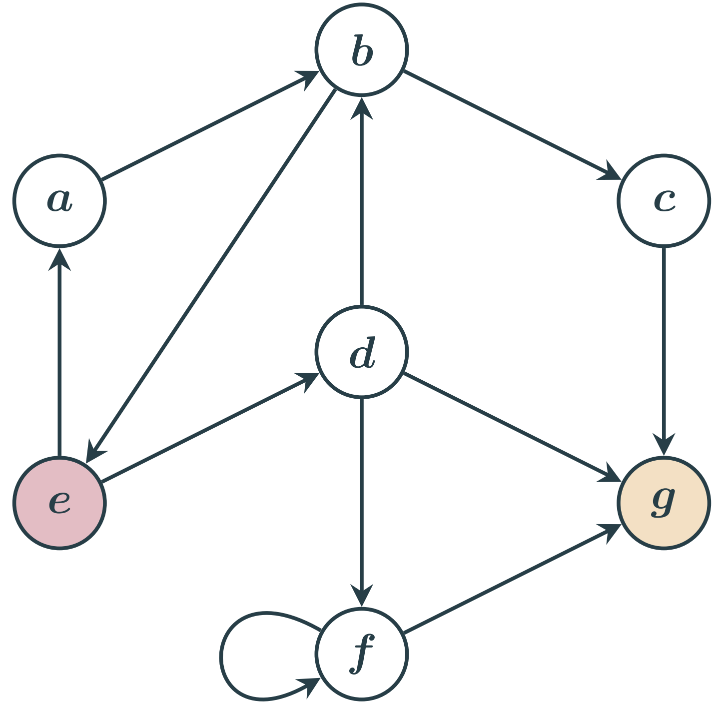
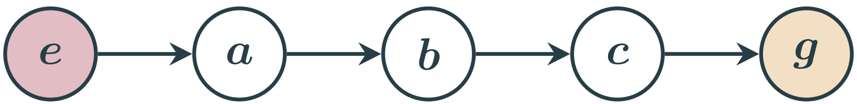
Paths in Graphs



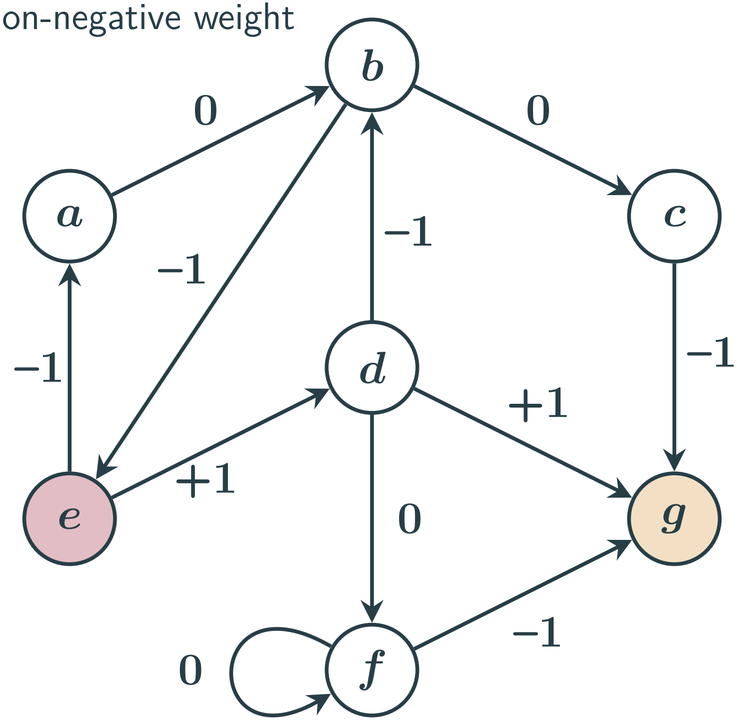
From e can you reach g?

Yes, just run BFS in linear time!



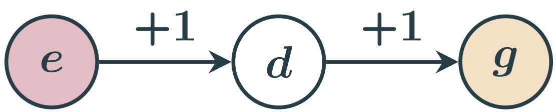
Never Negative Paths in $\{-1, 0, +1\}$-Weighted Graphs

all path prefixes have non-negative weight

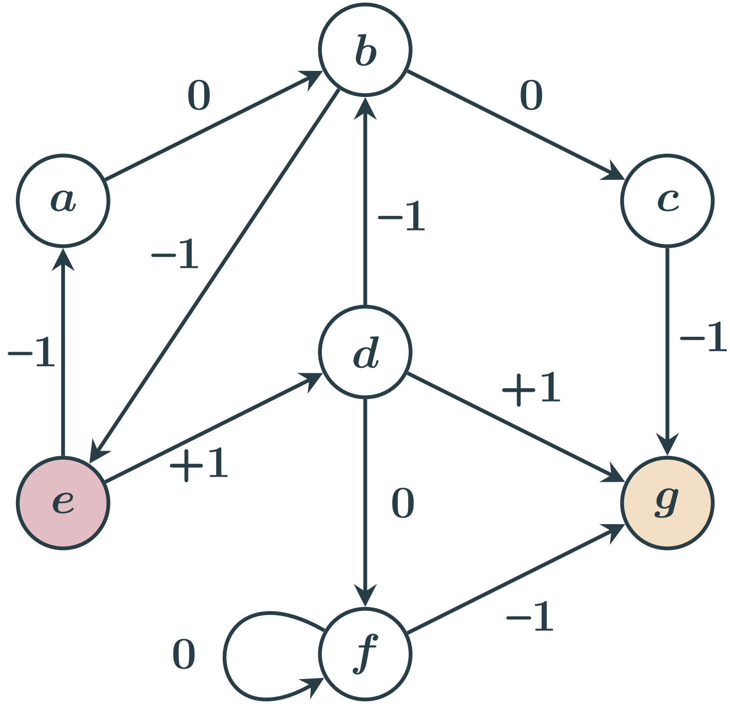


From e can you reach g with a never negative path?

Yes, modify your favourite *shortest* path algorithm!

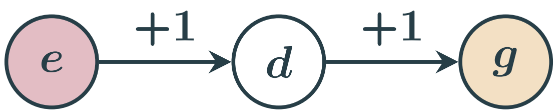


Coverability in 1-VASS

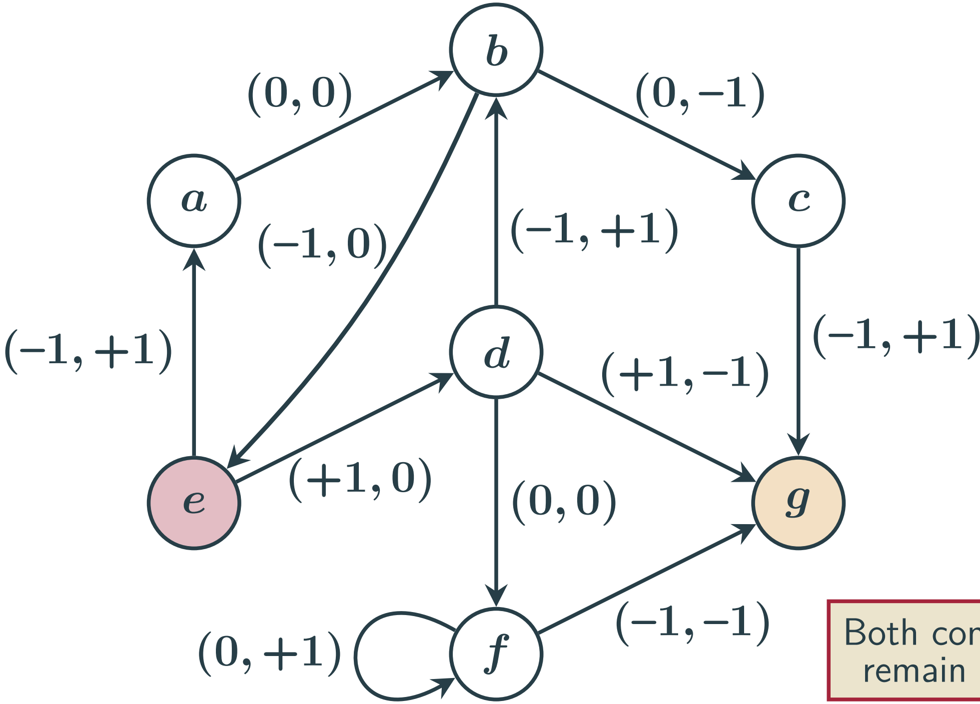


From e can you *cover* g ?

Yes, modify your favourite *shortest* path algorithm!



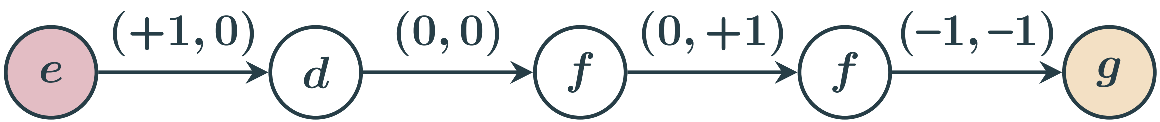
Coverability in 2-VASS



Both components must remain non-negative.

From e can you *cover* g ?

Yes … but it is not so straightforward!



Background

Theorem: Coverability in 1-VASS can be decided non-deterministic log-space.
[Valiant and Paterson '75]

Theorem: Coverability in d-VASS can be decided non-deterministic log-space, for every fixed $d \geq 1$. [Rackoff '78]

Theorem: Finding a path between two nodes in a (directed) graph is hard for (non-)deterministic log-space. [folklore]

Corollary: Coverability in d-VASS is complete for non-deterministic log-space, for every fixed $d \geq 1$.

What about the time needed to decide coverability?

This Presentation

Claim: Coverability in 2-VASS requires quadratic time*.

Proof: Reduction from finding a k-cycle in a graph.

Observation: Coverability in 2-VASS is harder than finding a path in a graph*.

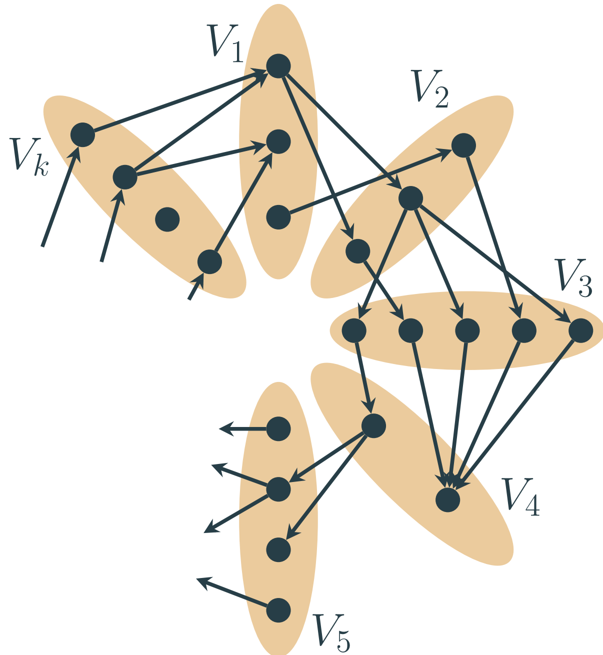
*subject to the k-cycle hypothesis.

k-Cycle Hypothesis

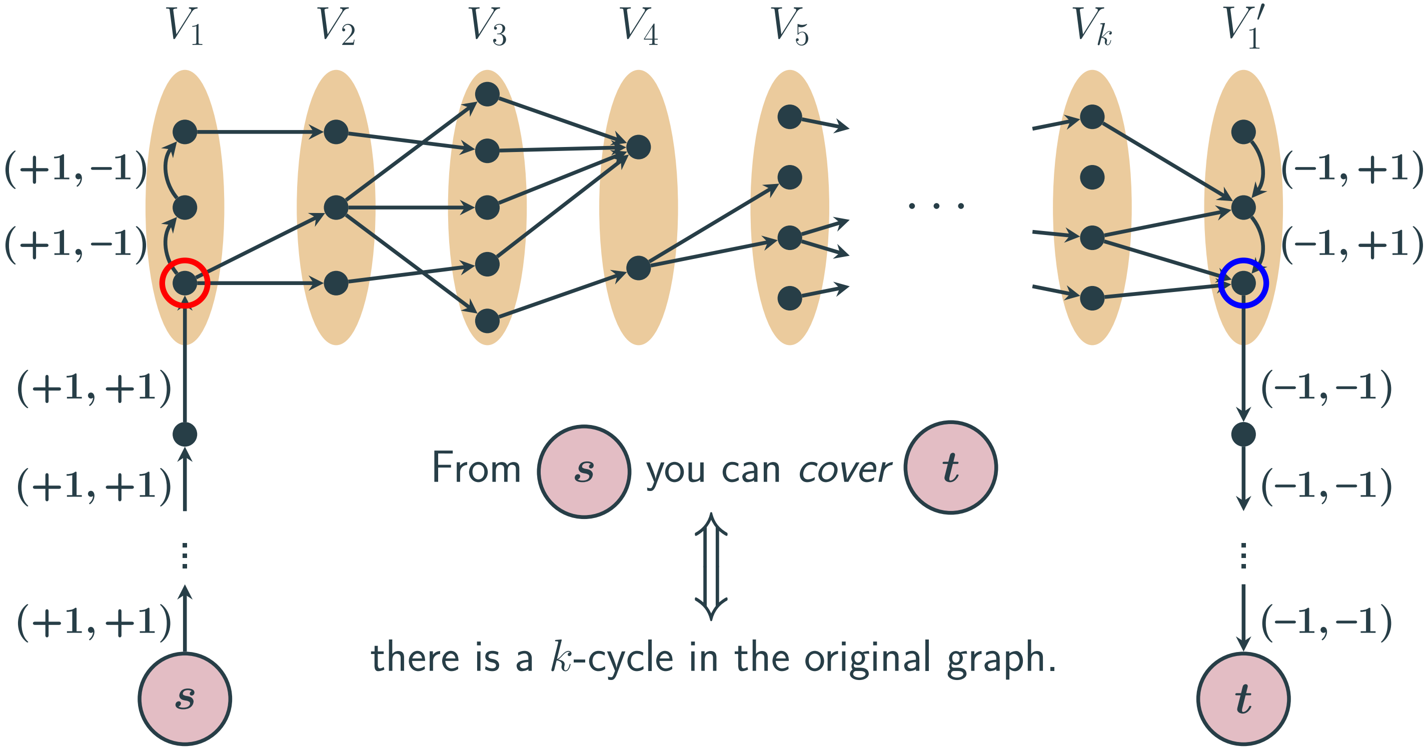
Hypothesis: Finding a k-cycle in a graph of m edges requires $\Omega(m^2)$-time.

It suffices to only consider **k-circle layered graphs:**

[Lincoln, Williams, and Williams '18]



Reduction Sketch



Suppose you leave V_1 via the i-th node and arrive at V_1' via the j-th node.

First component $\implies i \geq j$ and second component $\implies j \geq i$.

Coverability ensures that the start and ends nodes of the cycle match.

Conclusion

Hypothesis: Finding a k-cycle in a graph of m edges requires $\Omega(m^2)$-time.

$$+$$

Lemma: Linear-time reduction from finding a k-cycle in a graph to coverability in 2-VASS.

$$\Downarrow$$

Corollary: Assuming the k-cycle hypothesis, coverability in a 2-VASS of size n requires $\Omega(n^2)$-time.

Thank You!

Presented by Henry Sinclair-Banks
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