Approximate comparison of distance automata

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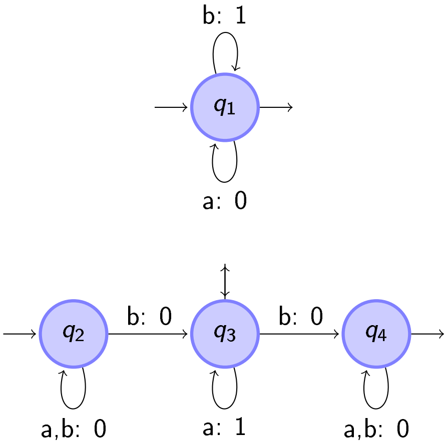
Distance automata

Distance automaton: Non deterministic finite automaton for which each transition is also labelled by a non-negative integer called the weight of the transition.

$$(\mathbb{A}, Q, I, T, E) \text{ with } E \subseteq (Q \times \mathbb{A} \times \mathbb{N} \times Q))$$



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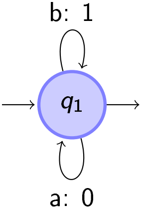
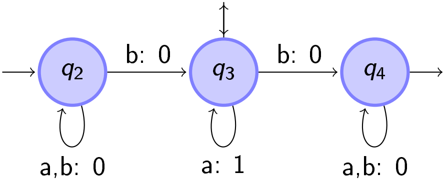


Weight of a run:
sum of the weights of the transitions



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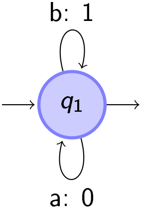
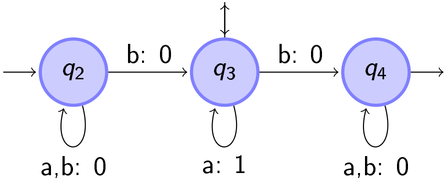


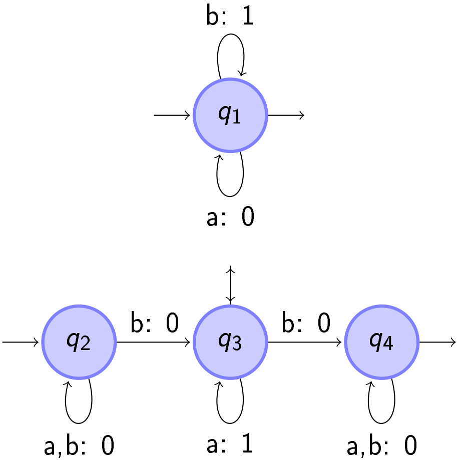


Weight of a run:
sum of the weights of the transitions

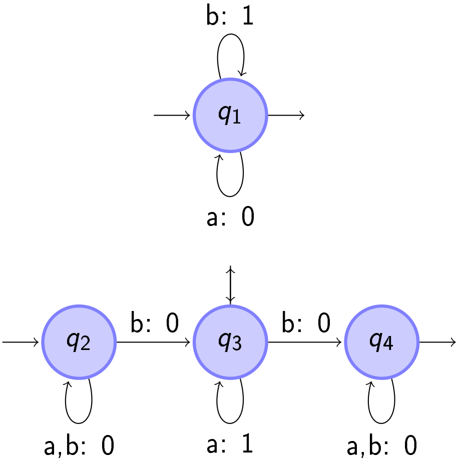
Computed function:

$$\begin{array}{lcl} \mathbb{A}^* & \to & \mathbb{N} \cup \{+\infty\} \\ w & \mapsto & \text{minimum of the weights of the runs labelled by } w \text{ going from an initial state to a final state } (+\infty \text{ if no such run}) \end{array}$$

Distance automata



Distance automata



$$a^{n_0} b a^{n_1} b \cdots b a^{n_k} \mapsto \min(n_0, n_1, \ldots, n_k, k)$$

Decision problems on comparison

f, g computed by distance automata : $\mathbb{A}^* \to \mathbb{N} \cup \{+\infty\}$

$f \leqslant g$ if for all words w, $f(w) \leqslant g(w)$

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Undecidable [Krob, 92]

Given f, g computed by distance automata, is $f \leqslant g$?

Decidable [Colcombet, 09]

Is there a polynomial P s.t $f \leqslant P \circ g$? (context of cost functions)

Generalisation of results by Hashiguchi, Leung and Simon

Theorem of affine domination

Proposition

Given f, g computed by distance automata, the two assertions are equivalent:

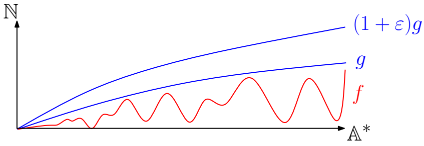
1. There is a polynomial P s.t $f \leqslant P \circ g$.
2. There is an integer a s.t $f \leqslant ag + a$.

Theorem

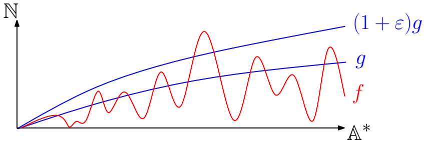
Given f, g computed by distance automata, one can decide if there is an integer a s.t $f \leqslant ag + a$.

Theorem of approximate comparison

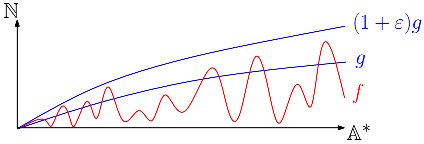
Input : f, g computed by distance automata and $\varepsilon > 0$



$\Longrightarrow$ YES



$\Longrightarrow$ NO



$\Longrightarrow$ YES or NO

Theorem: Existence of an algorithm having this behaviour.

Conclusion and further questions

Undecidable [Krob, 92]
$f \leqslant g$?

$\downarrow$

Algorithm of approximate comparison
EXPSPACE
(problem PSPACE-hard)

Decidable [Colcombet, 09]
Is there a polynomial P s.t
$f \leqslant P \circ g$?

$\Updownarrow$

Decidable
Is there an integer a s.t
$f \leqslant ag + a$?

Next steps

Capture other kinds of asymptotic behaviours
Case of max-$+$ automata